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Digital Radio Mondiale:
A ‘pseudo TEM-cell’ for receiver testing

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BRITISH BROADCASTING CORPORATION

Frontispiece: View of the Kingswood Pseudo TEM-cell

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Abstract

The inspiration for this work dates back to the 1990s, when the BBC embarked on the testing of a large number of portable FM radios. The objective was to determine the performance — and especially the RF sensitivity — under conditions as close as possible to the ‘real world’. To accomplish this, a ‘pseudo TEM-cell’ was constructed within a Belling-Lee screened room.

The same screened room still exists at Kingswood Warren, and there is now a need to measure the performance of DRM receivers on the HF band. It is hence a good time to reassess the pseudo TEM-cell, and to improve the accuracy of its calibration if possible. A further motive for carrying out this work is to demonstrate to receiver manufacturers that providing ‘in-house’ test facilities is not necessarily difficult or expensive.

This paper discusses calibration techniques, and also explains how to calculate the field generated within any such pseudo TEM-cell.

1. Introduction

The inspiration for this work dates back to the 1990s, when the BBC embarked on the testing of a large number of portable FM radios. The objective was to determine the performance — and especially the RF sensitivity — under realistic conditions. An obvious requirement was a well-defined electromagnetic field. To provide this, a 'pseudo TEM-cell' was constructed within a Belling-Lee screened room.

The original screened room still exists at Kingswood Warren, and there is now a need to measure the performance of DRM receivers on the HF band. It is hence a good time to reassess the pseudo TEM-cell, and to improve the accuracy of its calibration if possible. A further motive for this work is to encourage receiver manufacturers to carry out their own 'in-house' testing.

This report discusses calibration techniques, and also explains how to calculate the field generated within any such pseudo TEM-cell.

2. The Pseudo TEM-Cell at Kingswood

The pseudo TEM-cell at Kingswood was constructed inside a Belling-Lee screened room of approximate dimensions 2.4 m × 2.4 m × 3.2 m. The essential idea was to place an open transmission line close to the ceiling, and a second, identical, one close to the floor. Provided that the two lines were driven in anti-phase, there would be a reasonably constant leakage field over a significant volume of space — the 'test area'.

A consideration with the original design was the need to avoid the excitation of cavity resonances at Band II (88–108 MHz). For the present project, the maximum frequency of interest is 30 MHz. Although this is well outside the danger region for resonances, there seemed little point in radically altering the design: the only important change was to convert the lines from 75 Ω to 50 Ω characteristic impedance. The modified arrangement is sketched below and described overleaf:

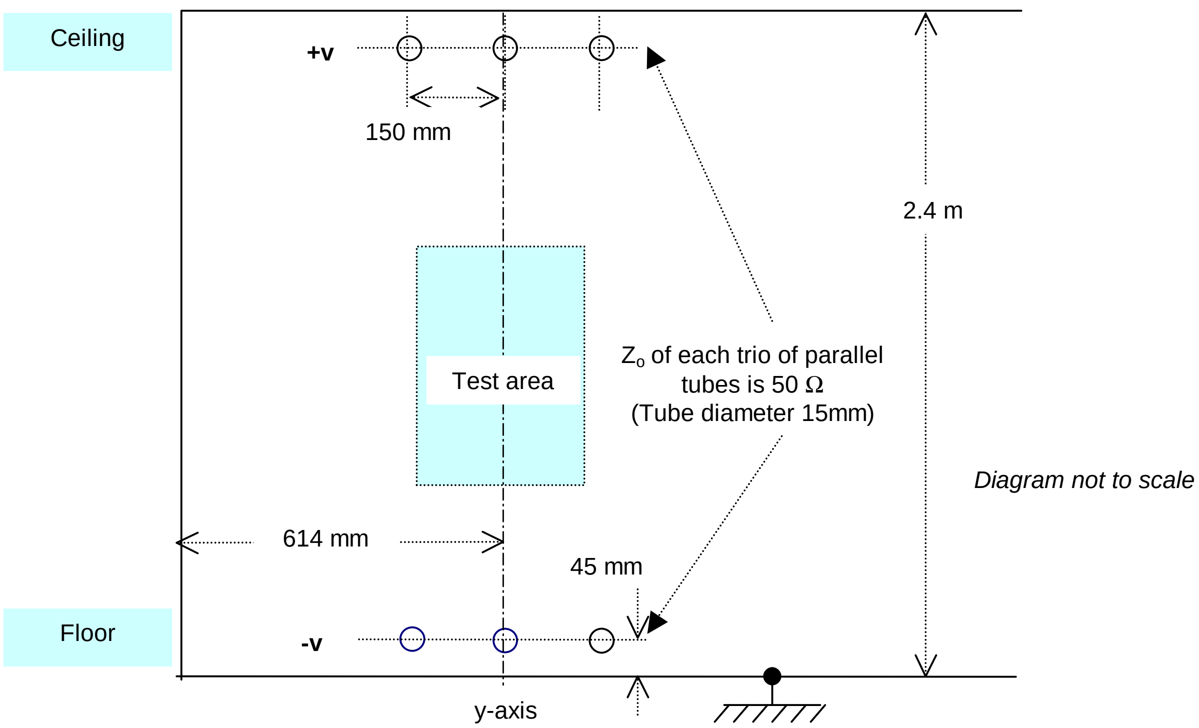


Figure 2.1: Cross-section of Screened Room, Transmission Lines End-on

- o Two transmission lines are fitted in the screened room: one above the floor, and the second below the ceiling. All surfaces of the room are conductive, and so the floor and ceiling behave as ground planes. Each line consists of three 15mm (outside diameter) copper plumbing pipes mounted with their centres 45 mm from the 'ground plane'. The pipes of each transmission line are coplanar and spaced by 150 mm, each pipe forming a transmission line of 150Ω characteristic impedance (Z_o).
- o The three tubes of each transmission line are brought together at the driven end, forming a composite transmission line with a Z_o of 50 Ω. The far ends are individually terminated in 150Ω resistors to ground.
- o The two resulting lines (floor and ceiling) are driven from an RF splitter with equal but anti-phase RF outputs, so providing a reasonably constant field in the central plane between the floor and ceiling. This is where the radio-under-test is positioned.[1]
- o The RF signal 'propagates' along the composite transmissions lines, and is absorbed in the 150Ω far-end terminations, minimising longitudinal modes in the room.

