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FM Radio: The Calculation of Distortion Arising from Multipath Propagation

R H M Poole

BRITISH BROADCASTING CORPORATION

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Ranulph Poole

Abstract

It has long been appreciated that FM reception is vulnerable to multipath propagation. The effect of an echo — for instance a reflection off a building — is to create high-order harmonic and intermodulation products on the audio output of the receiver. Such distortion is particularly unpleasant, and it follows that only low levels of multipath are tolerable. The situation is even worse when stereo signals are being broadcast.

This White Paper describes a simple model that was developed to quantify the harmonic distortion caused by a single echo. The model confirms that stereo signals suffer more as a result of multipath, although the distortion varies greatly as a function of echo delay. It would be straightforward to extend the model to include multiple echoes and audio intermodulation products.

The predictions of the model have been compared with the measurements made on actual receivers. Agreement is generally good for audio frequencies below 3 kHz. Higher frequencies can cause strange effects as a result of high-order harmonics entering the receiver's stereo decoder. Since the extent of this problem depends on the detailed design of the stereo decoder, a universal model cannot make accurate predictions.

FM Radio: The Calculation of Distortion Arising from Multipath Propagation

Ranulph Poole

1 Introduction

FM reception has long been recognised as vulnerable to multipath propagation. In order to obtain a feel for the problem, this White Paper describes a simple model for calculating the audio harmonic distortion arising from the presence of a single echo. Although such a model has obvious limitations, it is readily extended to cover multiple echoes and audio intermodulation distortion. However, even in its present form, the model is useful for illustrating the increased susceptibility of stereo reception.

Of course, any model must be reasonably representative of reality, and so this White Paper also compares the predicted results with measurements made on actual receivers.

2 The Stereo FM Broadcasting System

Stereo broadcasting in the UK has a long history, and it seems unnecessary to say much more about it. Nevertheless, a short summary is useful to clarify the terminology and to refresh the reader's mind.

Stereo broadcasting began in 1926, with two channels being transmitted experimentally from separate MF transmitters. Since the transmitters were in different parts of the country, one cannot help wondering how successful this was. **[1]** For many years, stereo remained merely a matter of scientific curiosity. However, by the late 1950s, stereo recordings were becoming available, and broadcasters were under pressure to devise more practical transmission techniques. 'Practical' implied the use of a single (FM) transmitter and a coding system that was compatible with existing mono equipment.

The system eventually adopted was developed by the General Electric Company, and approved by the US Federal Communications Commission in 1961. **[2]** The essential idea was to code the left and right signals into a sum channel that would deviate the FM transmitter just as before. A difference channel would also be derived, and this would be amplitude-modulated on to a 38 kHz suppressed carrier. The additional channel would form a so-called multiplex with the sum channel, and the levels would be controlled so as to keep the peak carrier deviation within 75 kHz. A 19 kHz pilot tone would also be included in the multiplex, to allow the receiver to regenerate the suppressed 38 kHz carrier.[1] The complete signal appears as below:

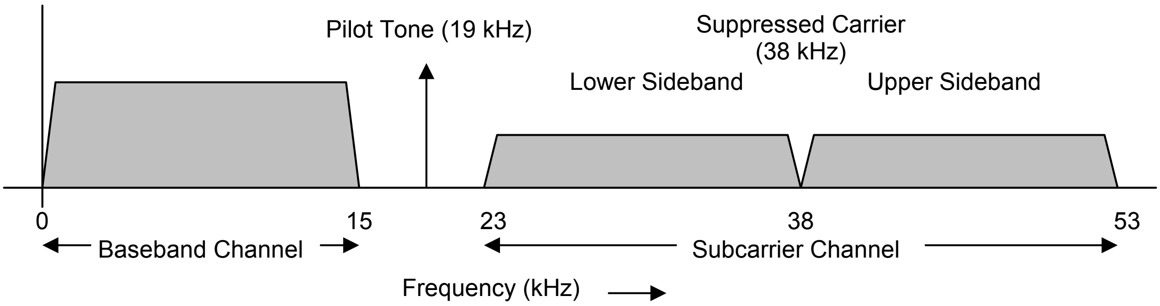


Figure 1: Spectrum of the Multiplex Signal

The convention is to refer to the left channel as 'A', and to the right channel as 'B', each with maximum values of 1. Then the instantaneous deviation of the transmitter as a fraction of the maximum is

$$0.9\{0.5(A+B) + 0.5(A-B)\sin\omega_s t + 0.1\sin\omega_s t/2\}$$

where ω_s is the angular frequency of the subcarrier.

1 The pilot tone is at 19 kHz rather than 38 kHz because it is then easier to extract from the remaining multiplex signal — there are no strong audio components nearby. Also, interfering noise components are likely to be less.

$(A + B)/2$ is often known as 'M' (for 'mono'), and $(A - B)/2$ as 'S' (for stereo). Nowadays, transmitters are lined up so that M = 1 gives rise to a peak deviation of 60.75 kHz, in the absence of the pilot. This leaves some space for additional features such as RDS (Radio Data System).

3 FM Signals and Multipath

The multiplex signal just described could in principle be used with either an AM or an FM transmission system. However, if AM had been chosen, the resulting RF channel bandwidth would have extended to 53 kHz either side of the carrier — considerably greater than the ±15 kHz needed for a mono signal alone. With an FM system, the channel bandwidth is deliberately chosen to be much greater than the modulation bandwidth, since this ensures good immunity to noise and interference. Adding the stereo multiplex components then creates less of an overhead.

In formal terms, the modulating signal (such as the stereo multiplex signal) is represented as

$x_m(t)$ where $|x_m(t)| \le 1$,

and the sinusoidal carrier is

$x_c(t) = a\cos(\omega_c t)$ where ω_c is the carrier's angular frequency, and a is its amplitude.

