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Echoes, Doppler and DVB-T receivers: some theory and practice

R.H.M. Poole

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Echoes, Doppler and DVB-T Receivers:
Some Theory and Practice

Ranulph Poole

Abstract

This report provides a summary of the work carried out at BBC R&D on the subject of DVB-T receivers and the effects of Doppler. The aim of the work was to extend the now familiar 'noise bucket' concept to include Doppler-related contributions.

Only the old 64 QAM 'UK Modulation Mode' is considered. This is partly because a mode change was not contemplated at the time the work was carried out, and partly because 64 QAM is relatively sensitive to Doppler. The effects of Doppler would not have been so easy to measure with 16 QAM or QPSK.

The receiver chosen for the measurements included a 'first generation' demodulator 'chip' of the type still used in many domestic receivers. Although modern chips could offer better performance, their more advanced features would have made analysis of the results more difficult.

The conclusion is that, where Doppler is present, equivalent noise contributions arise from linear interpolation, loss of orthogonality, and filtering of the channel state information. It is possible to calculate the first two of these satisfactorily, but the third needs to be demonstrated by experiment.

The report was prepared for the information of any person or organisation interested in this aspect of DVB-T performance.

Key Words

DVB-T, multipath, Doppler, channel equaliser, noise bucket.

White Papers are distributed freely on request.

Authorisation of the Chief Scientist is required for publication.

1. Introduction

Because of the increasing interest in mobile reception of digital television, BBC R&D has been carrying out work on the behaviour of DVB-T receivers in time-varying, or Doppler, channels. Provided that a multipath simulator is available, making measurements is not difficult. One can, for instance, introduce a 0 dB echo into the system and determine the minimum carrier-to-noise ratio needed for the receiver to provide an error-free output. One can then add a Doppler shift to the echo, and measure the change in carrier-to-noise requirement. If the Doppler shift is great enough, errors will be present even in the absence of added noise.

Such measurements generally show that the effect of Doppler increases rapidly with Doppler frequency. The difficulty lies in understanding why, and hence developing a suitable model for predicting Doppler performance. Were such a model to exist, it would be possible to say whether a particular receiver was working as well as it should do. It might also be possible to devise ways of improving the performance.

The approach adopted in this report is to adapt the now familiar 'noise bucket' model of DVB-T system performance. In essence, the model assumes that a receiver will fail if the amount of Gaussian noise at the input exceeds a certain level relative to the wanted signal. The noise can be 'real' thermal noise, or it can comprise, at least in part, the 'equivalent' noise of the various system impairments. Hence system impairments decrease the volume of the bucket that is available for thermal noise. In other words, the minimum carrier-to-noise requirement is greater.

The present challenge is to quantify the noise equivalent of a Doppler-shifted echo. At first sight, the problem is intractable; but the same was true for the earlier problem of calculating the equivalent noise of amplitude response errors. All that is necessary is some careful thought!

2. Echoes and the Channel Response

The DVB-T signal comprises a large number of carriers, each of which is digitally modulated at a low data rate defined by the symbol period. When multipath is present, the relative amplitudes and phases of the carriers are distorted. In the case of a single static echo of power –10 dB, say, the amplitude response will vary approximately sinusoidally across the channel. This is easy to demonstrate mathematically, as follows.

Suppose carrier m of the ensemble has frequency ω_m and amplitude a, and that an echo of amplitude b and delay τ is present. In the absence of any Doppler shift, the resultant signal at time t will be given by

$$r \sin \{\omega_m t + \varphi\} = a \sin \omega_m t + b \sin \{\omega_m (t + \tau)\}.$$

The amplitude r of the resultant is then:

$$r = \sqrt{\{a^2 + b^2 + 2b \cos \omega_m \tau\}},$$

$$\approx \sqrt{(a^2 + b^2)} + b \cos \omega_m \tau \text{ , according to the binomial expansion.}$$

This varies as a function of carrier frequency and echo delay, with maxima occurring when $\omega_m \tau$ is an integral multiple of 2π.

If the echo now suffers a constant Doppler shift of frequency $\delta\omega_m$, the resultant signal becomes

$$r \sin \{\omega_m t + \varphi\} = a \sin \omega_m t + b \sin \{(\omega_m + \delta\omega_m) (t + \tau)\}.$$

The expression for the resultant signal is the same, except that $(\omega_m + \delta\omega_m) (t + \tau)$ has replaced $\omega_m (t + \tau)$; in other words, $\omega_m t + \omega_m \tau$ has become $\omega_m t + \omega_m \tau + \delta\omega_m t + \delta\omega_m \tau$.

The term $\delta\omega_m t$ shows that there is a time dependence in addition to the original frequency dependence. Any one carrier now has an amplitude that varies as $\sqrt{\{a^2 + b^2 + 2b \cos \delta\omega_m t\}}$, and maxima occur at intervals corresponding to the reciprocal of the Doppler frequency $\delta\omega_m / 2\pi$. A '3-D' diagram illustrates this:

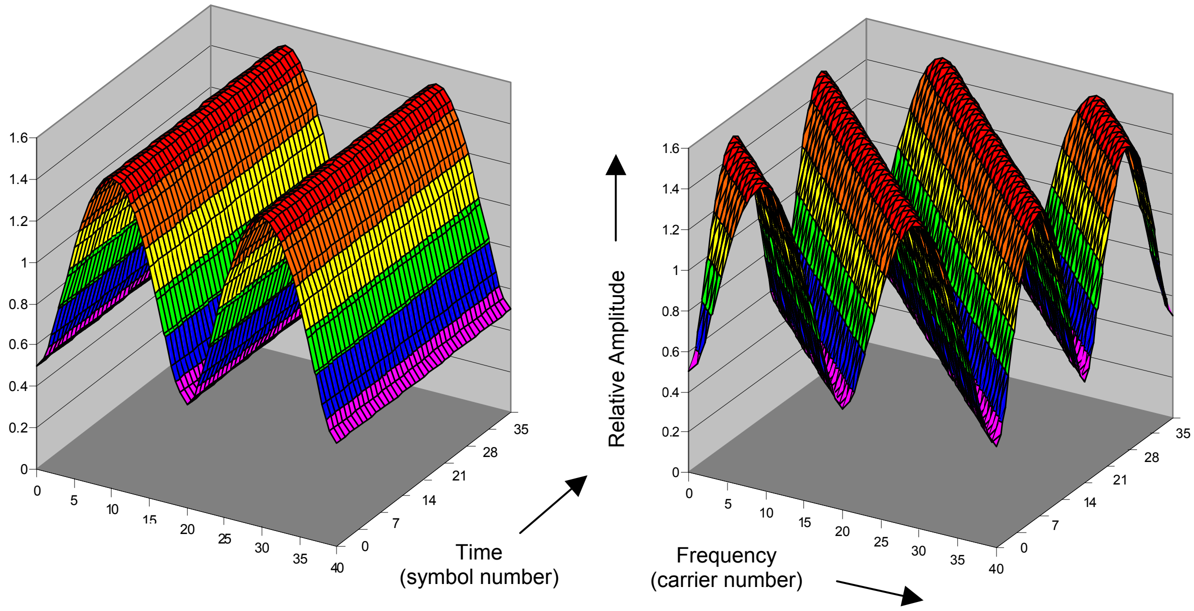


Figure 2.1: Illustration of the Effect of Doppler on the Channel Response

In the left-hand picture, the echo does not possess a Doppler shift, and the carrier amplitudes only vary with frequency: there is no time variation. With Doppler present, the result is shown in the right-hand picture, where the channel response appears to move diagonally across the time-frequency plane. The same component as before is projected on to the amplitude-frequency plane, but a component is also projected on the amplitude-time plane.