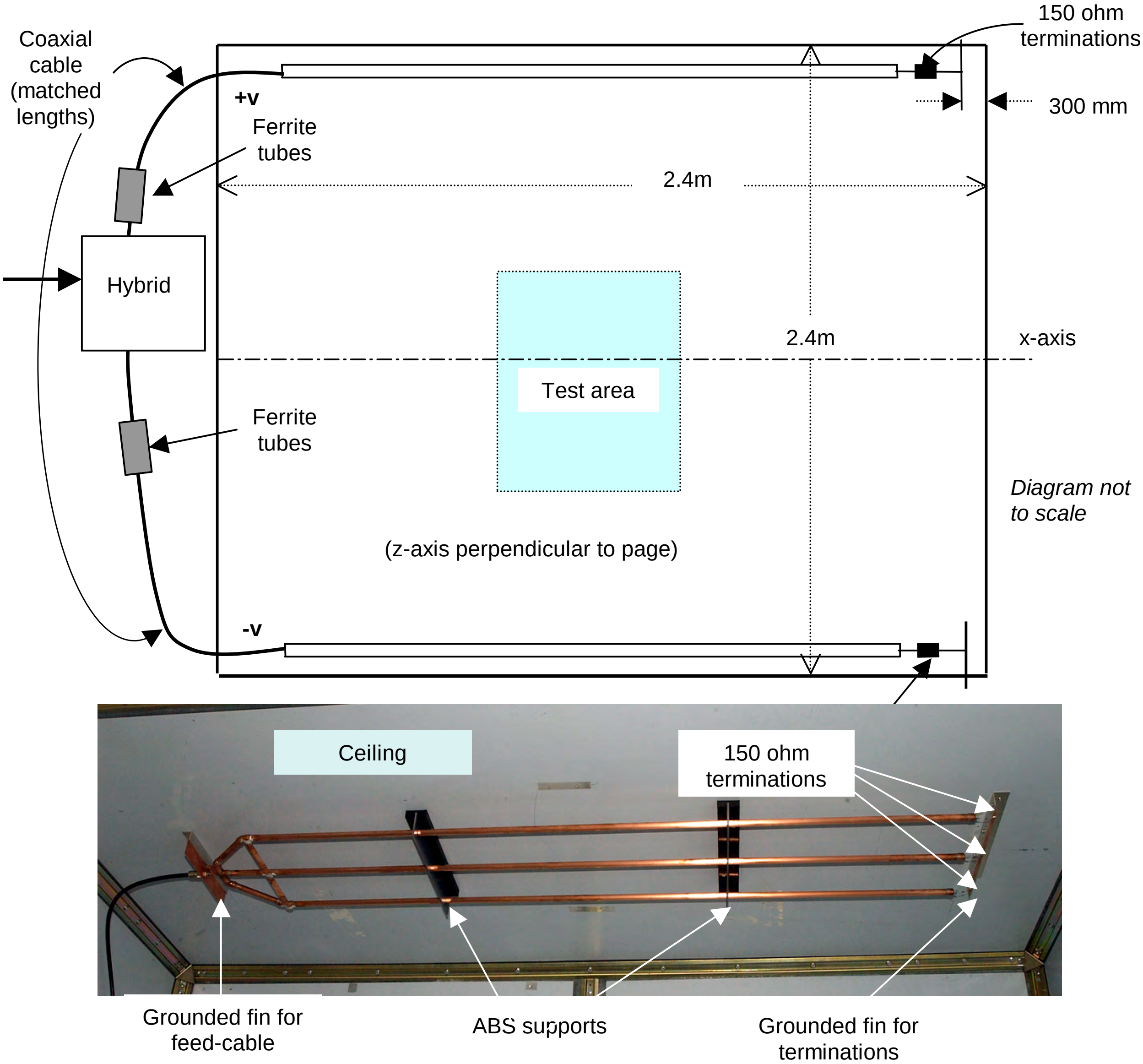


Figure 2.2: Cross-section of Screened Room, Transmission Lines Side-on

[1] Since there is nominally zero RF potential over this plane, there is the added bonus that test interconnections are less likely to disturb the measurements. Actually, the receiver-under-test would be placed on a wooden table whose surface was somewhat below the central plane. The telescopic antenna would then extend up through the central plane. It is therefore desirable for the electric field to be reasonably constant over a significant vertical distance.

3. Calibration of the Pseudo TEM-Cell

The pseudo TEM-cell was calibrated over the range 1 MHz and 100 MHz with the help of a professional G-TEM cell and a balanced E-field probe. These items are illustrated below:

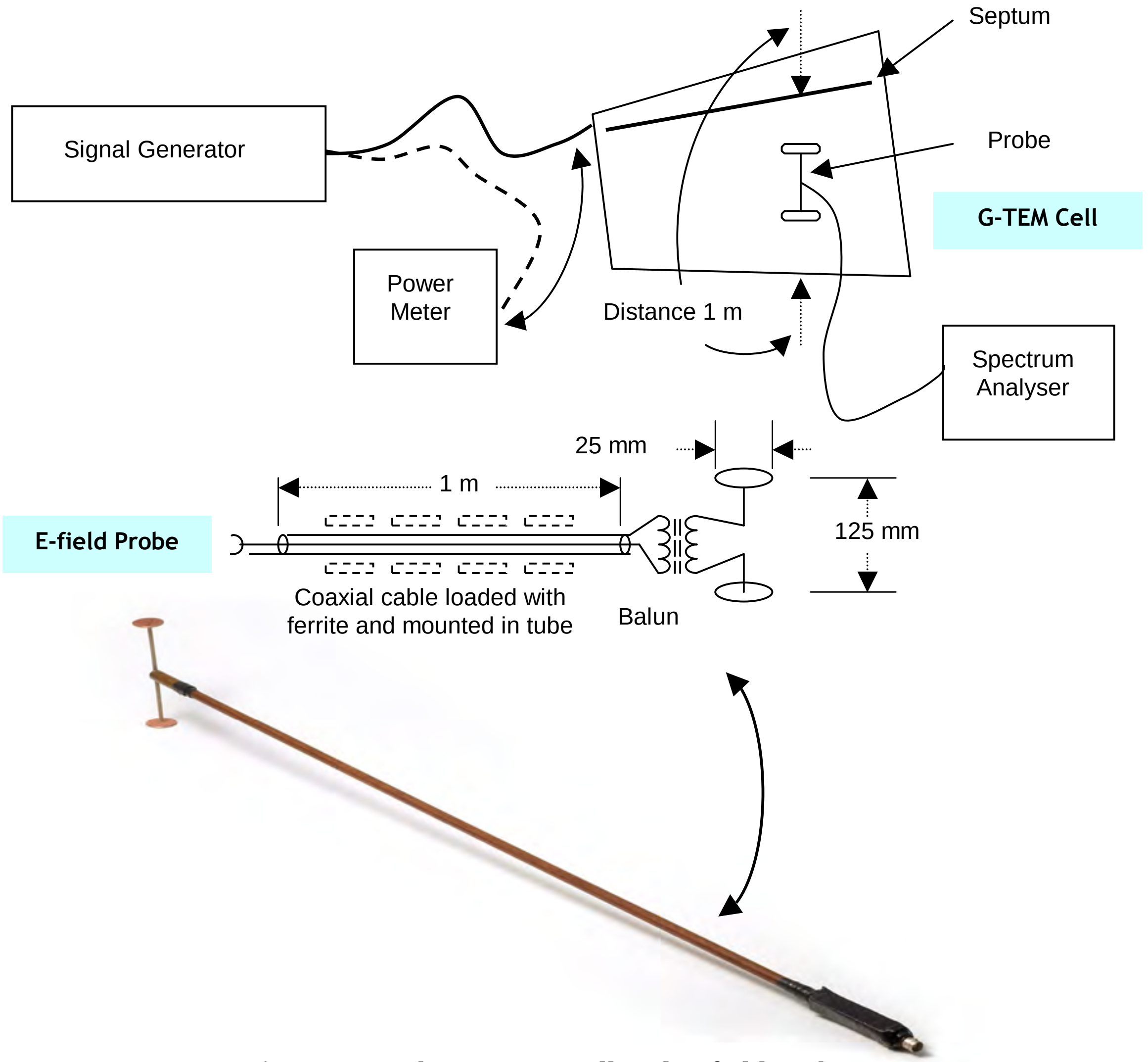


Figure 3.1: The G-TEM Cell and E-field Probe

The G-TEM cell is essentially an oversized co-axial transmission line with a rectangular cross-section. The inner conductor, or septum, takes the form of a plate which is terminated in the characteristic impedance (50 Ω) of the transmission line. Viewed sideways on, the G-TEM cell tapers so that, at the terminated end, there is a large distance between the septum and the floor. If the item under test is placed at a point where the septum is 1 metre from the floor, a potential of 1 V on the septum will give rise to a field-strength of 1 V per metre. Hence calculations are very straightforward.

Of course, it is important that any device used to measure a field does not significantly disturb that field. Such a probe was developed as part of the earlier project, to calibrate the pseudo TEM-cell at VHF. In essence, it comprises an electrically-short dipole at the end of a long, insulating tube. The dipole is connected via a balun transformer to a length of coaxial cable that runs down the length of the tube. At the far end of the tube is a BNC connector. Ferrite loading on the coaxial cable raises the common-mode impedance of the dipole. The purpose of the long tube is to keep the operator well away from the area of interest, and hence avoid the possibility of his affecting the field.

The first stage of the calibration procedure takes place with the probe placed in the G-TEM cell:

- The signal generator is set to the desired frequency. Its power is adjusted to a convenient level, say +10 dBm.
- The output of the E-field probe is measured with a device such as a spectrum analyser.
- The measurement is repeated with the E-field probe rotated through 180°.[2]
- The measurement is repeated with the probe disconnected from its associated cable.

For the second stage, the probe is transferred to the pseudo TEM-cell:

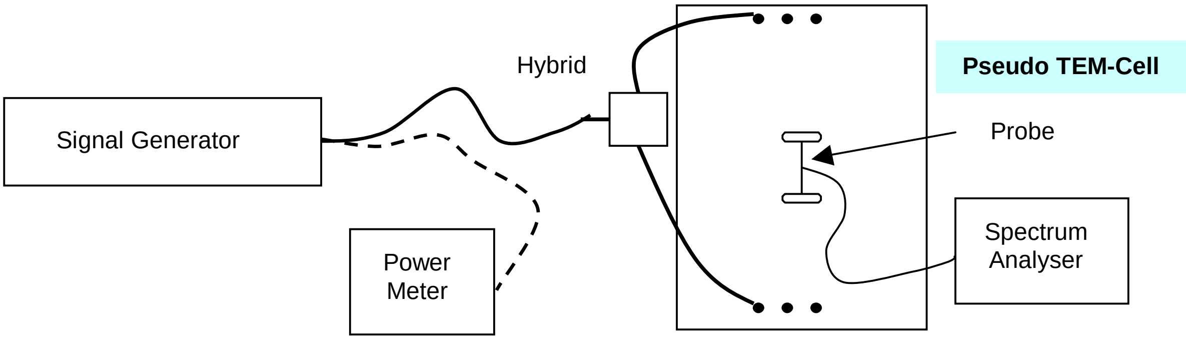


Figure 3.2: The Pseudo TEM-cell and E-field Probe

- The signal generator is set to the desired frequency and the power adjusted to the same level as before. It is important that the test equipment and interconnecting cables are left unchanged; the only difference is that the measurements are being made on the pseudo TEM-cell instead of the G-TEM cell.
- The output of the E-field probe is measured with the spectrum analyser, at 0° and180°, just as before.
- The difference (in dB) in measured level between stages 1 and 2 is recorded.