The modulator combines the carrier and modulating signal to give the transmitted signal:

$$y(t) = a\cos\left\{\int_0^t (\omega_c + \omega_d x_m(t))dt\right\}$$

When $x_m(t)$ is a simple sinusoidal signal $\cos(\omega_m t)$,

$$y(t) = a\cos\left\{\int_0^t (\omega_c + \omega_d \cos\omega_m(t))dt\right\} = a\cos\{\omega_c t + (\omega_d/\omega_m)\sin\omega_m t\}.$$

The quantity ω_d/ω_m is known as the modulation index, m. Since it is a ratio, it can be written in terms of the frequencies f_d and f_m:

$$m = f_d / f_m .$$

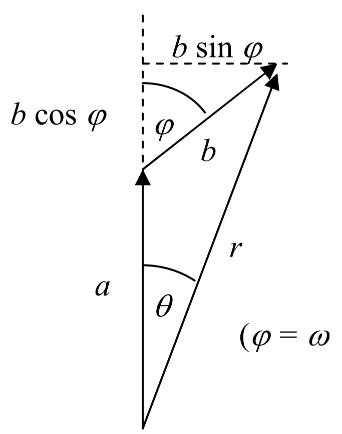
Even for this simplest possible modulation, the FM signal is quite complicated, since it involves mathematically difficult 'sines within cosines'. Early theoretical treatments involved heroic struggles with Bessel functions, which the author would not want to repeat. Fortunately, as will be shown, a simple PC spreadsheet can come to the rescue. For the moment, note that m, which represents the phase deviation of the carrier, is inversely proportional to the modulation frequency. This is why many of the advantages of the FM system decrease with f_m, and hence why the stereo multiplex signal is more susceptible to noise and distortion.

The next task is to understand what effect multipath — or an echo — has on the FM signal. For the plain carrier $x_c(t) = a\cos(\omega t)$, an echo of delay τ and amplitude b gives rise to a resultant signal

$$x_d(t) = a\cos(\omega t) + b\cos(\omega (t+\tau)).$$

At time $t = 0$, $x_d(0) = a\cos(0) + b\cos(0 + \omega\tau) = a + b\cos(\omega\tau)$.

This can be represented vectorially:



The wanted signal a and reflection $b \sin \varphi$ combine to give the resultant x_d, with amplitude r and phase shift θ.

By Pythagoras' theorem,

$$r^2 = (a + b\cos\varphi)^2 + (b\sin\varphi)^2$$

$$= a^2 + b^2 + 2ab\cos\varphi$$

or $r = \sqrt{a^2 + b^2 + 2ab\cos\omega\tau}$

Also, $\theta = \tan^{-1}(b\sin\omega\tau / (a + b\cos\omega\tau))$

Figure 2: Vector Addition of Wanted Signal and Echo

The variation of r with frequency gives the familiar response ripple associated with an echo; the amplitude of the signal reaching the receiver depends on the instantaneous frequency:

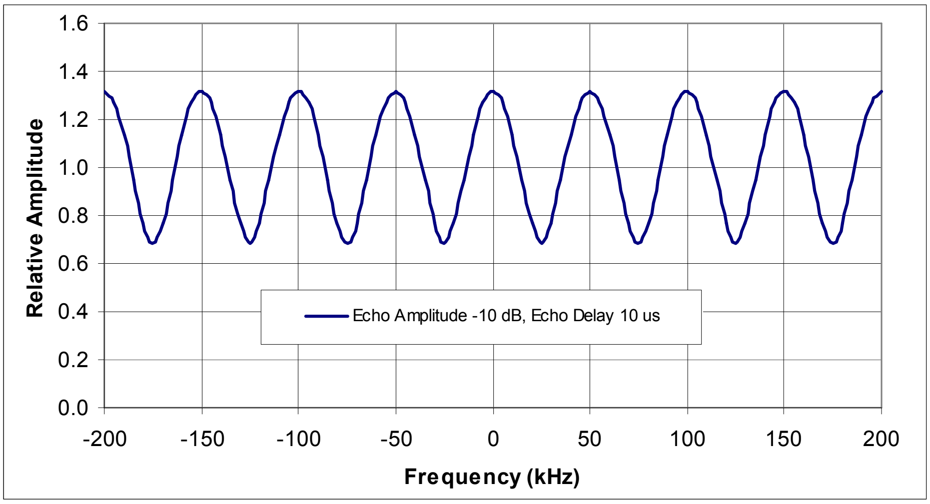


Figure 3: Amplitude Ripple Associated with an Echo

One of the advantages of an FM system is that amplitude variations can be removed from the signal by limiting. However, the phase variations remain, and these cause distortion of the demodulated signal:

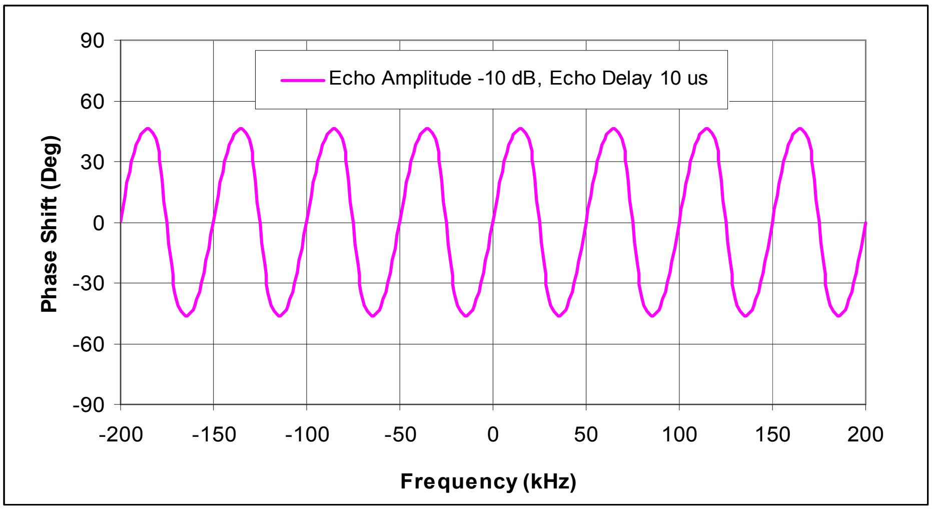


Figure 4: Phase Ripple Associated with an Echo

The increased vulnerability of the multiplex's subcarrier channel now becomes apparent. If the modulation comprises only a low frequency baseband signal, the phase deviation f_d / f_m is large, and a phase ripple of a few degrees cannot do much harm. On the other hand, within the subcarrier channel, f_d / f_m is small, and the same phase ripple has a greater effect.