The situation from the point of view of a single carrier is shown in a vector diagram:

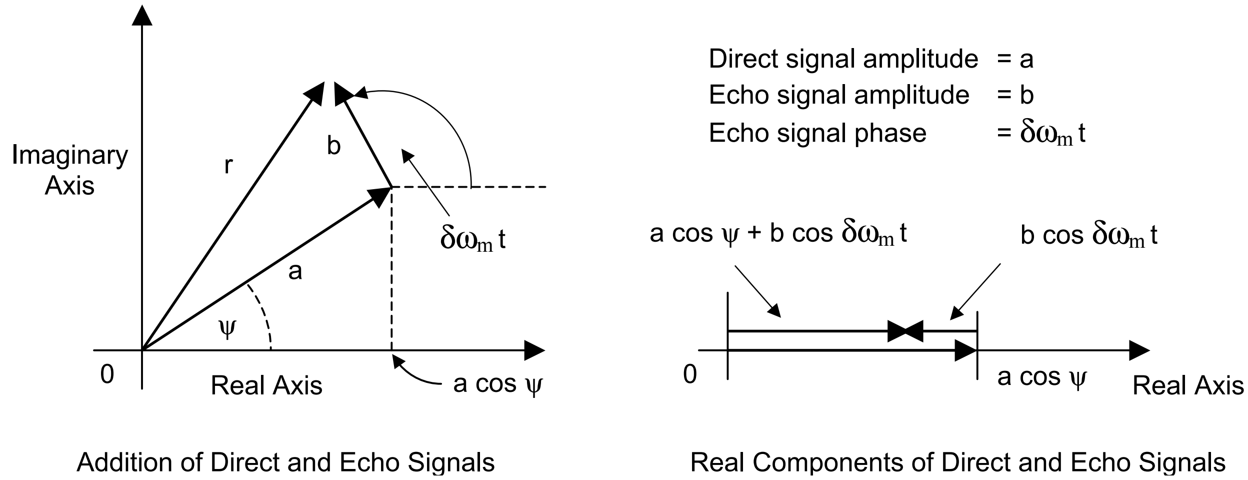


Figure 2.2 Addition of Direct and Echo Signal Vectors

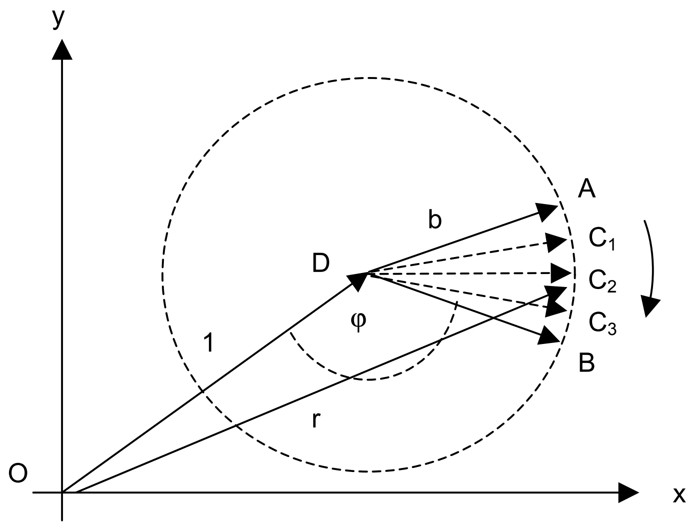
For convenience, the direct signal vector is taken as stationary, whilst the echo vector rotates once per period of $2\pi/\delta\omega_m$. Note that, although the behaviour of r is relatively complicated, the projection of r on to either the real or imaginary axis varies sinusoidally with time. This fact is useful when calculating the system failure point.

3. Doppler and Equivalent Noise

Before the distorted ensemble can be demodulated, it must be equalised. Equalisation is possible because the DVB-T signal incorporates 'scattered pilots': at any one time, one in twelve of the carriers possesses a reference amplitude and phase. Interpolation allows an appropriate correction to be made to the intermediate carriers. In fact, the situation is rather more complicated in that the pilots drift through the ensemble, changing positions at intervals of a symbol period. The pattern repeats every four symbols, and one in three of the carriers is a pilot at some time.

The net effect is that, in the absence of time variations, accurate channel equalisation is possible. Careful filtering of the pilot information allows 'perfect' equalisation for echo delays as great as one third of the symbol period, or about 70 μs. Where time variations are present, one in four symbols can be equalised perfectly, but some assumptions have to be made about the intermediate symbols.[1] The simplest thing to do is to adopt *linear interpolation*.

The diagram below is a repeat of Figure 2.2, but showing how the echo signal vector, and hence the resultant signal vector, varies as a function of time. Each unit of time is taken to be one symbol period.



The direct signal is represented by vector OD, of unity length, whilst the echo is represented by the vector of length b. Because of the Doppler shift, the tip of the echo vector describes a circle. In the course of four symbols, each of duration T_s, it moves between A and B.

Figure 3.1: Illustration of the Time Variation of a Single Carrier

Perfect equalisation is possible when the echo vector is in positions DA and DB, since the carrier is then carrying pilot information. However, there is no such information during the intermediate symbols when the echo is at DC_1, DC_2 or DC_3. The interpolator assumes, wrongly, that C_1, C_2 and C_3 lie on the chord AB. Calculating the error, or noise component, is not difficult, and is carried out in Appendix 1. Appendix 1 shows that the noise power relative to the total signal power is given by

$$P_{INT} = (17/8) \{b^2 / (1 - b^2)\} (2\pi\, \delta f_d\, T_s)^4 .$$

In other words, the noise increases with the fourth power of the Doppler frequency δf_d, and becomes very large as b^2 approaches 1 (a 0 dB echo).

A word of warning is necessary. Although the calculation is essentially correct, the interpolation noise is not even approximately Gaussian; hence it cannot be assumed that the noise has the same destructive power as true thermal noise.

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[1] In principle, 'perfect' temporal filtering could also be provided. Unfortunately, to do so would involve storing a large amount of information, and would make the demodulator prohibitively expensive to manufacture.

Equalising the ensemble with more or less precision is not the end of the story, however. An obvious feature of the Doppler-shifted echo component is that it is at the wrong frequency. The result of this error is 'loss of orthogonality' and consequent *intercarrier interference* (ICI). A detailed calculation of ICI is given in Appendix 2, but the essential arguments are as follows.

Suppose that the carrier frequencies are ω_m, ω_{m+1}, ω_{m+2} and so forth. The effect of the demodulation process is to multiply each carrier by a synchronous local oscillator signal, and then to integrate the result over the active symbol period. Consider the demodulation of the first of these carriers if the frequency is in error by $\delta\omega_m$. Before integration, the demodulator output is

$$V_{d1} = \cos \omega_m t \,.b \cos (\omega_m+\delta\omega_m)t ,$$

$$= (b/2) \{\cos (2\omega_m+\delta\omega_m)t + \cos \delta\omega_m t \} .$$

If V_{d1} is integrated over the active symbol period T_u, the output becomes

$$V_{i1} = \int k \, V_{d1} \, dt .$$

$$\approx \{k \, b \sin \delta\omega_m T_u/2\}/\delta\omega_m .$$

Thus, if the frequency of the first carrier is correct, V_{i1} equals $kT_u /2$. On the other hand, an increasing frequency error causes the output to fall away in accordance with the well-known 'sinc' function (sin x) / x, where x is $(\delta\omega_m T_u/2\}$.

The *power* associated with this carrier at the demodulator output is proportional to the square of the sinc function. Since the total power present is not affected by the frequency error, the implication is that the 'missing' power is appearing as noise on the remaining carriers. Thus the ICI component varies as $(1 - \text{sinc}^2 x)$. In other words,

$$P_{ICI} = b^2 \{1 - \text{sinc}^2 \, \pi \, \delta f_d \, T_u \} ,$$

where $\delta\omega_m / 2\pi$ has been written as f_d . This approximates to $(1/3) (\pi \, \delta f_d \, T_u)^2$ at low Doppler frequencies.