The calculation is now very straightforward. We know, from first principles, that 1 V RMS applied to the G-TEM cell results in a field-strength of 1 V/metre. Since the characteristic impedance of the G-TEM cell is 50 Ω, 0 dBm corresponds to 224 mV RMS and hence a field-strength of 224 mV/metre.

Suppose that, under identical test conditions, the output of the E-field probe falls by k dB when moved from the G-TEM cell to the pseudo TEM-cell. Then k dBm will be necessary to achieve 224 mV/metre in the pseudo TEM-cell. Alternatively, a level of

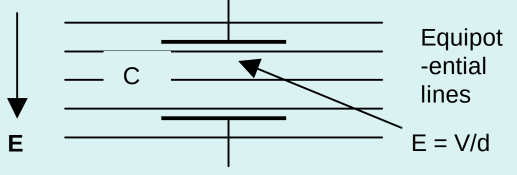
$k - (20 \log_{10} 224)$ dBm or $(k - 47)$ dBm

is required to generate a field of 1 mV/metre.

The measured results are shown overleaf.

The E-field Probe

The E-field probe is essentially a capacitor C. If its plates are separated by distance d, and V volts are applied between them, an E-field of V/d volts per metre exists between the plates. If this capacitor is now placed in an external field equal to V/d, the external field will not be affected:



If such a capacitor is placed in an alternating field, the EMF appearing across the plates will be given by V = E d. If the field is removed, and the plates driven by a generator of voltage V and frequency f, a current V j 2πf C will flow; hence the source impedance of the probe is 1 / j 2πf C.

If the probe is to drive a low impedance, its high source impedance must be taken into account when performing the calculations.

[2] The output should remain the same. If it does not, there is probably unwanted pick-up on the cable between the probe and the measuring device. The following measurement, with the probe disconnected from the cable, is to confirm that cable pick-up is insignificant. In principle, turning the probe through 90°, and observing a null in the output, would be just as effective. However, it is more difficult to determine exactly when the probe is horizontal.

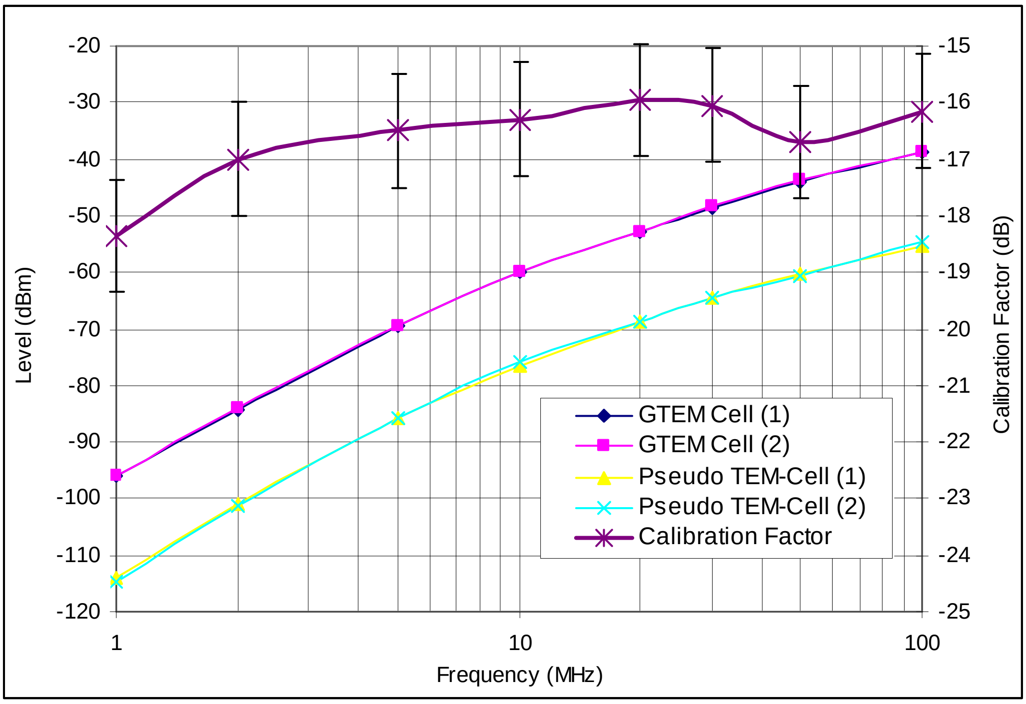


Figure 3.3: Calibration Results Obtained at Kingswood

Comments on the calibration measurements are as follows:

- All curves apart from 'Calibration Factor' (k) are the absolute power levels measured at the output of the E-field probe, when +10 dBm were applied to the TEM cell in question.
- Measurement runs '(1)' and '(2)' were made with the orientation of the probe being 0° and 180° respectively.[3]
- The absolute power levels are approximately proportional to frequency, as would be expected for a probe of this sort working into a low impedance — see the box on the previous page. Below 5 MHz, the level falls away more rapidly, because of losses in the probe's balun and so forth.
- The 'Calibration Factor' k is independent of frequency — again as expected, provided that losses in the hybrid and interconnections are negligible. The measurements at 1 MHz and 2 MHz are less reliable because of the low sensitivity of the probe at these frequencies. The average over 5 MHz to 100 MHz is –16.3 dB, with variations of less than ±0.5 dB.
- The calibration factor applies to the field at the centre of the test-bench in the pseudo TEM-cell, but does not vary significantly over the 'Test Area' shown in the diagrams. Measurements of the variations with position are presented later.

It appears, from the above work, that the pseudo TEM-cell may be used with confidence. The calibration factor k is –16.3 dB, and so a power level of 16.3 – 47 dBm, or –30.7 dBm, will generate an E-field of 1 mV/metre.

4. Variation of Field with Position in the Pseudo TEM-Cell

The previous work has established the calibration of the pseudo TEM-cell in the centre of the test area. However, it is also important to know what variations there are within the test area. Measurements were therefore carried out by moving the probe along each of the three possible axes and recording the relative levels. In fact, for the sake of curiosity, the field was measured outside the test area as well. The results are shown overleaf.

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[3] The results are almost identical, and so the plots are difficult to distinguish in Figure 3.3.