4 A Spreadsheet Calculator

As explained earlier, traditional calculations on FM systems include Bessel functions and worse, and are very tedious. However, with the advent of PC spreadsheet programs such as *Excel*, the mathematics can be performed with very little pain.

When confronted with similar problems in the past, the author has used the spreadsheet to generate a waveform that represents the modulated carrier. The Fourier analysis tool then converts this 'time domain' representation into the frequency domain — rather like a spectrum analyser display in the real world. With the signal in this form, it can be multiplied by the 'channel' — the gain and phase information already calculated. It only remains to apply an inverse Fourier analysis and 'demodulate' the distorted signal.

This is a valid technique, but the spreadsheet can struggle with the large amount of data generated during the calculations. The author therefore tried an alternative approach, keeping the signal in the time domain at all times. A flow diagram of the new spreadsheet is given below. The numbering of each step corresponds to that of the following description.

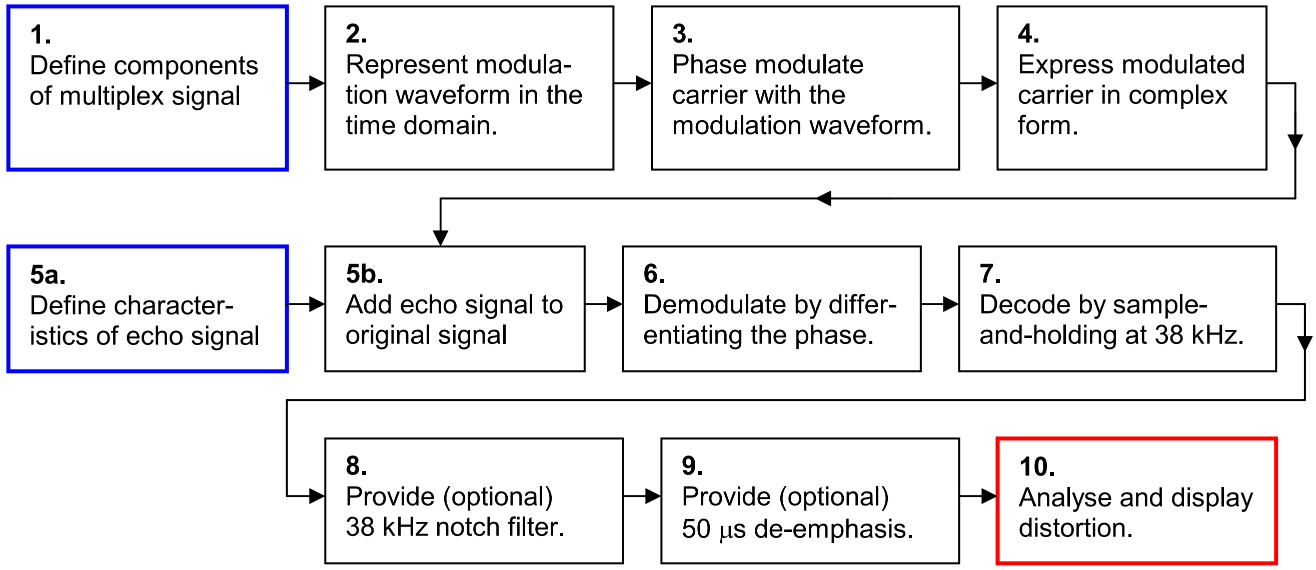


Figure 5: Flow Diagram to Illustrate Operation of the Spreadsheet

1. The baseband signal is a sinusoid of 594 Hz or a harmonic (n) thereof. The reason for this seemingly awkward frequency will become apparent later. When required, a subcarrier channel is also generated, comprising sidebands at $f_s \pm (n \times 594)$. There is no pilot tone. It is assumed that the multiplex signal will be the 'worst case' of left-only; i.e. half the available frequency deviation is caused by the baseband component and half by the subcarrier channel. However, other multiplex signals could easily be created if desired.

2. The modulation waveform is described by 1024 samples over 1 cycle of 594 Hz — or actually a convenient 16 samples during 1 cycle of 38 kHz. This choice is a relic of the original spreadsheet incorporating Fourier analysis, where the number of samples must be a power of 2. Retaining 1024 samples allows Fourier analysis of the final (decoded) audio signal if required.

3. Modulation of the carrier takes place in accordance with the formula given in Section 3:

$$y(t) = a\cos\{\omega_c t + (\omega_d/\omega_m)\sin\omega_m t\}.$$

For convenience, the carrier frequency ω_c is set to zero, and the amplitude a to 1. As previously described, there are three sin ω_m terms, corresponding to the baseband audio and the two subcarrier channel sidebands. The modulation indices ω_d/ω_m of the individual components are chosen so that the deviation of the subcarrier channel equals that of the baseband; i.e. A = B.

4. The modulated carrier is expressed in complex form, so that it can be added to the echo and then demodulated:

$$y(t) = \cos\{(\omega_d/\omega_m)\sin\omega_m t\} + j\sin\{(\omega_d/\omega_m)\sin\omega_m t\}.$$

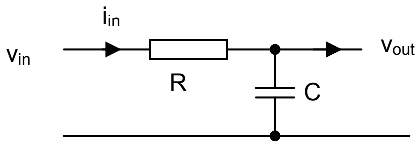
5. The multipath signal is assumed to comprise the direct signal and a single echo only. The echo signal is simply $y(t)$ already calculated, but delayed by an appropriate number of samples and multiplied by the relative amplitude. In principle, any number of further echoes could be added in the same way.

6. The demodulator measures the difference in phase between adjacent samples and scales these appropriately. This is equivalent to differentiation and is the inverse of the integration implicit in the modulation process. A difficulty arises in the ambiguity between phases of $(\pi + \delta)$ and $(-\pi + \delta)$ radians. A 'pi-catcher' is therefore included to add or subtract 2π if the phase appears to change by more than possible between samples.

7. The stereo decoder samples the demodulated multiplex waveform at intervals of 1/(38 kHz), at the peaks of the subcarrier signal. It holds each sample until the next sample arrives, in a manner equivalent to the way many commercial decoders work.