The derivation of this formula has assumed that the frequency of the direct signal is unaffected by the Doppler shift. In practice, the automatic frequency control (AFC) in the receiver will take some sort of average of the direct and Doppler shifted frequencies. For instance, if the direct and echo signals are of equal level, half the Doppler shift will transfer to the direct signal and half will remain on the echo. On the assumption that the shift of the direct signal is proportional to the amplitude of the echo, the expression for the ICI becomes

$$P_{ICI} = \{(1/3) (\pi \, \delta f_d \, T_u)^2 \} \{2b^2 / (1 + b)^2 \} .$$

It is interesting that the AFC action is helpful for large echoes: in the case of a 0 dB echo, sharing the Doppler shift with the direct signal halves the total P_{ICI}. However, where the echo is small, P_{ICI} is doubled.

Finally, there is a 'third contribution' which Doppler introduces to the total system noise. An important feature of the DVB-T system is the forward error correction — the addition of redundancy to the transmitted data. The Viterbi decoder in the receiver makes use of the redundancy to reconstruct the original data stream when errors are present. However, to do this efficiently, the decoder needs to know the reliability of the data associated with each carrier; that is, the *channel state information* (CSI). The problem arises because the CSI must be filtered, or averaged, to reduce the effect of thermal noise. Where Doppler is present, the filtering has the unhelpful consequence of making the CSI unreliable: the CSI contains unwanted history. Calculation of the third contribution is difficult — at least for this author! — but measurement is possible, as will be seen.

4. Doppler and the Noise Bucket

The 'noise bucket' concept has been used extensively in previous discussions on DVB-T system impairments. Since the concept is so fundamental to the understanding of system performance, Reference **[1]** devotes much energy to the subject. As mentioned in the Introduction, the size of the noise bucket, Q, corresponds to the minimum amount of Gaussian noise that results in 'system failure'. The convention adopted in this Report is to define Q as being relative to the power of the direct signal path — not the total power of the direct and echo paths.

'System failure' is perhaps a misnomer, because it more convenient to use a 'reference bit error ratio' (BER_{REF}) instead. BER_{REF} is generally taken as 2×10^{-4} following the Viterbi decoder. Such a value results in the receiver providing a clean output, since its Reed-Solomon decoder will remove the remaining errors. The BER would have to be substantially worse for errors to emerge.

Where a Doppler-shifted echo is present, the calculated noise bucket contributions are as follows:

Interpolation errors

The noise increases as the fourth power of the Doppler frequency. The full expression for the predicted interpolation noise, as derived in Appendix 1, is

$$P_{INT} = (17/8)\ \{b^2 / (1 - b^2)\}(2\pi\ \delta f_d\ T_s)^4\ ,$$

where b^2 is the relative echo power, δf_d is the Doppler frequency, and T_s is the overall symbol period. However, because P_{INT} is non-Gaussian, the derivation cannot say how much of this noise is necessary to achieve BER_{REF}. A way of overcoming the problem is to calculate the Doppler frequency for failure, δf_{fail}, and then to take the interpolation noise as

$$P_{INT} = Q\ (\delta f_d / \delta f_{fail})^4\ .$$

Calculation of δf_{fail} is not trivial, but was achieved during the course of the investigation. The surprising result is that δf_{fail} is generally close to the value that would be expected for truly Gaussian noise. This rule breaks down for large echo powers and less robust modulation modes (64 QAM). Unfortunately, it does not follow that P_{INT} will combine linearly with the other noise contributions in the noise bucket.

Intercarrier interference resulting from the loss of orthogonality was calculated in **[1]** as

$$P_{ICI} = \{(1/3)\ (\pi\ \delta f_d\ T_u)^2\}\ \{2b^2 / (1 + b)^2\}\ ,$$

where T_u is the useful part of the symbol period; that is, the overall period T_s minus the guard interval.[2] It should be possible to add this quantity directly to the noise bucket, since the interference is AWGN to all intents and purposes.

The ***'third contribution'*** resulting from the Viterbi decoder being fed with out-of-date information is difficult — perhaps impossible — to calculate analytically. For the purposes of the following noise bucket illustration, it can be thought of as part of the 'system noise'. When the experimental results are presented, the 'third contribution' can be isolated by the simple expedient of showing what happens if the CSI filtering is altered.

Apart from the 'special' noise bucket contributions arising from Doppler distortion, there are still the 'normal' ones associated with static echoes and other system impairments. These are summarised overleaf; Reference **[1]** gives further details.

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2 For the UK modulation mode, the difference between T_u (224 µs) and T_s (231 µs) is largely academic.

Channel difficulty is associated with the presence of echoes, and is a consequence of an uneven channel response. In essence, response variations increase the BER for a given amount of AWGN. The equivalent noise depends somewhat on the implementation of the demodulator, but approximates to

$$R = 0.0093\, b^2 .$$

An important point is that R, for a given echo, is only a fixed quantity if the noise bucket is full. This is generally true in experimental work, since impairments are measured indirectly by topping up the noise bucket with AWGN until it overflows. The difference between Q and the amount of 'top-up' AWGN represents the equivalent noise of the impairment. However, channel difficulty is really a noise *multiplier* — it makes any existing noise in the channel worse. If there is no existing noise, then R is zero.

The nature of R results in a complication when interpolation noise is present. In the absence of other impairments, the failure point is reached when P_{INT} equals Q. Because the calculation of P_{INT} automatically takes into account the channel response — or, rather, because P_{INT} is the result of the channel response — there is no associated R component. If P_{INT} fills the bucket, R is zero. On the other hand, if only AWGN is present at the failure point and P_{INT} is zero, the channel difficulty has maximum effect. In other words,

$$R = 0.0093\, b^2 (Q - P_{INT}) / Q .$$

R is reduced by an amount equivalent to the fraction of the bucket occupied by P_{INT} .

Echo noise results from the fraction of the echo existing outside the guard interval. This fraction is uncorrelated with the direct signal and appears as Gaussian noise. Echo noise is not relevant to the present Report, as only short-delay echoes are considered. However, where longer delay echoes are present, echo noise often dominates.

System noise, S, refers to any other noise introduced by system imperfections. Examples include true thermal noise and phase noise introduced by local oscillators. As mentioned before, S can include the Doppler 'third contribution' for the purposes of illustration.

Gaussian noise, A, is the additional noise required to top up the noise bucket once all the other contributions are there. This is given by

$$A = Q - \{P_{INT} + P_{ICI} + R + S + \ldots\} .$$

The noise bucket diagram finally appears as below:

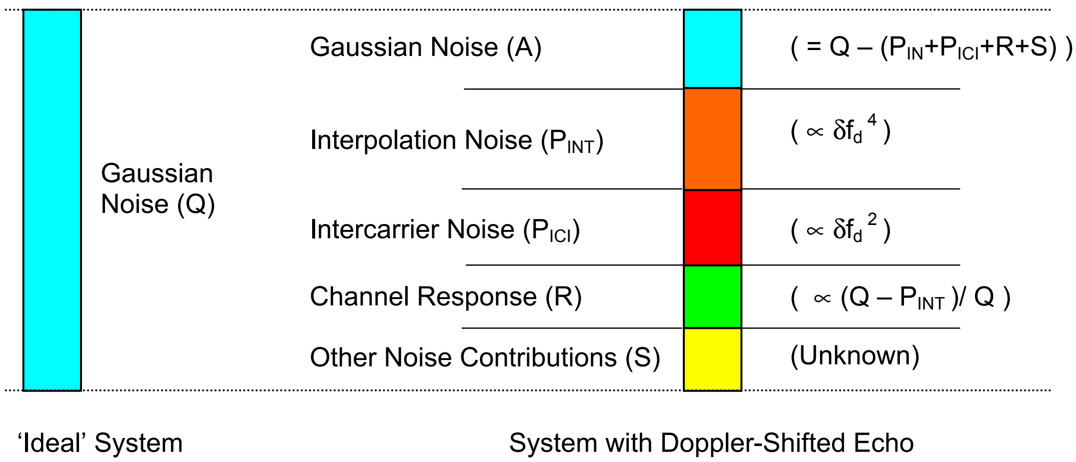

Figure 4.1: Noise Buckets Associated with a Doppler-Impaired DVB-T System

In a practical experimental system, Q is determined by increasing the amount of AWGN in an otherwise unimpaired channel until BER_{REF} is achieved. Q is then simply the ratio of noise power (N) to signal power (C). When another impairment is introduced, its noise equivalent is not usually measured directly. Instead, the change in N required to maintain BER_{REF} is found — in other words, the equivalent noise degradation (END). The equivalent noise floor (ENF) is deduced by means of the relationship