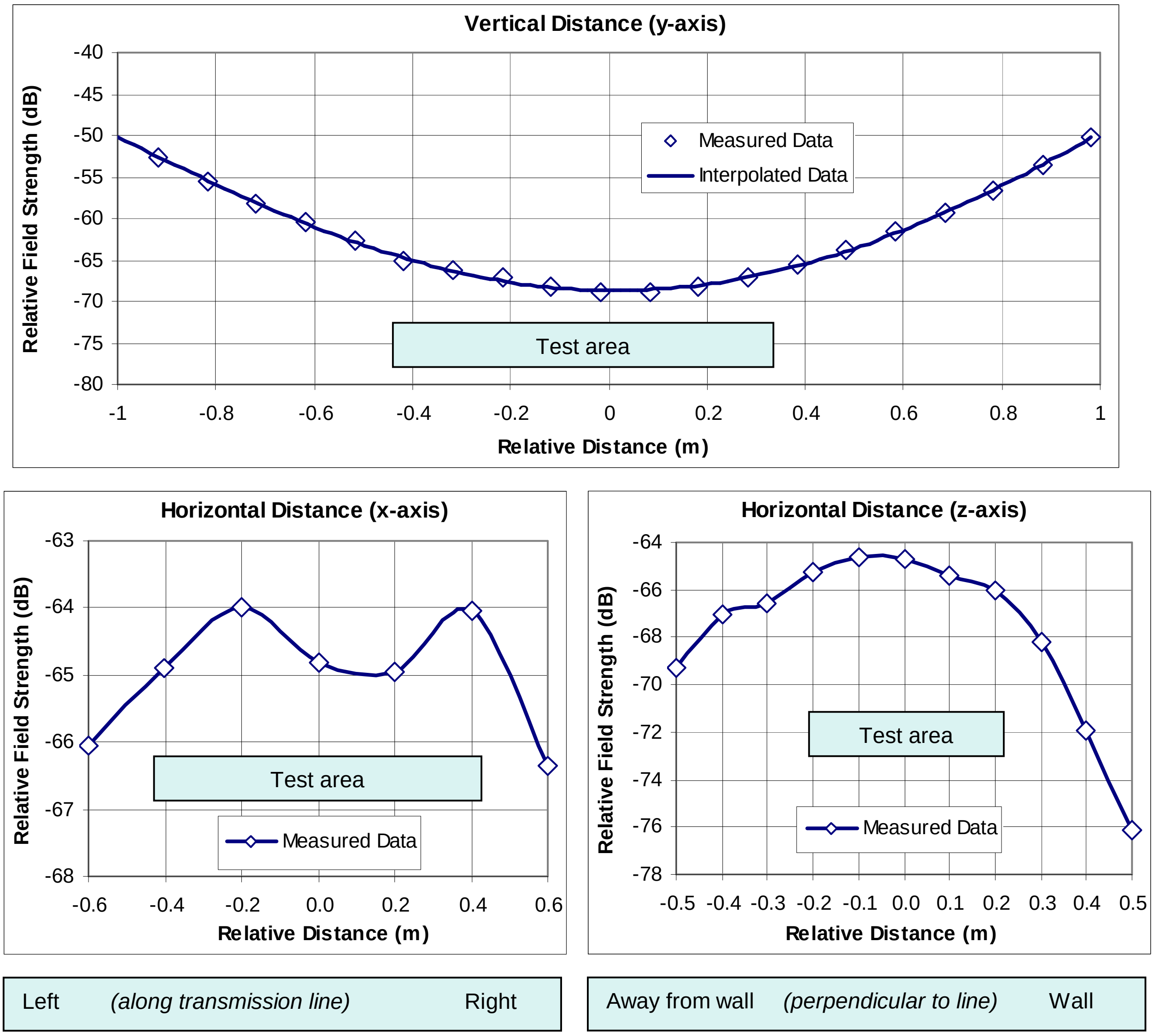


Figure 4.1: Variation of Field with Position

The 'horizontal' measurements were made with the probe at the height of the wooden test-bench; that is, 0.74 m above the floor, or y = –0.48 m. Variations in this plane are well controlled, and generally within ±1 dB over the area of interest. The field falls away rapidly as the rear wall of the screened room is approached, as might be expected.

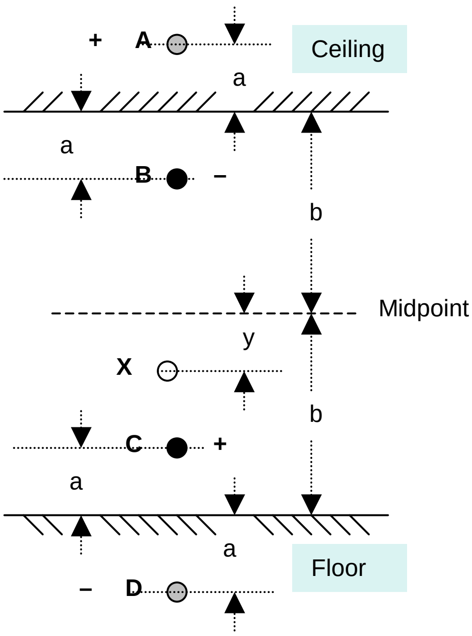
Note that the surface of the test-bench does *not* correspond to y = 0, and hence there are significant variations with height above the bench. It might have been better if the bench could have been raised somewhat. On the other hand, the telescopic antennas used with portable radios are usually of sufficient length that there would always be practical difficulties in keeping the field-strength nearly constant over the area of interest.

5. Calculating the Field-Strength within the Pseudo TEM-cell: Stage 1

The above method of calibrating the pseudo TEM-cell seems to be fairly foolproof: it is only necessary to measure one distance and one relative power level to a reasonable degree of accuracy, and the job is done. Even so, it would be more satisfactory to make an independent check of some sort. The original plan was to use a Potomac field-strength measuring receiver, just as had been done when the system was calibrated for VHF/FM measurements. Unfortunately, the minimum frequency of the receiver is 45 MHz and, worse, its two rod antennas each need to be extended to 1.7 m to make a measurement at that frequency. Clearly, this method is impracticable.

An alternative is to calculate the field from first principles. At first sight, this is a daunting prospect, but with a little perseverance it can be done. The treatment below is such an attempt. *An important limitation of the chosen method is that it is 'quasi-static':* it assumes that the electric charge on a transmission line is constant, and does not vary with time. This limitation becomes obvious when considering transmission line reflections in the walls of the screened room. Propagation times then disturb the phase relationship between the direct field generated by the transmission line and the secondary field caused by the reflection. If the dimensions of the room are great enough, the reflection could tend to cancel instead of add, and vice versa.

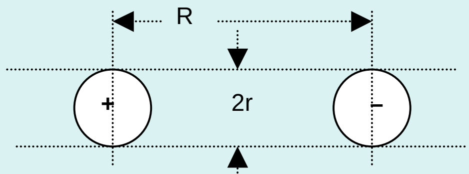
However, one could argue that the quasi TEM-cell should not be used at frequencies where the dimensions are comparable to a wavelength. What is more, the calibration results shown in Figure 3.3 do not indicate any systematic variation with frequency. The quasi-static approach seems to be worth a try.



B and C are the 'real' transmission lines, whereas A and D are their images in the ceiling and floor respectively.

Transmission-Line Parameters

For the balanced transmission line in the illustration, the capacitance C per unit length, the inductance L per unit length and the impedance Z_0 are as given below:



$$C = \frac{\pi\,\varepsilon_r \varepsilon_0}{\cosh^{-1}(R/2r)}$$

$$L = \frac{\mu_r \mu_0 \cosh^{-1}(R/2r)}{\pi}$$

$$Z_0 = \sqrt{\frac{\mu_r}{\varepsilon_r}} \cdot \frac{376\cosh^{-1}(R/2r)}{\pi}$$

$R = 0.09$, $r = 0.0075$; thus $\cosh^{-1}(R/2r) = 2.478$.

$\varepsilon_0 = 1/(36\pi \times 10^9)$, $\mu_0 = 4\pi \times 10^{-7}$; $\varepsilon_r = \mu_r = 1$.

Z_0 (the characteristic impedance) is about 300 Ω.

Figure 5.1: Transmission Lines within Pseudo TEM-cell

The diagram above illustrates the arrangement in the screened room. The actual transmission lines are B and C, but their reflections form images A and D. An alternative interpretation is that pairs A,B and C,D comprise balanced transmission lines whose characteristics are as given in the box 'Transmission Line Parameters'.[4] The observer is positioned at point X.

Let us calculate the electric field due to transmission line C. Fundamental theory states that, at distance *r* from a point charge *q*, the electric field is given by

$$\mathrm{E} = \frac{q}{4\pi\varepsilon_r\varepsilon_0 r^2}\hat{\mathrm{r}} \qquad \text{where } \hat{\mathrm{r}} \text{ is a unit vector.}^{5}$$

[4] C and D must have equal and opposite charges, and must be equally spaced from the conducting floor, if the floor is to remain at earth potential.

[5] That is, it defines the direction of **r**, but is dimensionless.

The transmission line has its charge uniformly distributed along it, and so an integration is needed to determine the field at distance h from the line:

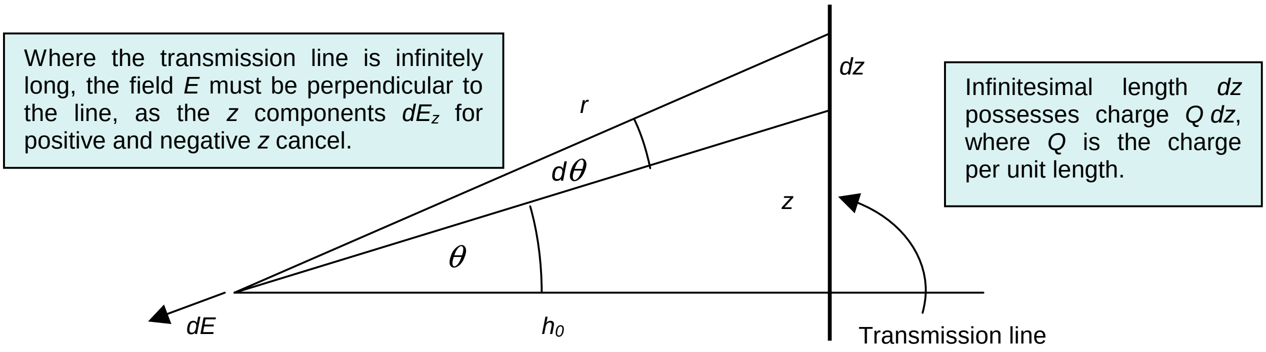


Figure 5.2: Model for Calculating the Field Due to a Transmission Line

The diagram shows that the incremental electric field dE_h in the h direction is given by

$$dE_h = \frac{1}{4\pi\varepsilon_r\varepsilon_0 r^2} Q.dz \cos\theta$$

where $Q.dz$ is the incremental charge associated with dz .