8. The staircase waveform emerging from the stereo decoder needs filtering. The first filter performs a running average over 16 samples, the effect of which is to create a notch at 38 kHz. In the absence of a distorting channel, the emerging baseband audio now looks identical to that applied to the modulator.

9. A second filter mimics the 50 μs de-emphasis normally included in a receiver. Performing this function in the time domain is not difficult but requires some thought:



R and C comprise the standard de-emphasis network.

$i_{in} = (v_{in} - v_{out})/ R$, and $dv_{out}/dt = i_{in}/C$.

Hence $v_{out} = v_{in} - RC\, dv_{out}/dt$.

In other words, the output sample equals the input sample minus the difference between adjacent output samples scaled by the RC time-constant. This integration process needs a little time to settle down, and so the spreadsheet actually starts 50 samples before the first sample of interest.

10. The distortion residual is obtained by subtracting the original 'reference' baseband signal from the final output. The 'Solver' tool is useful for adjusting the reference amplitude and phase to minimise the fundamental component. An example plot is shown below:

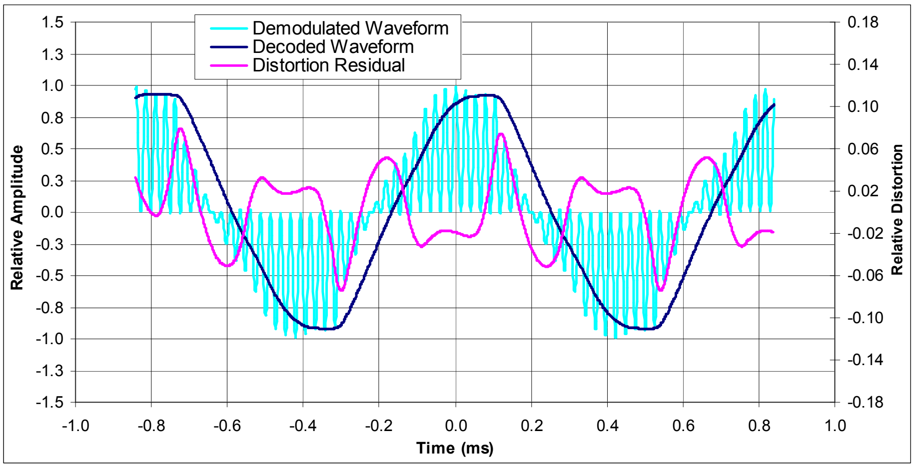
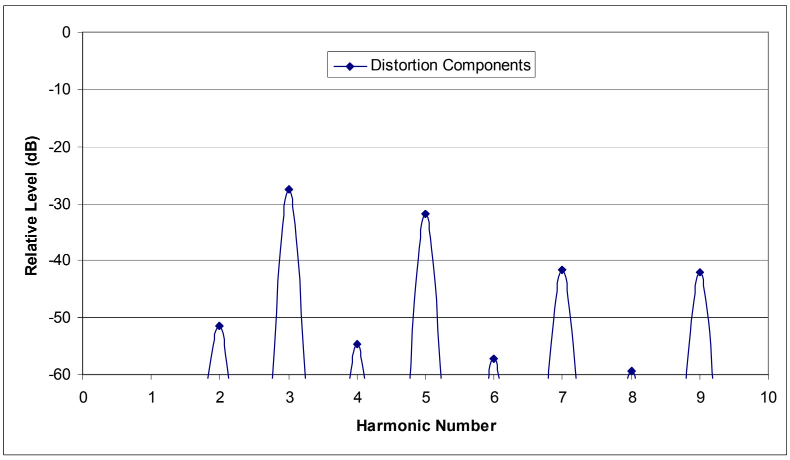


Figure 6: Example of the Waveforms Generated by the Spreadsheet

The spreadsheet also analyses the harmonic distortion. It does this by multiplying the residual waveform by the sine and cosine versions of the harmonic in question. The resulting samples are then summed. However, because 1024 (2^{10}) samples have been chosen, the residual can also be analysed by *Excel's* Fourier analysis tool — if it works. The plot appears as below:

The harmonics appear as they might on a spectrum analyser display. '0 dB' corresponds to the level of the fundamental component before being nulled out.

Figure 7:
Example of the Harmonic Plot



5. The Laboratory Set-up

The model so far developed is only useful if the results bear some relation to those of a real system. Such a 'real system' was set up in the lab as illustrated below:

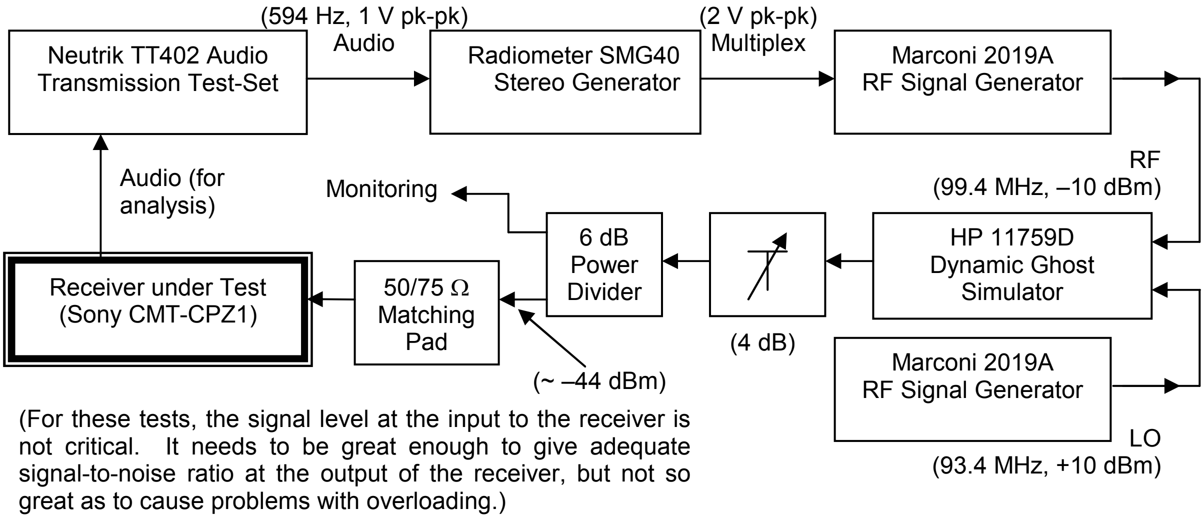


Figure 8: Test Arrangements for the Measurement of Multipath Distortion

The setting-up procedure is as follows:

- The RF signal generator is set to 'FM', 'External Modulation' and '60.8 kHz' peak deviation. Its output level should be –10 dBm to suit the Dynamic Ghost Simulator.
- The stereo generator is set to 'External' (audio), 'Left-only' and '0 µs' (pre-emphasis)[2].
- The Neutrik audio generator is set to 594 Hz and approximately 1 V pk.-pk. This level should cause the meter of the stereo generator to read about '4'.
- The output of the RF signal generator is temporarily connected to a modulation analyser such as the HP 8901A. The level control on the stereo generator is set precisely for 60.8 kHz peak deviation *with the pilot tone switched off*. With the pilot tone only present, the peak deviation should be 6.08 kHz.
- The ghost simulator is set to enable only the primary signal path, with 0 dB attenuation. Although the RF level applied to the receiver is not critical, it should be about –44 dBm at the monitoring point (before the 50 Ω/75 Ω transition).
- The audio level from the receiver headphone socket should be set (by means of the volume control) to about 0 dBu. The weighted signal-to-noise ratio[3] should be at least 50 dB. If not, further adjustments can be made to the RF and audio output levels.

All that remains is to select the required multipath profile and measure the total harmonic distortion (THD). It is advisable to enable the Neutrik's various filters — 400 Hz HPF, 30 kHz LPF and 22 Hz – 22 kHz BPF — to avoid artefacts such as residual pilot tone from affecting the results. A spectrum analyser may be used to measure the levels of the individual harmonics.

6. Measured and Calculated Results

To start with, the ghost simulator was set up to generate an echo at –5 dB and a 'typical' delay of 25 µs. The echo phase was 0°, meaning that the channel response was symmetrical about the carrier frequency. Three different receivers were tried: the already-mentioned Sony, a Technics ST-G70L (a late-1980s hi-fi tuner) and a Studer A-764 (a late-1980s precision tuner). The results are shown overleaf, where they are compared with the predictions of the spreadsheet.

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2 With this stereo generator, selecting pre-emphasis reduces the audio output level — an inconvenient feature.

3 The weighting provided by the Neutrik test-set is CCIR Rec. 468-3.

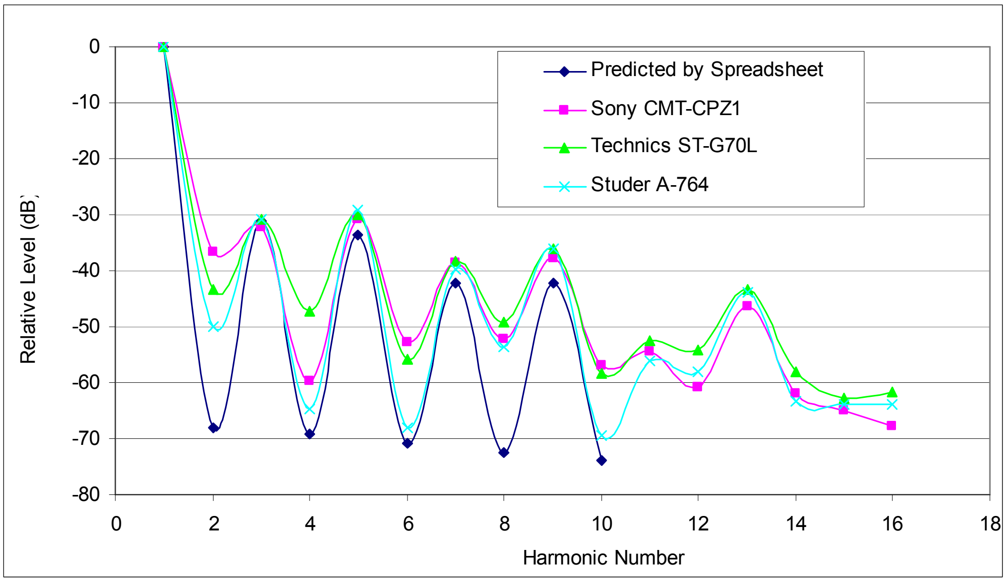

Figure 9: Multipath Harmonic Distortion: Predicted and Measured

There are several comments to make:

- The harmonics are, of course, discrete. They are shown joined up to improve clarity.
- The fundamental (Harmonic Number 1) is normalised to 0 dB in all cases.
- According to the spreadsheet, distortion is essentially confined to the odd harmonics. This is as expected for a channel response that is symmetrical about the carrier frequency.
- The 'real' tuners — the Sony in particular —generate greater even harmonic levels than predicted. This is attributable to harmonic distortion inherent in the tuners, and has nothing to do with multipath: the Sony gives a THD of nearly 1% with a clean RF signal.

This echo profile was used during later listening tests as the 'Large Multipath' channel. Although the measured total harmonic distortion (THD) was a modest 4.4%, the presence of high-order products gave rise to very poor sound quality. By contrast, the 'Small Multipath' channel included a single echo at –15 dB. The THD had fallen to 1.2%, but the sound quality was still deemed objectionable on some programme material.

As the spreadsheet calculator was giving promising results, it was used to explore the effects of varying the echo power, delay and phase in turn, the default parameters being –5 dB, 25 µs, 0°. Plots are shown below, along with 'real' measurements made on the Sony tuner. The mono signal was obtained by switching the stereo generator to 'L = R', and setting the deviation to 60.8 kHz.