$$ENF = Q \{ 1 - 1/ END \} .$$

The channel difficulty (R) can be measured in this way, provided that the Doppler shift is zero. When Doppler is present, R must be reduced by a fraction P_{INT}/Q, for the reason discussed above.

5. Experimental Arrangements

The calculations must now be tested against reality. The standard experimental arrangements are shown below:

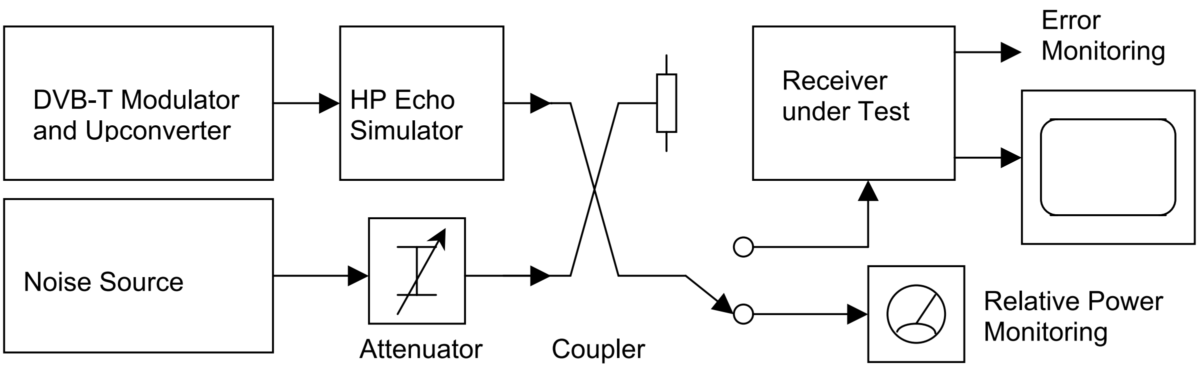

Figure 5.1: Experimental Arrangements for Measuring Equivalent Noise Degradation

The DVB-T signal is generated by a standard modulator and upconverter, and passed through the echo simulator. The simulator can add up to five independent echo paths, each with Doppler if required. Gaussian noise is added to the DVB-T signal, and the composite signal fed to the receiver under test.

The system is calibrated by generating a 'clean' DVB-T signal, with the echo paths disabled, and adjusting the noise attenuator until the receiver output reaches the appropriate BER. Under these conditions the ratio of noise power to DVB-T power corresponds to the size of the noise bucket. One or more echo paths are then introduced and the noise attenuation increased until the same BER is restored.[3] The difference in attenuator settings (in dB) represents the equivalent noise degradation (END) of the system. As stated in the previous Section, the equivalent noise floor (ENF) is readily calculated from the END. Note that the usual convention is to relate the END to the *total* signal power, and not to the power of the direct signal alone. To avoid confusion, the END determined in the way just described will be designated END_{DIR}.

Where not all the echo paths of the simulator are needed, the test arrangements can be made even simpler. One path is set to the maximum possible delay of 186 μs. Receivers are not capable of equalising such a channel, and so all the echo power appears as Gaussian noise. In effect, the simulator contains a built-in noise source, and the external noise source is not necessary. System calibration is easy, since the simulator provides a direct read-out of relative echo (or noise) power.

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[3] It is theoretically possible that the attenuation will need to be reduced if a more robust modulation mode is used; that is, the channel difficulty noise could be negative!

6. Measured Results for Single Echoes

When prediction and measurement are to be compared, the Reader is requested not to expect perfect agreement — the situation is too complicated for that!

'Single echo' measurements were performed on a 'first generation' DVB-T receiver. Although the performance of this was known to be less than ideal, it had the virtue of not containing any esoteric mechanisms that might make the results difficult to interpret. It was also convenient to use, and representative of many domestic receivers still in use. The measurements are shown in the plots below as squares, whilst the calculated noise contributions — and the sum of the contributions — appear as continuous curves. If all the contributions have been correctly taken into account, their sum should correspond to the measured total noise.[4]

Also seen in the plots are the 'predicted' and 'measured' failure points. The predicted failure point is equivalent to Q, and is the noise power giving rise to BER_{REF} in an unimpaired channel; whilst the measured failure point is the Doppler frequency giving rise to BER_{REF} in a noise-free channel.

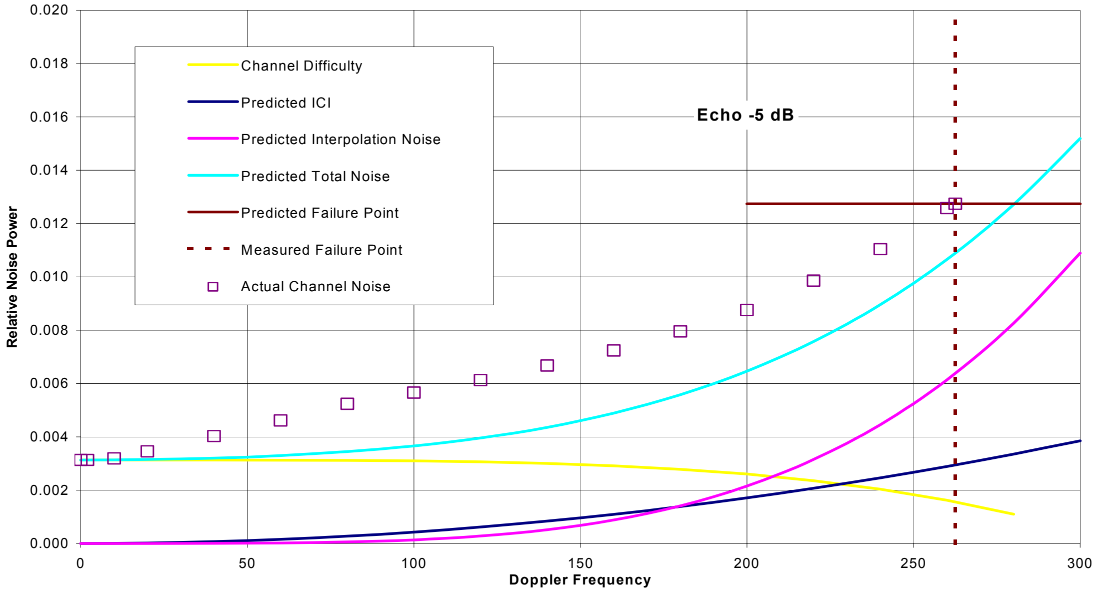

Figure 6.1: Noise Contributions for a –5 dB Echo

When presented with a 'moderate' –5 dB echo, the receiver fails at only slightly less than the Doppler frequency predicted by the model. However, the equivalent noise increases more rapidly than expected at low Doppler frequencies, and is nearly double the calculated value at 50 Hz. The additional noise can be attributed to the 'third contribution' associated with the CSI filtering.