So $$dE_h = \frac{1}{4\pi\varepsilon_r\varepsilon_0} \frac{\cos^2\theta}{h_0{}^2} \frac{h_0\, d\theta}{\cos^2\theta} \cos\theta$$

$$= \frac{Q}{4\pi\varepsilon_r\varepsilon_0 h_0} \cos\theta\, d\theta$$

$$r = \frac{h_0}{\cos\theta} \qquad dz = \frac{r\, d\theta}{\cos\theta}$$

Thus $$dz = \frac{h_0\, d\theta}{\cos^2\theta}$$

Integrating, $$\int_{-\pi/2}^{\pi/2} dE_h = \frac{Q}{4\pi\varepsilon_r\varepsilon_0 h_0} \left[\sin\theta\right]_{-\pi/2}^{\pi/2} \quad = \frac{Q}{2\pi\varepsilon_r\varepsilon_0 h_0}$$.[6]

The next step is to express Q (and hence E) in terms of the capacitance per unit length C and the voltage V by using the relationship $Q = CV$:

$$E_h = C \frac{V}{2\pi\varepsilon_r\varepsilon_0 h_0} \quad = \frac{2\pi\varepsilon_r\varepsilon_0}{\cosh^{-1}(R/2r)} \frac{V}{2\pi\varepsilon_r\varepsilon_0 h_0} \ , \qquad \text{since } C = \frac{2\pi\varepsilon_r\varepsilon_0}{\cosh^{-1}(R/2r)} .$$

As $\cosh^{-1}(R/2r)$ equals 2.478 for this particular line,

$$V/\left(2.478\, h_0\right) \ .$$

Note that the value of C used above is *twice* that shown in the 'Transmission Line Parameters' box on the previous page. This is because the box gives the capacitance between the two conductors, not that between a single conductor and earth. By the same token, the impedance of the single line is 150 Ω rather than the 300 Ω given in the box:

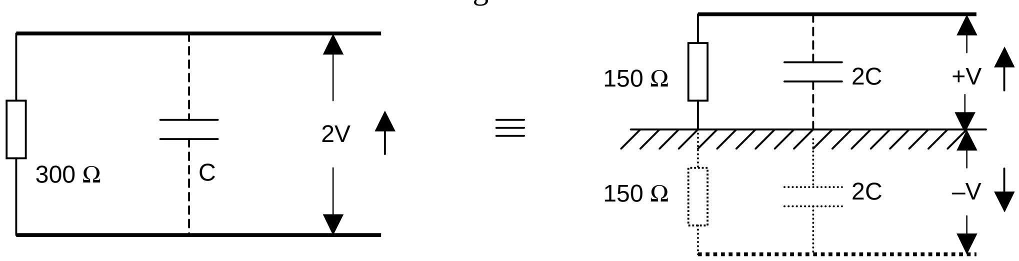


Figure 5.3: Equivalence of 300 Ω Balanced and 150 Ω Unbalanced Transmission Lines

[6] It is possible to derive this expression more straightforwardly by quoting Gauss's divergence theorem: the integral of the electric field over a surface equals the charge enclosed divided by $\varepsilon_r\ \varepsilon_0$. A cylinder of radius h_0 and unit length possesses surface area $2\pi h_0$ and encloses charge Q. Since by symmetry the field E must be constant over the surface of the cylinder and perpendicular to it, $E = Q/(2\pi\, \varepsilon_r\, \varepsilon_0\, h_0)$.

The arrangement in the pseudo TEM-cell uses three such lines in parallel, to form a 50 Ω unbalanced transmission line. Hence the final expression for the field is

$$E_h = 3\,V / \left(2.478\,h_0\right) .$$

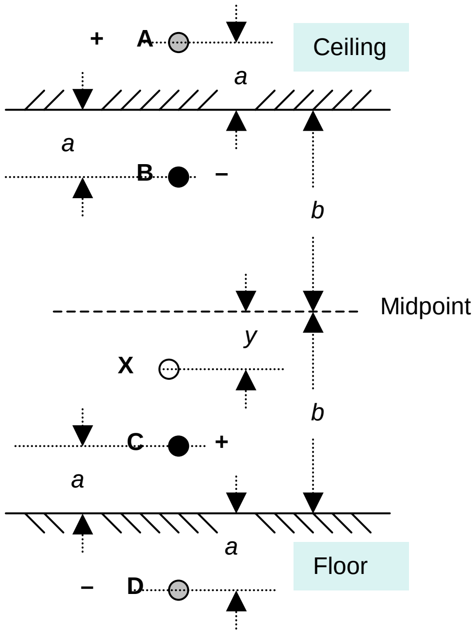


Figure 5.1: (Repeat)

Where the situation is as shown in Figure 5.1 — with two 'real' transmission lines B and C, and two images A and D — the net field is the sum of the four contributions:

$$E_X = \frac{3\,V}{2.478}\left(\frac{1}{XC} - \frac{1}{XD} + \frac{1}{XB} - \frac{1}{XA}\right) \text{, or}$$

$$E_X = \frac{3\,V}{2.478}\left(\frac{1}{b-y-a} - \frac{1}{b-y+a} + \frac{1}{b+y-a} - \frac{1}{b+y+a}\right) .$$

The negative signs arise because the image must possess the opposite charge to the real transmission line.

If a is much smaller than b ± y, a useful simplification is:

$$E_X = \frac{3\,V}{2.478}\;2a\left(\frac{1}{(b-y)^2} + \frac{1}{(b+y)^2}\right) .$$

Note that the field due to a transmission line and its image falls off as the inverse *square* of the distance, not as a simple reciprocal. Either the full expression or its approximation can now be entered into a spreadsheet, and the calculation made.

6. Calculating the Field-Strength, Stage 2: The Chamber of Mirrors

Unfortunately, the situation is more complicated than suggested by the work above, because there are further reflections, or images, to take into account. For instance, transmission line C is not only reflected in the floor to provide image D, but is also reflected in the ceiling to provide an image C'. C' will appear at a distance $2b - a$ above the ceiling, or $3b - a + y$ from X. Similarly, the primary image D will be reflected in the ceiling to give a secondary image D' at $3b + a + y$ from X.

To make matters more complicated, there will be further reflections of the images: any image appearing above the ceiling, for instance, will then be reflected in the floor. An analogy is viewing oneself in a mirror when there is a further mirror behind one. The full expression for the net field is

$$E_X = \frac{3\,V}{2.478}\;2a\left(\underbrace{\frac{1}{(b-y)^2} + \frac{1}{(b+y)^2}}_{primary} + \underbrace{\frac{1}{(3b-y)^2} + \frac{1}{(3b+y)^2}}_{1^{st}\ reflections} + \underbrace{\frac{1}{(5b-y)^2} + \frac{1}{(5b+y)^2}}_{2^{nd}\ reflections} \cdots\right) .$$

It might be possible to sum this series analytically, but the author's knowledge of mathematics is inadequate. Fortunately, for the second reflection and beyond, y in each denominator becomes negligible, and the expression reduces to

$$E_X = \frac{3\,V}{2.478}\;2a\left(\frac{1}{(b-y)^2} + \frac{1}{(b+y)^2} + \frac{1}{(3b-y)^2} + \frac{1}{(3b+y)^2} + \frac{0.245}{b^2}\right) .$$

Second- and higher-order reflections

Note that the reflection contributions are quite small: at worst, when $y = 0$, the 1^{st} reflections contribute about 1 dB, and all remaining reflections a further dB.

Reflections in the floor and ceiling are unavoidable in any pseudo TEM-cell of this type. However, 'self-inflicted wounds' are always possible, obvious candidates being reflections in a sidewall. Indeed, as the previous illustrations show, the Kingswood pseudo TEM-cell has such a sidewall at a distance of about 0.6 m from the transmission lines. The calculations are not particularly difficult, but they are tedious; hence they are presented in the Appendix 1. Fortunately, adding typical dimensions to the calculated formula shows that the effect of the reflections is small.[7]

7. Calculating the Field-Strength, Stage 3: Short Transmission Lines

So far, the work has shown that it is straightforward to calculate the 'basic' field existing between two pairs of infinite transmission lines in a pseudo TEM-cell configuration. The work has also discussed multiple reflections within the TEM-cell. Although, in theory, reflections make the problem much more complicated, in practice they can probably be ignored or estimated. However, a more serious concern is the actual length of the transmission lines, whose dimensions are likely to be comparable with their separation: in the case of the Kingswood TEM-cell, the separation is actually greater than the length.

Once again, the calculations are relegated to the Appendix, but the general principles can be understood from the diagram below:

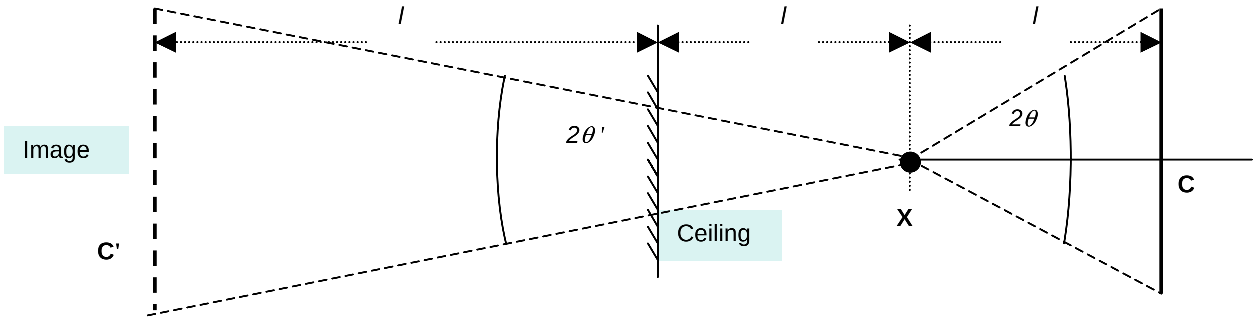


Figure 7.1: Illustration of the Angles Subtended by a Transmission Line and Its Reflection

The 'real' transmission line C subtends an angle 2θ at X. The previous calculations have used the model shown in Figure 5.2, where θ is $\pi/2$ (for an infinite line). If θ is $\pi/4$ — approximately true for the Kingswood TEM-cell — the field due to the transmission line is reduced by 3 dB. Image reflections, such as C' above, are reduced still further, as the subtended angle $2\theta'$ is less.