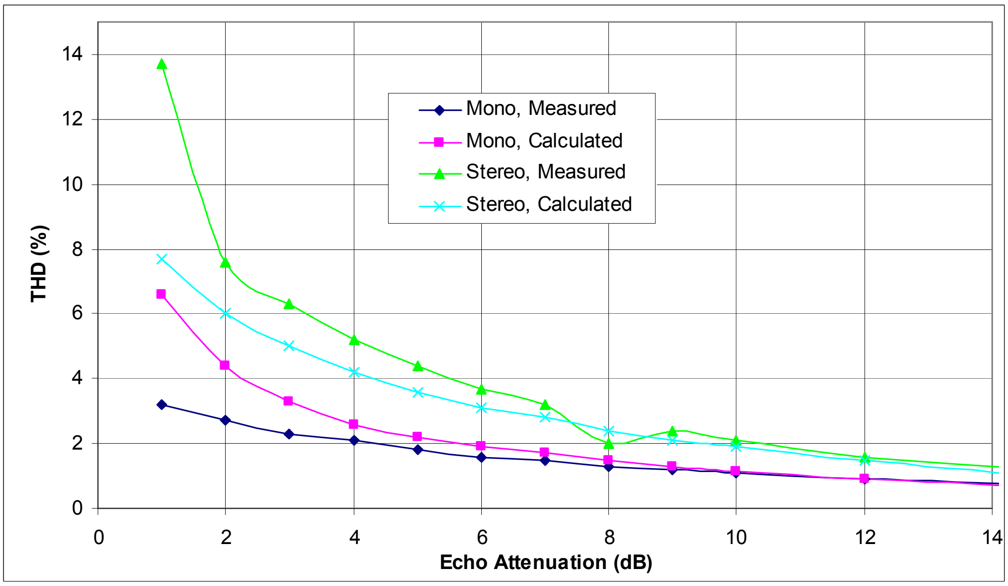

Figure 10: THD, Predicted and Measured: Echo Attenuation Varied

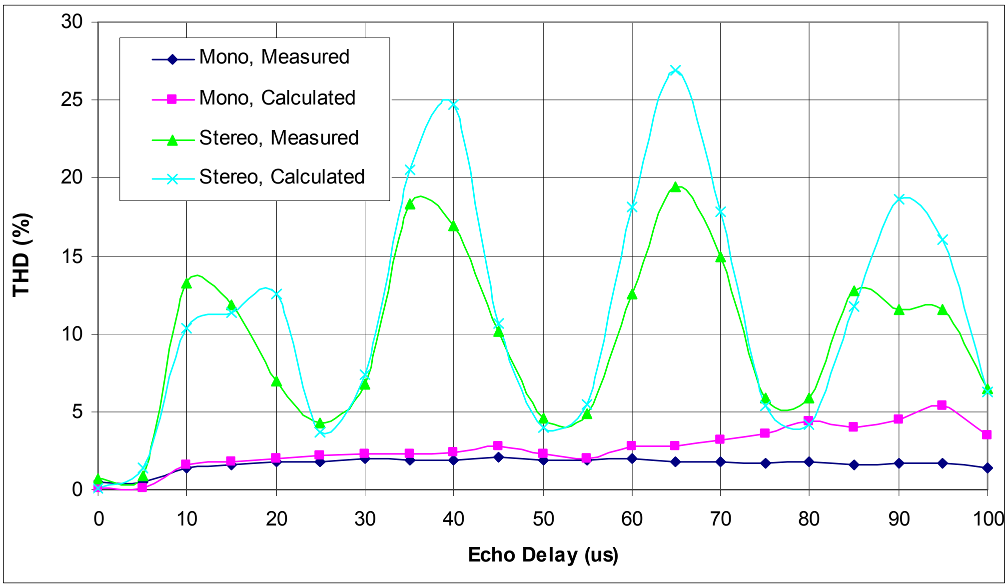

Figure 11: THD, Predicted and Measured: Echo Delay Varied

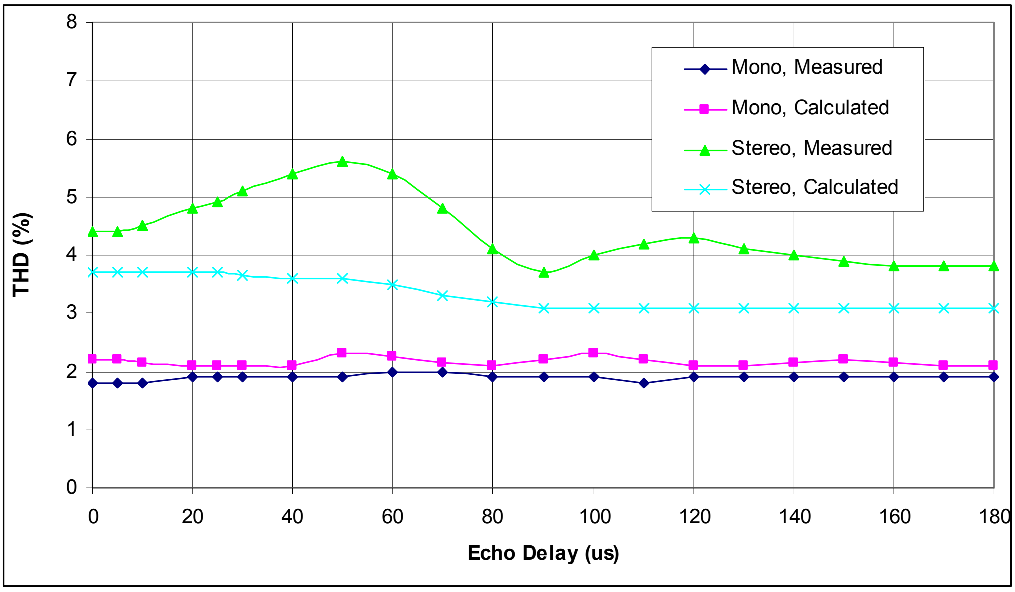

Figure 12: THD, Predicted and Measured: Echo Phase Varied

Some comments are as follows:

- Predicted and measured values generally agree well, despite the challenging amounts of multipath distortion.
- Stereo THD is always worse than mono THD, but not always greatly so.
- The most dramatic differences between mono and stereo THD occur as the echo delay varies. The mono THD is well behaved, but the stereo THD shows peaks separated by 25 µs — presumably when the subcarrier channel takes 'direct hits' from the peak phase errors. Distortion is then gross, and the output of the receiver would be unusable.
- The phasing of the 38 kHz subcarrier is critical.[4]
- The echo phase has only a minor effect on the THD for this particular echo. It is likely to have more effect where the echo delay is such that the THD is a maximum.
- The measured mono THD figures are lower than the calculated values. Partly this can be ascribed to HF loss in the receiver.