The results for a 'large' –1 dB echo, shown overleaf, are particularly interesting. The 'third contribution' is now so pronounced that it completely dominates P_{ICI} and P_{INT}. Also the poor performance in a static echo channel means that the 'channel difficulty' noise alone more than half-fills the noise bucket. The consequence is that the measured failure point is well below the expected value. Modern demodulators are much more satisfactory in this respect, and can achieve a failure point of greater than 200 Hz.

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[4] Of course, the total equivalent noise is all that can be measured: it is not usually possible to 'turn off' one of the contributions.

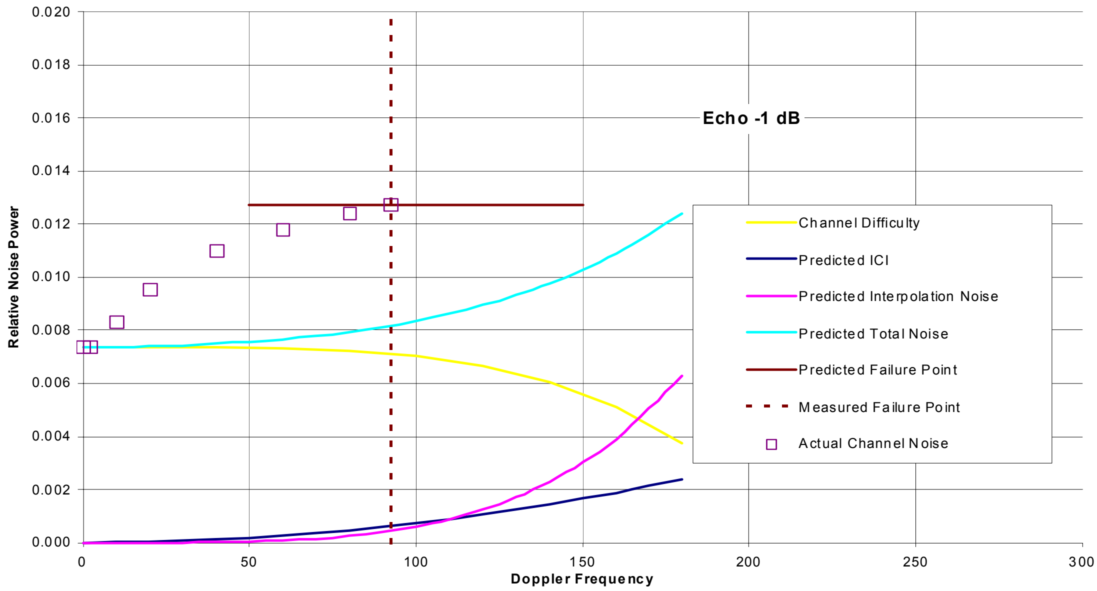

Figure 6.2: Noise Contributions for a –1 dB Echo

Finally, the results are given for a 'small' –10 dB echo:

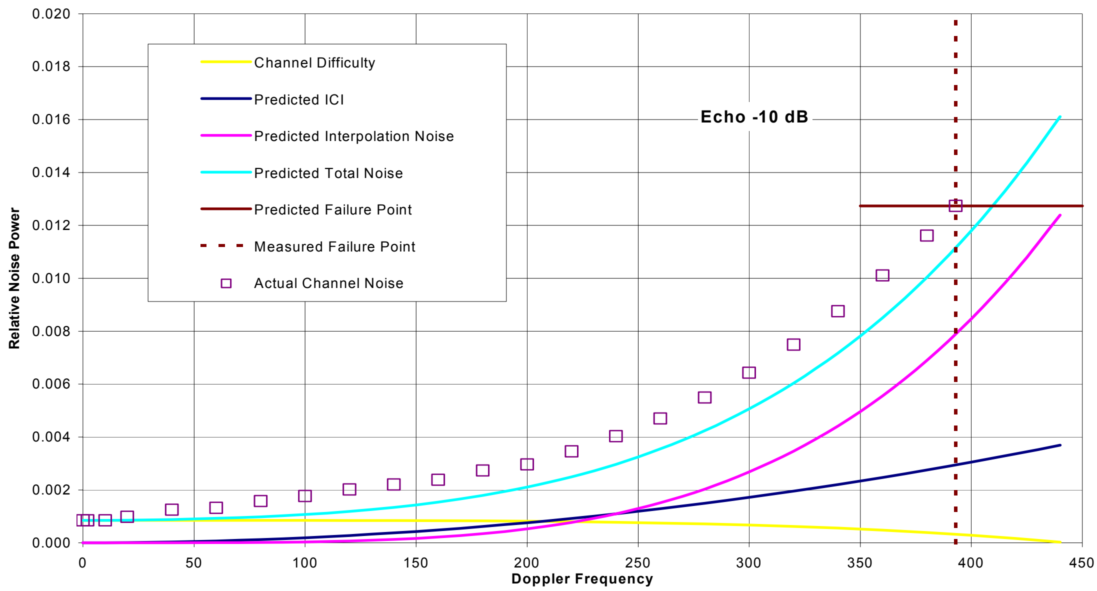

Figure 6.3: Noise Contributions for a –10 dB Echo

The 'third contribution' is now small, and hence the measured total noise is close the sum of calculated contributions. For 'small' echoes, CSI filtering — assumed responsible for the 'third contribution' — is less of an issue, since response variations across the channel are also small. If the CSI is wrong, it cannot do too much harm.

In summary, the total measured noise is always greater than the sum of the calculated individual contributions. The difference is ascribed to the 'third contribution' arising from the CSI filtering. Even so, the measured failure point can usually be predicted with reasonable accuracy by ignoring the 'third contribution'. This is because the known Doppler contributions increase rapidly with frequency, whereas the 'third contribution' remains nearly constant.

7. The Effect of Channel State Filtering

So far, it has been assumed that filtering of the CSI is responsible for the 'third contribution' of Doppler dependent noise. This assertion is plausible, but needs to be demonstrated conclusively. At first sight, it might seem nearly impossible to do this. However, the receiver chosen for the measurements has provision for altering the effective 'gain' of the CSI filter. Options of 1/2, 1/8, 1/32 or 1/128 are available: the smaller the fraction, the heavier the filtering.[5] The default setting, which was used for the previous measurements, is 1/8.

The following three plots show the effect of changing the CSI filter gain from 1/32 through to 1/2. A 'moderate' –5 dB echo was used throughout:

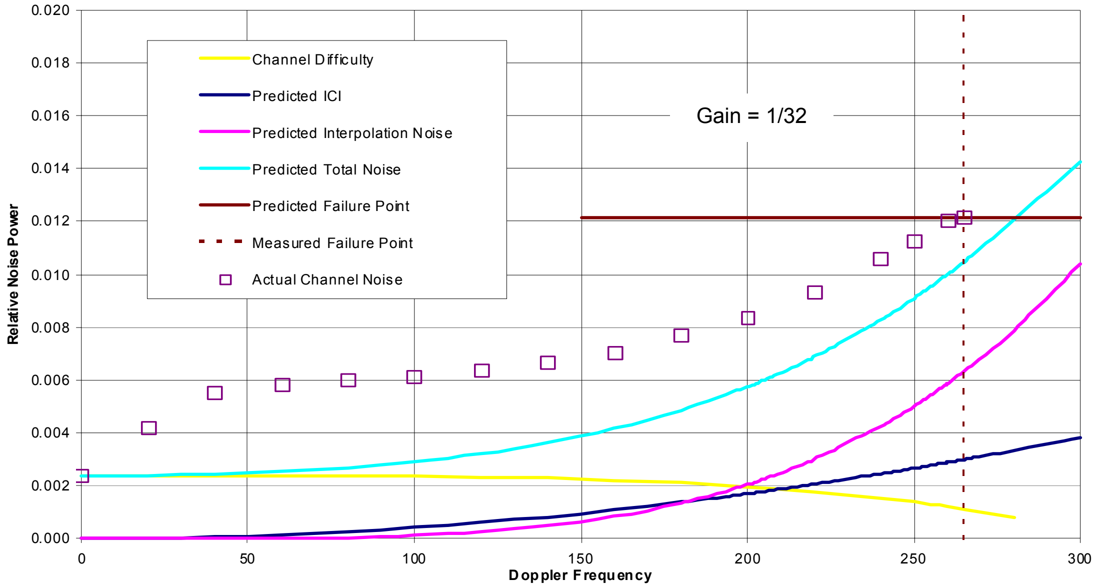

Figure 7.1: Receiver Performance with Low Filter Gain

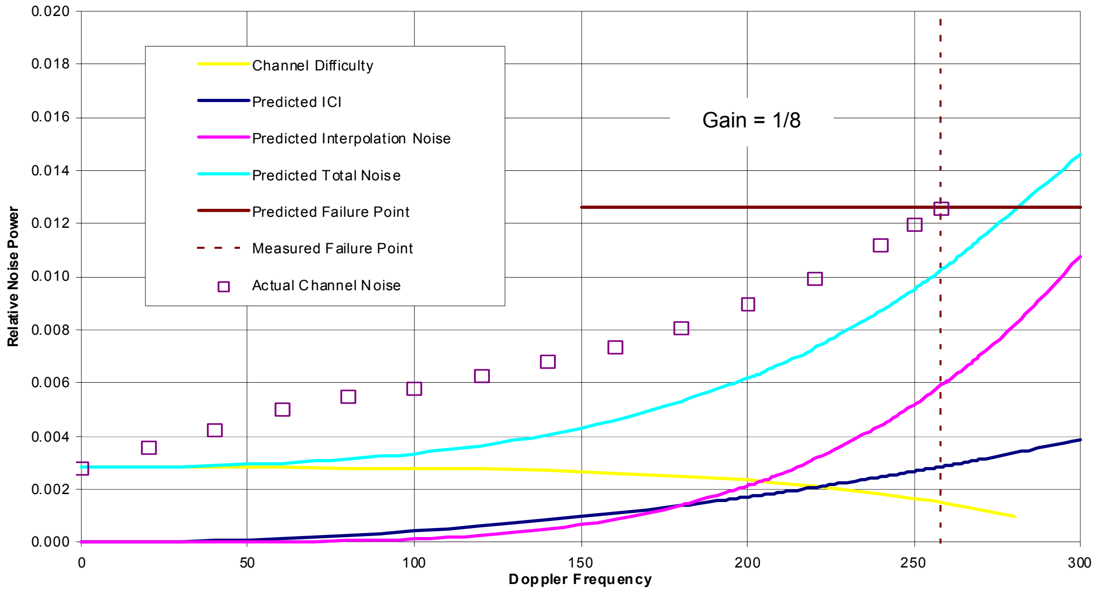

Figure 7.2: Receiver Performance with Medium Filter Gain

[5] The fraction refers to the amount of information taken from the current symbol. For instance, if the gain is set to 1/2, the filtered CSI takes half its value from the current symbol, 1/4 from the preceding symbol, 1/8 from the symbol before that, and so forth.

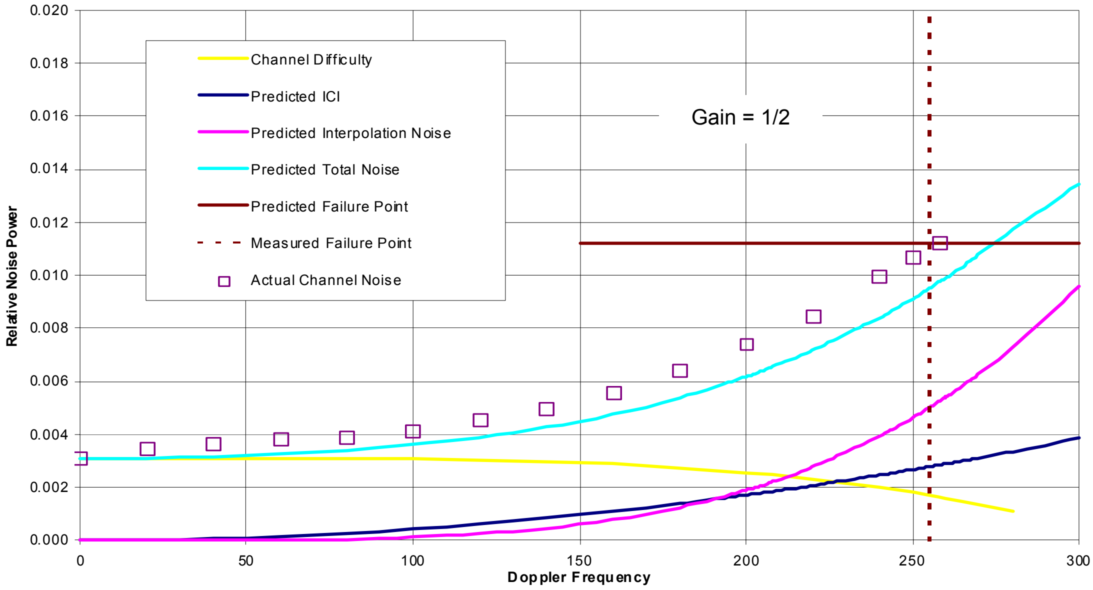

Figure 7.3: Receiver Performance with High Filter Gain

This sequence of plots clearly shows that the 'third contribution' is associated with the CSI filtering: heavy filtering results in a large contribution. Although it was not possible to remove the filtering completely, it seems reasonable to believe that, if one could, the experimental points and calculated total noise would agree very closely. Note also that the noise bucket size decreases slightly with increase in filter gain. This is because the increased CSI noise makes the receiver less tolerant in a Gaussian channel.[6]

8. Echoes Outside the Guard Interval

All the work so far described has been concerned with the simplest case of a single echo within the guard interval. If the echo is outside the guard interval, there is a further component to add to the noise bucket — the noise resulting from the lack of correlation between the direct and echo signals. As explained in **[1]**, this noise contribution approximates to 0.009 times the echo power for every microsecond that the echo delay exceeds the guard interval. It has no dependence on the Doppler frequency.

The plot overleaf shows the results for a –10 dB echo of 13.05 μs delay. If the noise bucket concept is valid, the measured noise should equal that of a 3.5 μs echo plus the lack-of-correlation component. The light blue curve shows the previously measured results for the 3.5 μs echo. Since there was some experimental error, the curve is not perfectly smooth: the results are presented 'warts and all'! The horizontal dark green line shows the calculated lack-of-correlation noise, whilst the purple curve is the sum of the two.

Agreement between the expected total noise and that actually measured is good. A striking feature of the results is the large proportion of noise attributable to lack of correlation. Despite the echo delay exceeding the guard interval by only 6 μs, this component equals about half the total noise at the failure point. At lower Doppler frequencies, it is correspondingly greater. Thus Doppler is only likely to be significant if the affected echoes fall within the guard interval.

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[6] Actually, the trend tends to reverse at very low filter gains — something that is particularly apparent at the 1/128 filter setting. However, this reflects a limitation in the mathematics carried out by the demodulator.