At first sight, ignoring the finite length of the transmission lines would seem to introduce serious errors. However, as the calculations in the Appendix show, this is not true. The transmission lines always occur as pairs; for instance, C is paired with its equal-and-opposite image D. What is measured at X is the small difference between the two large fields associated with C and D, and this is affected disproportionately by the difference in angles subtended. In fact, shortening the transmission line (within reason) does not reduce the net field significantly.

In the same way, reflections also always occur in pairs, and so their net field is not reduced as much as might be expected from the small angle $2\theta'$. However, there is *some* reduction, and this is still useful in ensuring that the already-small reflections become even more insignificant.

[7] This statement is only correct if the observer (X) is in line with the transmission lines A,B, C,D. The field falls away rapidly if X approaches the wall responsible for the reflection.

8. The Proof of the Pudding...

At long last, it is possible to compare calculated and measured values of field-strength for the Kingswood quasi TEM-cell. In the plot below, the discrete points represent the measured data, just as in Figure 4.1. The calculations take into account the following factors:

- The probe calibration: a field of 1 V/metre gives rise to an output of –45.45 dBm.
- The power supplied to each transmission line: +5.9 dBm.[8]
- The fields generated by the two 'real' and two 'image' transmission lines B,C and A,D.
- The fields resulting from the first-order reflections of the above.
- The finite length of the transmission lines and their reflections.

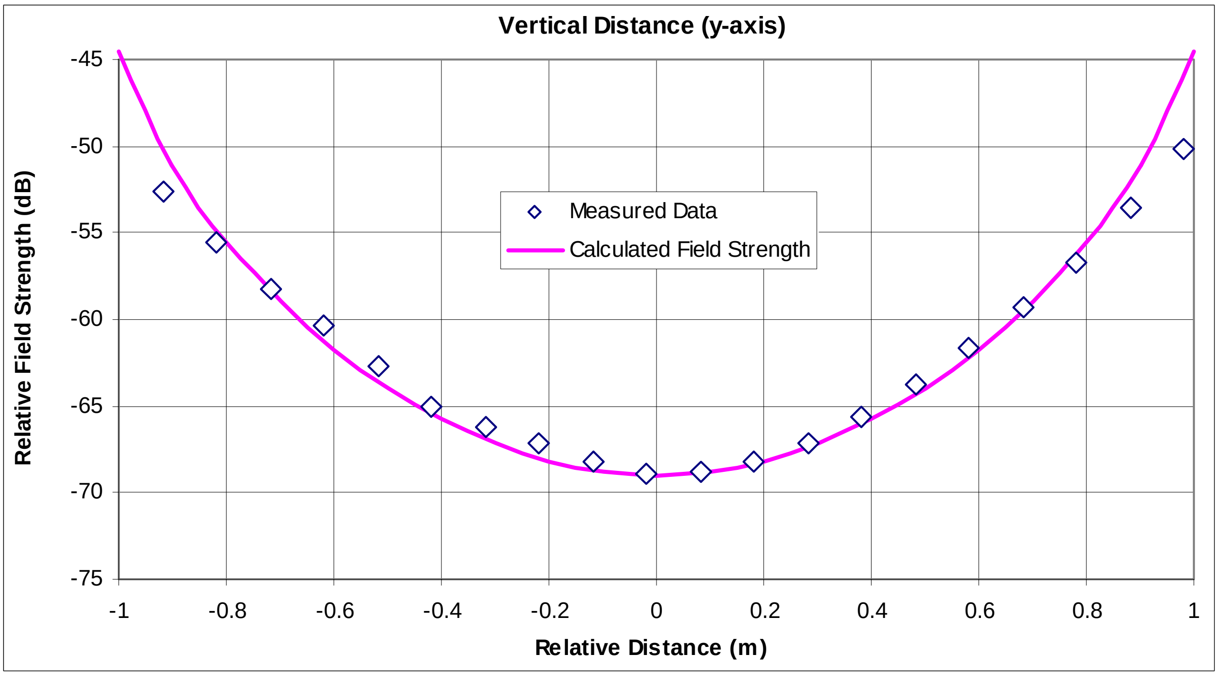


Figure 8.1: Comparison of Measured and Calculated Data

The agreement between the measured and calculated results is quite good, and mostly within the estimated 1 dB experimental error. There is some discrepancy at large distances, where the probe is close to the transmission lines. This is because the calculations assume that each line is single, whereas in fact it is a triplet, as shown in Figure 2.1. It would be straightforward to make the calculations more representative — the reader is invited to try!

It is also interesting to see how much difference the allowances for finite line length and first-order reflections make. Figure 8.2 overleaf gives the theoretical results for an infinite line complete with floor and ceiling reflections, and for a finite line without reflections. In both cases, the results are very similar to the 'true' calculations, where both reflections and finite line length are taken into account. Errors are within a dB or so.

This is a comforting result for anyone mistrustful of the previous theory. It is arguable how sophisticated the model should be made — especially as complexity carries the risk of mistakes — but a simple-minded approach generates nearly the right answers.

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[8] The generator was set to +10 dBm and the power measured at the inputs to the transmission lines. A theoretical 3 dB is lost in the splitter, the remaining 1 dB disappearing in the various cables and interconnections.

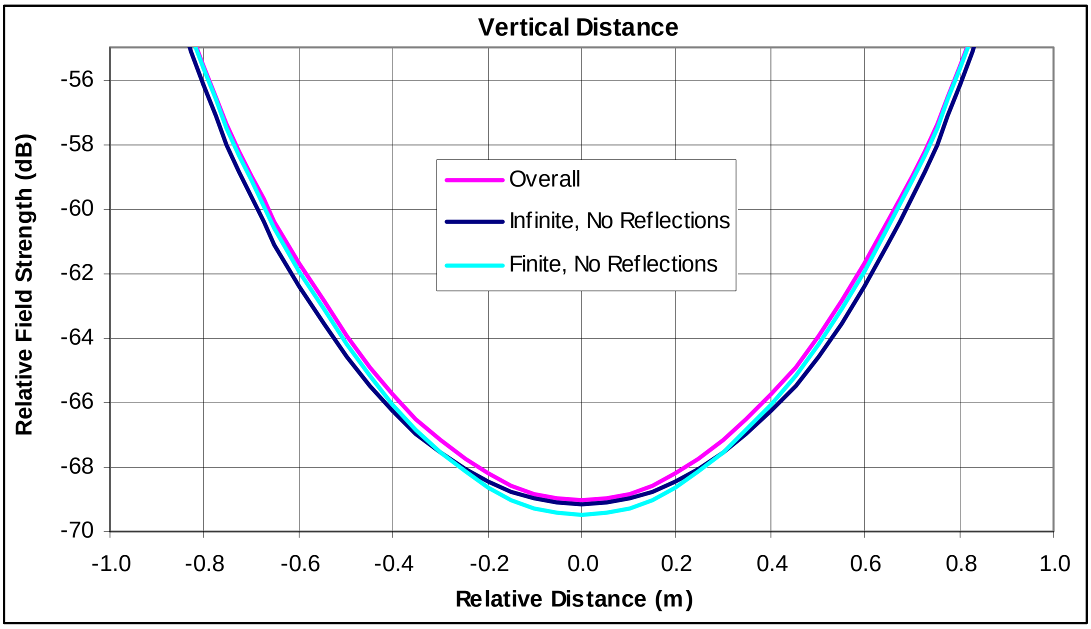

Figure 8.2: Effect of Finite Line Length and Reflections

9. Discussion and Conclusions

This report has taken a look at the calibration of pseudo TEM-cells, and shown how this can be carried out both in theory and in practice. Provided one has access to a G-TEM cell, practical calibration involves a very straightforward substitution method: it is only necessary to compare the fields within the two cells. If a G-TEM cell is not available, an oversize capacitor could be fabricated instead:

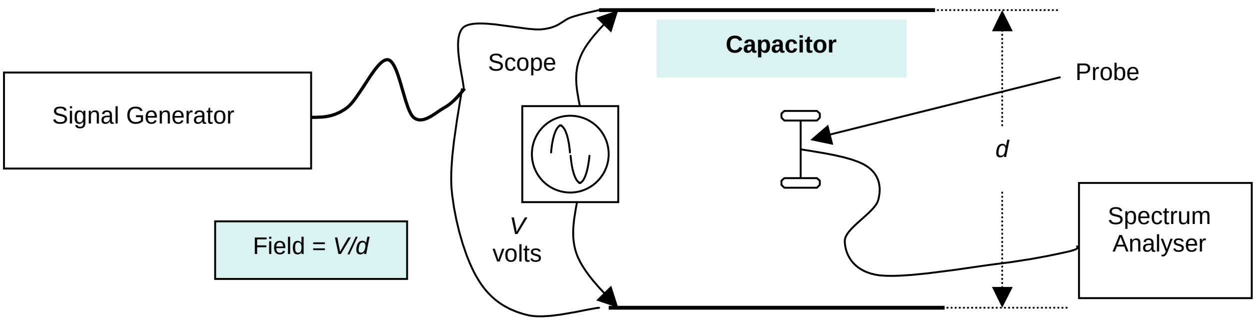

Figure 9.1: Calibrating an E-field Probe with a Capacitor

The pseudo TEM-cell has the possible advantage of generating both electric and magnetic fields, so providing an approximation to radiation in free-space. Its disadvantage is that the field is only uniform in one direction — along the transmission lines — and is not particularly uniform in the important vertical direction. If a portable radio with a 1 m telescopic antenna were to be tested in the Kingswood TEM-cell, the field would vary by at least 5 dB over the length of the antenna. Hence there would be obvious calibration uncertainties.