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[4] This issue is easy to explore in the spreadsheet, which allows the sampling of the 'decoder' to be offset by a multiple of 22.5° at 38 kHz. Some thought also had to be given to decoder phasing in the basic design of the spreadsheet, because the differentiation process employed by the 'demodulator' can introduce timing errors.

It is interesting to look at the nature of the distortion introduced by an echo. The waveforms associated with the 'standard' echo of –5 dB, 25 μs and 0° are as follows:

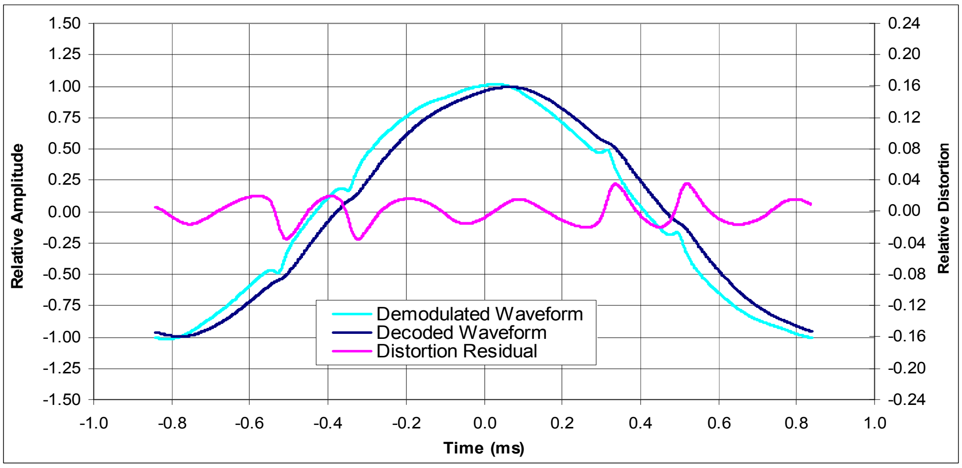

Figure 13: Waveforms Associated with the 'Standard' Echo, Mono Reception

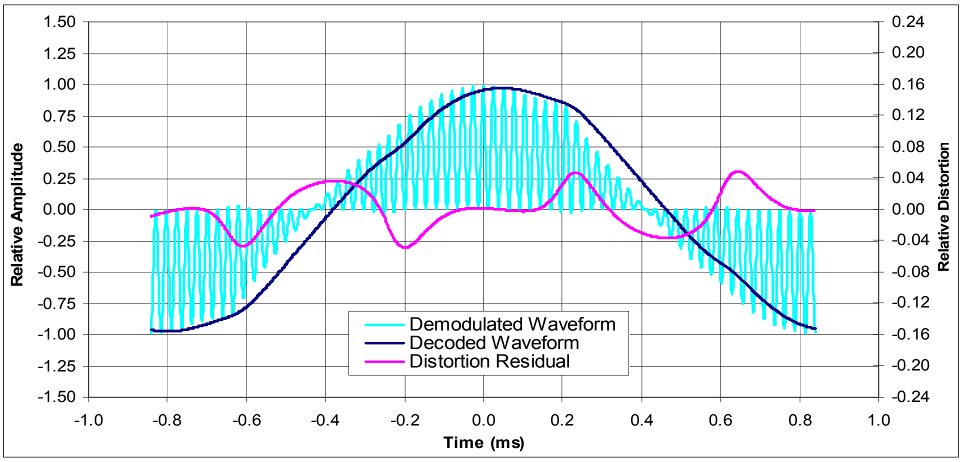

Figure 14: Waveforms Associated with the 'Standard' Echo, Stereo Reception

The mono waveform shows the typical 'coggles' associated with the phase error peaks in the channel. These coggles are more spread out on the stereo waveform, as the deviation of the baseband signal is only half as great. However, the subcarrier channel now also introduces distortion, and the THD is greater (3.7% rather than 2.2%).

Although the distortion would be unpleasant to listen to, the 25 μs echo is relatively benign, and the waveforms are at least recognisable. A 'difficult' echo of 40 μs is a different matter:

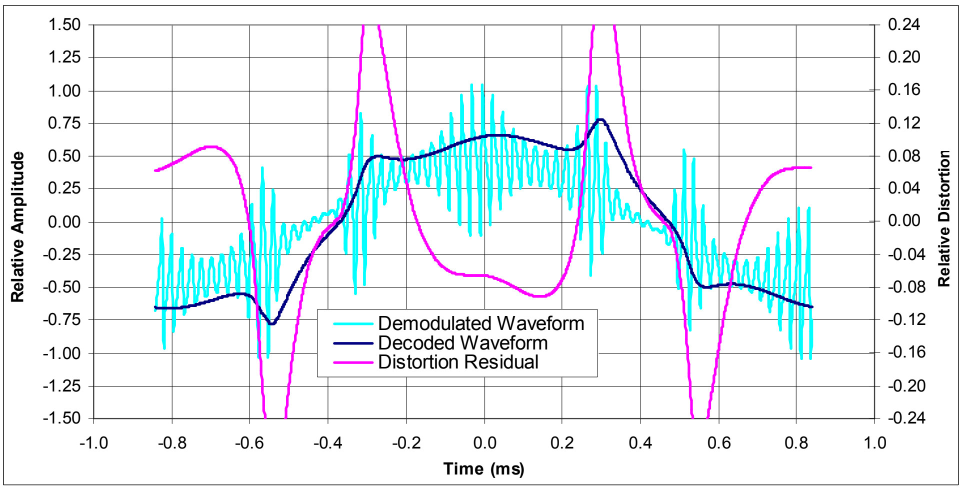

Figure 15: Waveforms Associated with a 40 μs Echo, Stereo Reception

The waveform is now unrecognisable, and the output of the receiver would be unusable. By contrast, the effect on mono reception is unspectacular:

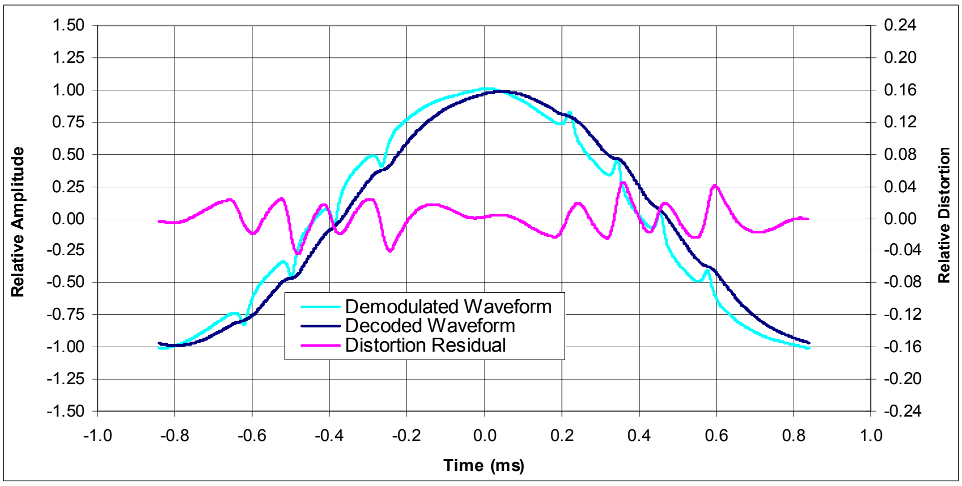

Figure 16: Waveforms Associated with a 40 µs Echo, Mono Reception

7. A Limitation of the Model

As the previous section shows, the spreadsheet calculator gives a good prediction of the distortion resulting from the presence of multipath. Only the simplest cases have been considered — a single baseband tone and a single echo. However, a small amount of work would be sufficient to increase the scope of the calculator if that were deemed necessary.

Unfortunately, there is a more fundamental weakness that arises when the echo is large and the baseband tone is high in frequency. In the real world, baseband harmonics can then fall within the subcarrier channel, so giving rise to a further range of distortion products. Even worse, the harmonics can upset the pilot tone circuitry to the extent that the receiver no longer recognises the presence of a stereo signal. Because the new distortion products are unlikely to be harmonically related to the original tone, calculations are difficult: the spreadsheet is only effective where the time-span is such as to incorporate an integral number of cycles of each signal component.

To show how severe the problem can be, a left-only tone of 6.18 kHz was used to deviate the generator by 60.75 kHz peak. The echo path parameters were –5 dB, 25 µs and 0°. A spectrum analyser placed on the audio output of the receiver gave the following results:

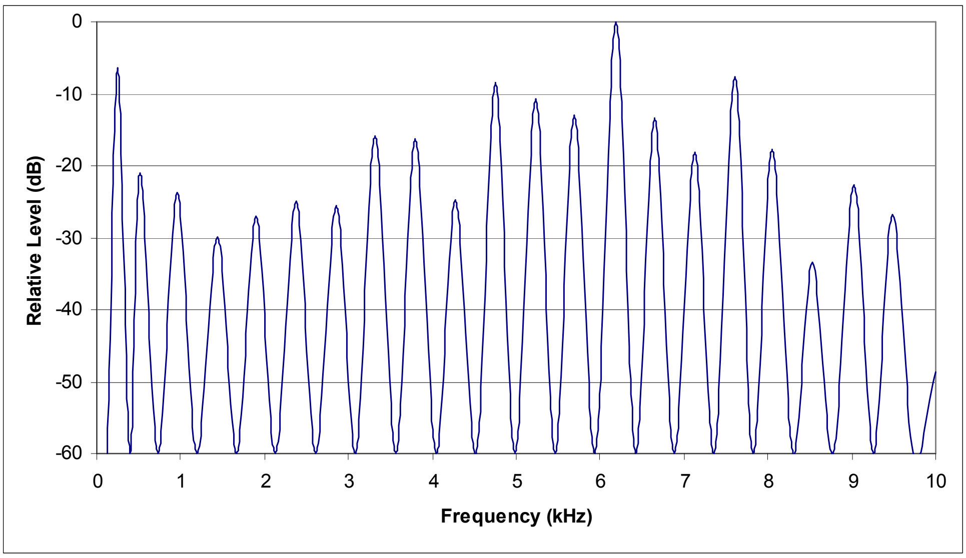

Figure 17: Distortion Products Resulting from a 6.18 kHz Left-only Tone

The spurs are spaced by a little under 500 Hz. It appears that the third harmonic of the baseband tone is mixing with the pilot tone (19 kHz – (3 × 6.18) kHz = 460 Hz). The corrupted pilot tone is then transferring the unwanted sidebands on to the wanted signal in the decoding process.

Admittedly, this is an extreme example: in practice, such a combination of baseband tone and echo is unlikely. However, low levels of these anharmonic spurs would be very audible, and it would be useful to be able to calculate them.

8. Conclusion

This White Paper has described a model for calculating the audio distortion arising when an FM system is subject to multipath propagation. The model has concentrated on the simple case of a single echo and a single audio tone, but could easily be extended to more complicated situations. The predicted levels of harmonic distortion agree well with measurements made on a typical receiver.

Both model and measurements show that multipath distortion levels are worse for stereo reception than for mono, and considerably so for unfavourable delays.

The model, in its present form, cannot predict the levels of anharmonic products arising from interference to the pilot tone and subcarrier recovery. Calculation is difficult, in part because the model would need to take into account the implementation of the stereo decoder. The model would not then be universal.

9. Acknowledgements

The author would like to thank John Salter for reading through the draft document and making useful comments.

10. References

1. PHILLIPS, G J, 1967. ‘Stereophonic Broadcasting in Britain.’ Electronics and Power, August 1967.

2. CSICSATKA, A, and LINZ, R M, 1962. ‘The New Stereo FM Broadcasting System — How to Understand the FCC Specifications and Generate the Composite Signal.’ Journal of the Audio Engineering Society, January 1962, Volume 10, Number 1.