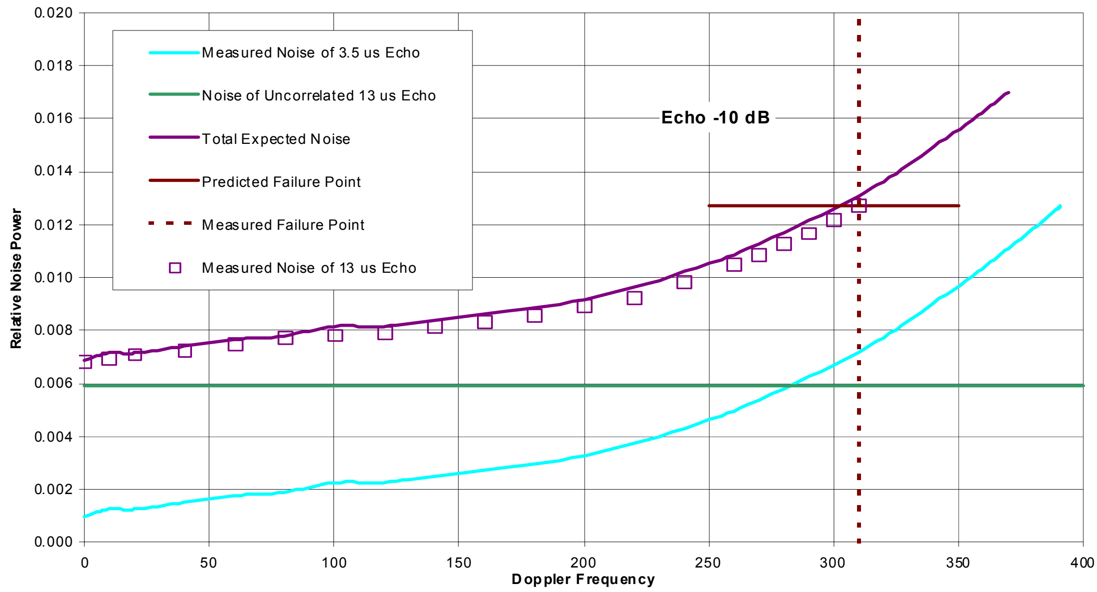

Figure 8.1: Noise Associated with a Doppler-shifted, 13.05 µs Delay Echo

9. Multiple Echoes

The subject of multiple echoes is too great to treat satisfactorily in the present Report. However, a single example of a two-echo test is presented below. The aim is to demonstrate that the noise bucket model remains valid.

The measurement method made use of the standard experimental set-up, and was as follows. To start with, the echoes were removed and the noise attenuation set to achieve BER_{REF}. The noise was then reduced by 3 dB. An echo of appropriate delay and Doppler shift was chosen, and its power set to give BER_{REF} once again. In other words, the impairment introduced by the echo corresponded to an END of 3 dB, or a half-full noise bucket. The echo power was now reduced slightly, and a second echo added at such a power as to maintain 3 dB END. By repeating the process, it was straightforward to list pairs of echo powers corresponding to the fixed END.

One of the echoes chosen was within the guard interval, and the other outside:

- Echo 1 3.5 µs delay, 200 Hz Doppler –6.7 dB for 3 dB END
- Echo 2 13.05 µs delay, 125 Hz Doppler –11.0 dB for 3 dB END

For the noise bucket concept to be strictly valid, the noise power associated with each echo should be proportional to the echo power; also, the two noise powers must add to give the total noise power. This implies that a plot of Echo 2 power versus Echo 1 power, for constant END, should be a straight line. Furthermore, the two noise contributions, when added together, should be independent of Echo 1 power. Figure 9.1 overleaf illustrates what actually happens.

The dark blue squares correspond to the measured echo power combinations when expressed in linear terms, whilst the purple squares show the implied total noise power. To calculate the total noise power, the noise of each echo alone was taken as 0.0063 for 3 dB END[7], and then scaled as appropriate. For instance, the noise power would be 0.0063 with Echo 1 set to –6.7 dB, and half that for a setting of –9.7 dB. The noise powers of the two echoes were then added.

[7] For the receiver in question, BER_{REF} in a Gaussian channel was achieved with a noise attenuation of 19 dB. Hence an END of 3 dB corresponds to an attenuation of 22 dB, or 0.0063 in linear terms.

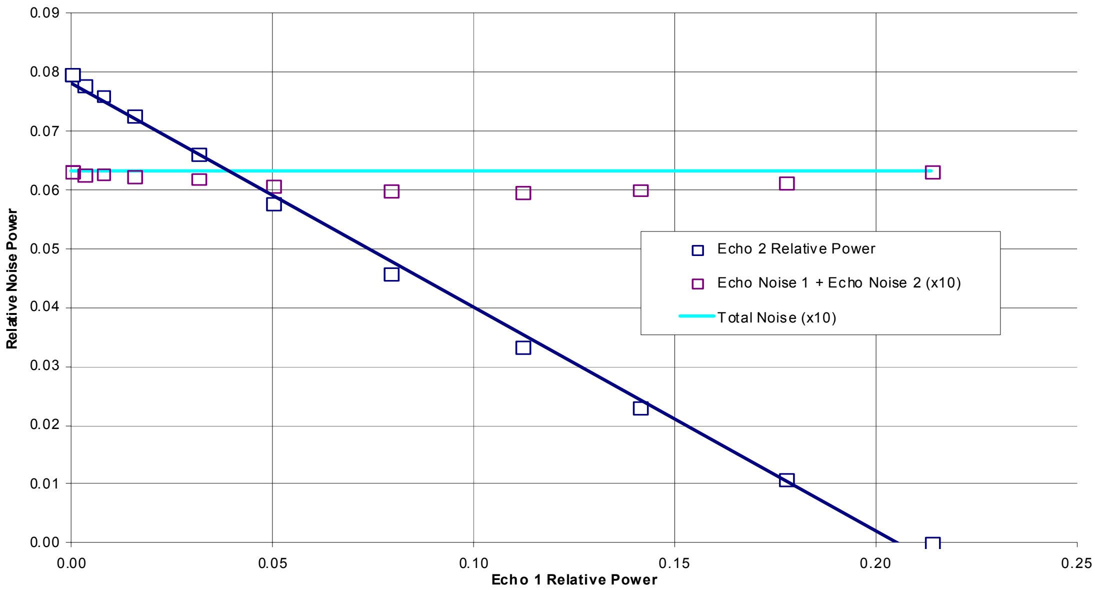

Figure 9.1: Echo Powers and Noise Contributions Corresponding to 3 dB END

Despite the large total impairment, the system is closely linear: a good straight line can be drawn through the dark blue squares, and the purple squares are near to the constant total noise shown by the light blue line. Correspondence is not exact, as the expressions for P_{ICI} and P_{INT} show that the equivalent noise of an echo is not strictly proportional to the echo power.

10. Conclusion

This report has described an investigation into the behaviour of DVB-T receivers handling 64 QAM in time-varying channels. The conclusions are as follows:

- Models have been developed to predict the equivalent noise of interpolation errors and intercarrier interference.
- However, practical measurement shows that there is a significant 'third contribution' resulting from the demodulator's channel state filtering.
- The above Doppler-related noise contributions can be added to the other, more familiar, contributions already in the noise bucket.
- Although linear interpolation is a crude technique, it does not seriously limit the performance of the demodulator: the demodulator should still accommodate a 0 dB echo shifted by 200 Hz.[8] The results would be even better for 16 QAM and QPSK modes.
- The limited evidence so far available suggests that, when a multiple echo signal is present, the noise contributions of the component echoes can be added in the noise bucket.[9]
- Any small discrepancies between noise bucket theory and practical measurement could be the result of the non-Gaussian nature of interpolation noise.

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[8] A Doppler shift of 200 Hz corresponds to a speed of 100 metres per second (180 miles per hour) at 600 MHz — hardly a significant limitation for most purposes. In any case, the failure point could never be increased beyond about 400 Hz because of, firstly, the finite symbol period and, secondly, the ICI component.

[9] This statement should probably include a limitation on the total echo power present: the models developed in earlier reports were only valid if there were no nulls in the channel response.

11. Acknowledgements

The following provided valuable contributions to this report:

- Jonathan Stott and Chris Nokes, who offered their theoretical insights;
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- John Salter, who advised on the experimental work and provided helpful comments concerning the document.