If only the electric field is required, it is tempting to use the oversize capacitor approach, as suggested for the calibration of the E-field probe. The field between the plates would then be sensibly uniform, and easy to calculate.

Another intriguing possibility is to use the stray field from a physically-small capacitor. As illustrated overleaf, the magnetic analogy is the H-field generated by a small loop.

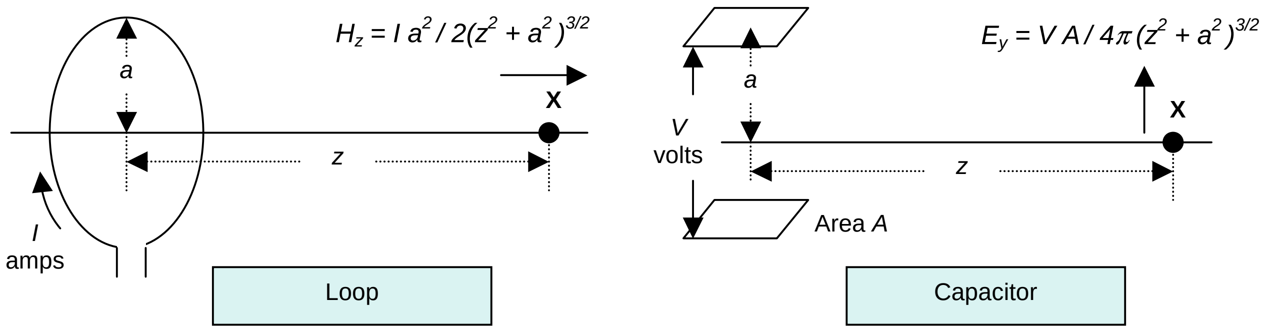

Figure 9.2: Physically-small Loops and Capacitors

It is hoped that this report has demystified the subject of pseudo TEM-cells — and, if nothing else, has offered food for thought!

10. Acknowledgements

The author would like to thank John Salter, Jonathan Stott and Nick Wells for their helpful comments, and John Sykes for commissioning this work.

Appendix 1a: The Chamber of Mirrors Elaborated

Reflections in the floor and ceiling are unavoidable in any pseudo TEM-cell of the type being discussed. However, as mentioned in Section 6, reflections in a sidewall could also be troublesome: the Kingswood pseudo TEM-cell has a sidewall about 0.6 m from the transmission lines. The situation for the lower transmission line pair (C and D) is given below:

From the previous theory, the field at X due to image C' is

$$E_{C'X} = -\frac{3\,V}{2.478\,r}$$

However, this field is in the direction CX, whereas only the vertical component is of interest. $E_{C'X}$ must therefore be multiplied by $\sin\theta_{C'}$.

$$E_{C'Xv} = -\frac{3\,V}{2.478r}\sin\theta_{C'}$$

Putting $r^2 = (b-y-a)^2 + 4c^2$ and $\sin\theta_{C'} = (b-y-a)/r$:

$$E_{C'Xv} = -\frac{3\,V}{2.478}\cdot\frac{(b-y-a)}{(b-y-a)^2+4c^2}$$

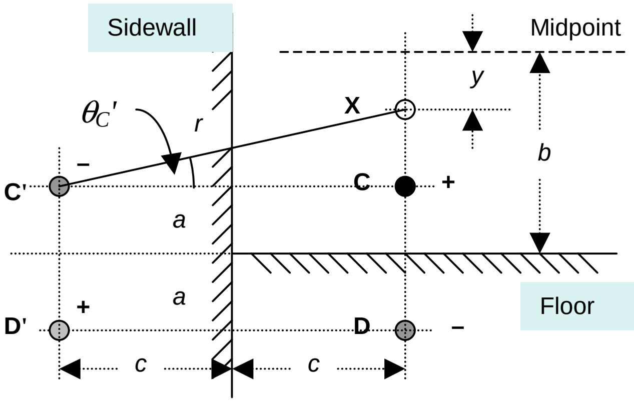

Figure A1.1: Model for Calculating the Effect of a Sidewall

The calculation for the contribution due to D' is carried out in just the same way, giving a total of

$$E_{C'Xv} + E_{D'Xv} = -\frac{3\,V}{2.478}\cdot\frac{(b-y-a)}{(b-y-a)^2+4c^2} + \frac{3\,V}{2.478}\cdot\frac{(b-y+a)}{(b-y+a)^2+4c^2}\ .$$

Where *a* (the transmission line spacing from the floor) is small, this reduces to

$$E_{C'Xv} + E_{D'Xv} = \frac{3\,V}{2.478}\cdot\frac{2a}{(b-y)^2+4c^2}\ .$$

Finally, the contributions due to A' and B' — the images of the upper transmission line — must be added:

$$E_{A'Xv} + E_{B'Xv} + E_{C'Xv} + E_{D'Xv} = \frac{3\,V}{2.478}\cdot 2a\left(\frac{1}{(b+y)^2+4c^2} + \frac{1}{(b-y)^2+4c^2}\right) \text{ (for small a).}$$

Needless to say, the complete story is even more complicated, as there are further images resulting from the higher-order reflections. Fortunately, for typical dimensions, the first-order sidewall reflections make only small contributions, and higher-order reflections are negligible.[9]

As will be discussed in Appendix 1b, 'real' pseudo TEM-cells make use of fairly short lengths of transmission line, whereas all the previous work has assumed the transmission lines to be infinite. Where the transmission lines are confined, there are further reflections in the end-walls. Calculations are trickier, because the images need to be taken as lines rather than points, and an integration is necessary in each case. However, it is normally possible to convince oneself that the 'centre of gravity' of the image is sufficiently distant for the associated field to be negligible.

[9] The approximate formula applicable when *a* is small is actually pessimistic. If a is taken fully into account, the sidewall contributions tend to cancel, and the total can even change sign.

Appendix 1b: Using Short Transmission Lines

According to Section 5, the field due to an infinite transmission line carrying charge Q per unit length is as follows:

$$\int_{-\pi/2}^{\pi/2} dE_h = \frac{Q}{4\pi\varepsilon_r\varepsilon_0 h_0}\left[\sin\theta\right]_{-\pi/2}^{\pi/2}$$

In terms of the voltage V on the line, this becomes

$$E_h = \frac{V}{4\cosh^{-1}(R/2r)}\left[\sin\theta\right]_{-\pi/2}^{\pi/2} ,$$

where R and r represent the transmission line dimensions as before. For the 50 Ω line used at Kingswood, this equation becomes

$$E_h = \frac{3V}{2\times 2.478\,h_0}\left[\sin\theta\right]_{-\pi/2}^{\pi/2} .$$

This is simply the original equation

$$E_h = \frac{3V}{2.478\,h_0}$$

with the integration limits $\pm\pi/2$ retained.

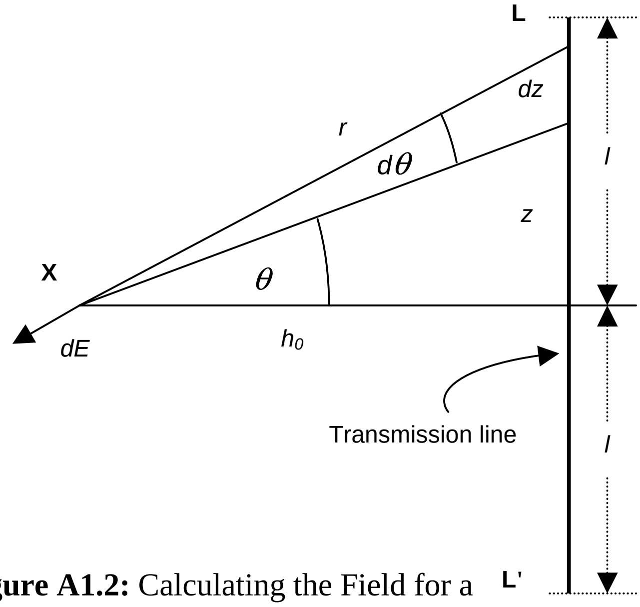

Figure A1.2: Calculating the Field for a Finite Transmission Line

For the short transmission line, it is only necessary to replace the limits with

$$\theta = \pm\sin^{-1} l/r \; = \pm\sin^{-1} l\Big/\sqrt{(l^2 + h_0{}^2)} \; .$$

Where $l = h$, as is approximately the case for the Kingswood pseudo TEM-cell, half-way between the floor and the ceiling, $\sin\theta = 1/\sqrt{2}$, and so the field is 3 dB below that for infinite lines.

The actual pseudo TEM-cell comprises four transmission lines — two 'real' ones B and C, and two images A and D. In each case, the contribution to the overall field E_X must be multiplied by the appropriate value of $\sin\theta$:

$$E_X = \frac{3V}{2.478}\left(\frac{\sin\theta_{LXC}}{XC} - \frac{\sin\theta_{LXD}}{XD} + \frac{\sin\theta_{LXB}}{XB} - \frac{\sin\theta_{LXA}}{XA}\right) \text{, or}$$

$$E_X = \frac{3V}{2.478}\left(\frac{l\Big/\sqrt{((b-y-a)^2+l^2)}}{b-y-a} - \frac{l\Big/\sqrt{((b-y+a)^2+l^2)}}{b-y+a} + \frac{l\Big/\sqrt{((b+y-a)^2+l^2)}}{b+y-a} - \frac{l\Big/\sqrt{((b+y+a)^2+l^2)}}{b+y+a}\right) .$$

If the results of this analysis are plotted, an interesting point to emerge is that E_X is not greatly affected by the finite length of the transmission lines, even though the individual contributions are. In other words, adding the $\sin\theta$ factors does not appreciably change E_X. The reason is that E_X is made up of the small differences between much larger quantities: the fields due to A and B are nearly the same, as are those due to C and D. Hence the slight differences in the associated $\sin\theta$ factors have a disproportionate effect.

Appendix 2: The Magnetic Field Generated within the Pseudo TEM-Cell

The work discussed in this report has been concerned with the electric field (E) within the pseudo TEM-cell. However, there could also be interest in the magnetic field (H), as HF receivers could in principle use rod or loop antennas. It is fairly obvious that the transmission lines will generate some sort of magnetic field, as current is flowing in them. It might also be obvious that E/H will equal a characteristic impedance Z_0, but does Z_0 refer to the characteristic impedance of the transmission line or to free space? (Free space would be the preferred candidate.) A calculation is needed!

The calculation of magnetic fields is very similar to that for electric fields. (See Section 5.) Assume a current I flows around the transmission line pair AB:

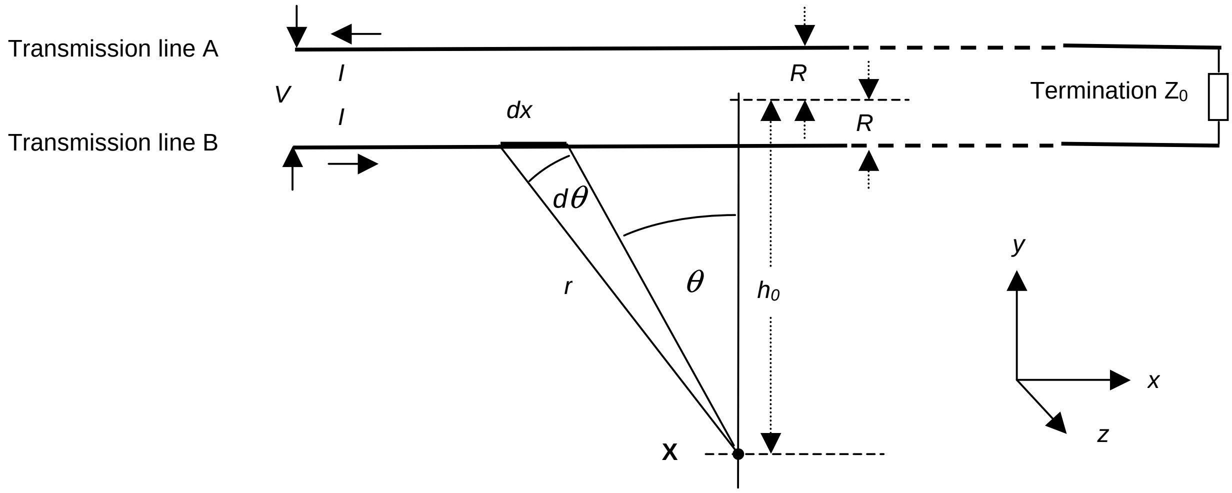

Figure A2.1: Model for Calculating the Magnetic Field Due to a Transmission Line

The incremental magnetic field dH_z in the z direction is given by

$$dH_z = \frac{I}{4\pi r^2}\,\hat{\mathbf{r}}\times\mathbf{dx}\ , \qquad \text{where } I \text{ is the current flowing through } dx.$$

Hence

$$dH_z = \frac{I}{4\pi r^2}\,\cos\theta\, dx\ ,$$

$$= \frac{I}{4\pi}\frac{\cos^2\theta}{(h_0 \pm R)^2}\cos\theta\,\frac{(h_0 \pm R)d\theta}{\cos^2\theta}$$

$$= \frac{I}{4\pi(h_0 \pm R)}\cos\theta\, d\theta$$

$$\cos\theta = \frac{(h_0 \pm R)}{r}\text{, or}\quad r = \frac{(h_0 \pm R)}{\cos\theta}$$

$$dx = \frac{r\,d\theta}{\cos\theta}\ ,\quad \text{or}\quad dx = \frac{(h_0 \pm R)d\theta}{\cos^2\theta}$$

Integrating,

$$\int_{-\pi/2}^{\pi/2} dH_z = \frac{I}{4\pi(h_0 \pm R)}\left[\sin\theta\right]_{-\pi/2}^{\pi/2}$$

$$= \frac{I}{2\pi(h_0 \pm R)}$$

The distance ($h_0 + R$) represents the upper transmission line A, and ($h_0 - R$) the lower transmission line B. The overall magnetic field is the sum of the contributions from A and B:

$$H_z = \frac{I}{2\pi(h_0 - R)} + \frac{I}{2\pi(h_0 + R)} \quad = \frac{I}{2\pi}\frac{2h_0}{(h_0^2 - R^2)}$$

Since $V/I = Z_0$ (the transmission line characteristic impedance),

$$H_z = \frac{V}{\pi Z_0}\frac{h_0}{(h_0^2 - R^2)}\ .$$

As shown in Section 5, the electric field E_x is given by

$$E_x = \frac{Q}{2\pi\varepsilon_r\varepsilon_0\left(h_0 - R\right)} + \frac{Q}{2\pi\varepsilon_r\varepsilon_0\left(h_0 + R\right)} \quad = \frac{Q}{\pi\varepsilon_r\varepsilon_0}\,\frac{h_0}{\left(h_0^{\,2} - R^2\right)} \ ,$$

where Q is the charge per unit length. Since $Q = CV$, where C is the capacitance per unit length, and C is given by the equation

$$C = \frac{\pi\varepsilon_r\varepsilon_0}{\cosh^{-1}\left(R/2r\right)} \ ,$$

$$E_x = V\,\frac{\pi\varepsilon_r\varepsilon_0}{\cosh^{-1}\left(R/2r\right)}\cdot\frac{h_0}{\pi\varepsilon_r\varepsilon_0\left(h_0^{\,2} - R^2\right)} \ .$$

This gives

$$\frac{E_x}{H_z} = \left\{\frac{V}{\cosh^{-1}\left(R/2r\right)}\cdot\frac{h_0}{\left(h_0^{\,2} - R^2\right)}\right\}\Bigg/\left\{\frac{V}{\pi Z_0}\,\frac{h_0}{\left(h_0^{\,2} - R^2\right)}\right\}$$

$$= \frac{\pi Z_0}{\cosh^{-1}\left(R/2r\right)} \ .$$

Since Z_0 for a transmission line is given by

$$Z_0 = \frac{\cosh^{-1}\left(R/2r\right)}{\pi}\sqrt{\frac{\mu_r\mu_0}{\varepsilon_r\varepsilon_0}} \ ,$$

$$\frac{E_x}{H_z} = \sqrt{\frac{\mu_r\mu_0}{\varepsilon_r\varepsilon_0}} \ .$$

This is simply the impedance of the medium within the TEM-cell. For air, μ_r and ε_r are closely equal to 1, and E_x/H_z equals 376 Ω, the impedance of free space.

Note that, although the characteristic impedance of the transmission line does not determine the ratio of E_x to H_z, the transmission line must be accurately terminated. If, for instance, the line is being used at low frequencies and is left unterminated, no current will flow through it, and there will therefore be no H_z.