12. References

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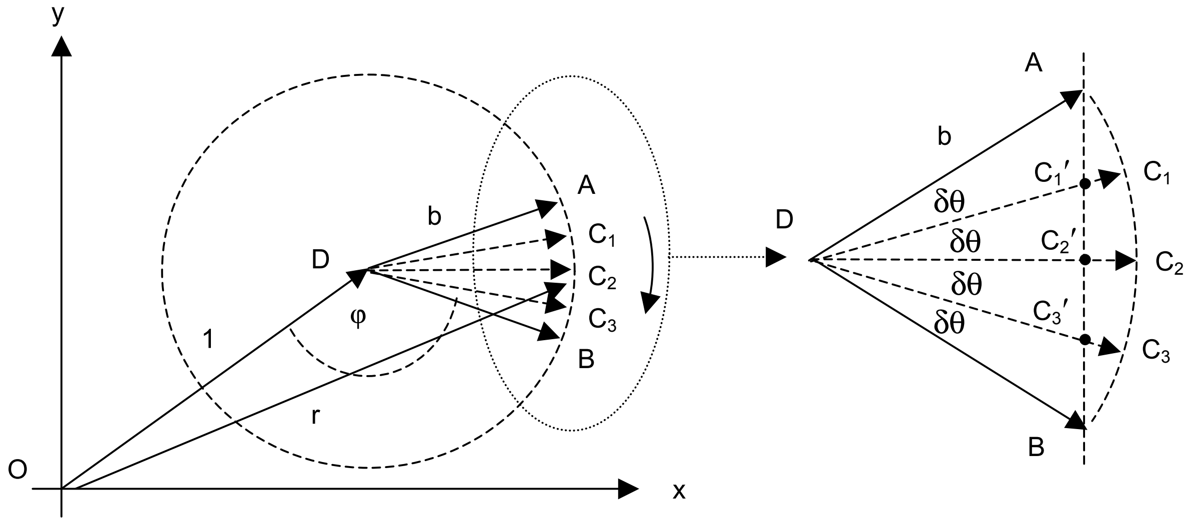
Appendix 1. Interpolation Noise and the Fourth Power Law

Section 2 of this Report showed how the presence of a Doppler-shifted echo causes time-variations in the channel response. Section 3 then described in outline how the channel equalisation process introduces interpolation errors. These errors amount to a noise signal whose power is proportional to the fourth power of the Doppler frequency. The result quoted was

$$P_{INT} \;=\; (17/8)\ \{b^2 / (1 - b^2)\}(2\pi\ \delta f_d\ T_s)^4 ,$$

where b is the relative echo amplitude, δf_d is the Doppler shift of the echo, and T_s is the symbol period. A full derivation of this expression is presented below.

The following diagram is based on Figure 3.1, but presents an expanded view of the interpolation errors. It is assumed that the phase of the echo changes by a constant $\delta\theta$ per symbol.



The direct signal is represented by vector OD, of unity length, whilst the echo is represented by the vector of length b. Because of the Doppler shift, the tip of the echo vector describes a circle. In the course of four symbols, it moves between A and B.

The interpolation errors are represented by distances $C_1' C_1$, $C_2' C_2$ and $C_3' C_3$.

Figure A1.1: The Generation of Interpolation Errors

An interpolation error arises because the demodulator knows the exact position of the resultant signal vector at A, and four symbols later when it has reached B. However, the demodulator has to guess where the vector is during the intermediate symbols, when it is actually at C_1, C_2 or C_3. By applying linear interpolation, the demodulator guesses that it is at C_1', C_2' or C_3'. Distances $C_1 C_1'$ and so forth represent the interpolation errors, which may be calculated by simple trigonometry:

$$C_2 C_2' \;=\; b - b\cos 2\delta\theta \;=\; b\ (1 - \cos 2\delta\theta) \;\approx\; b\ 2\delta\theta^2 , \text{ for small } \delta\theta.$$

$$C_1 C_1' \;=\; D\,C_1 - D\,C_1' , \text{ where } D\,C_1 = b \text{ and } D\,C_1' = D\,C_2' / \cos\delta\theta .$$

Since $D\,C_2' = b\cos 2\delta\theta$,

$$C_1 C_1' \;=\; b - (b\cos 2\delta\theta) / \cos\delta\theta \;=\; b\ (1 - \cos 2\delta\theta / \cos\delta\theta) ,$$

$$\approx\; b\ \{1 - (1 - 2\delta\theta^2)\ (1 + (1/2)\ \delta\theta^2)\} = b\ (3/2)\ \delta\theta^2 , \text{ for small } \delta\theta.$$

Similarly,

$$C_3 C_3' \;\approx\; b\ (3/2)\ \delta\theta^2 .$$

The noise powers equal the squares of these interpolation errors, giving a total power of

$$(1/4)\, b^2\, \delta\theta^4\, \{(3/2)^2 + 2^2 + (3/2)^2\} \;=\; (17/8)\, b^2\, \delta\theta^4 ,$$

averaged over four symbols. $\delta\theta$ equals $2\pi \times$ Doppler shift $\delta f_d \times$ symbol period T_s , so the noise power becomes

$$(17/8)\, b^2\, (2\pi\, \delta f_d\, T_s)^4 .$$

However, this expression is only true when b is small, since the action of the channel equaliser is to normalise the resultant vector r to unity. In the process, the interpolation error is changed by a corresponding amount. The interpolation noise after equalisation is given by

$$P_{INT} \;=\; (17/8)\, (b^2/ r^2)\, (2\pi\, \delta f_d\, T_s)^4 , \text{ where } r^2 \;=\; 1 + b^2 - 2b \cos \varphi .$$

Since φ is varying, the expression needs to be integrated over the range $\varphi = 0 \rightarrow 2\pi$; so

$$P_{INT} \;=\; \int \{(17/8)\, b^2\, (2\pi\, \delta f_d\, T_s)^4 / (1 + b^2 - 2b \cos \varphi)\}(d\varphi/ 2\pi) .$$

The integral $\int (d\varphi/ 2\pi) / (1 + b^2 - 2b \cos \varphi)$ equals $(1 - b^2)$, and the expression for the noise power becomes

$$P_{INT} \;=\; (17/8)\, \{b^2 / (1 - b^2)\}(2\pi\, \delta f_d\, T_s)^4 .$$

Note that the noise power increases with the fourth power of the Doppler frequency. The $1 - b^2$ term in the denominator results in the noise power increasing rapidly as b approaches unity.

As an example, if the Doppler frequency is 100 Hz, and the echo power is –12 dB, the formula predicts that the noise should be –42 dBc. A more sophisticated calculation, taking into account the averaging that takes place over each symbol, gives a value 0.5 dB less. However, this small error makes little practical difference.

Unfortunately, interpolation noise is not even approximately Gaussian, and there is no guarantee that the failure point can be calculated by simply adding P_{INT} to the noise bucket. Fortunately, calculation shows that P_{INT} is similar in effect to Gaussian noise, provided that the echo power does not approach 0 dB. The errors are less for the more robust modulation modes.

It is obvious that the formula is of no use for 0 dB echoes, since the $1 - b^2$ term in the denominator is then zero. In reality, averaging over the symbol period prevents this term from ever being zero. Furthermore, even if a carrier was zero for an entire symbol, the very large noise spike could never result in more than six bit errors (for 64 QAM). It is possible to modify the $1 - b^2$ term so as to give better agreement with 'reality'. However, it is probably better to accept that 0 dB echo performance is a complicated matter, unsuitable for representation by a simple model!

Appendix 2. Loss of Orthogonality

Section 3 of this Report showed how the presence of a Doppler-shifted echo causes intercarrier interference (ICI), as a result of the echo component being at the wrong frequency. The result quoted was

$$P_{ICI} = \{(1/3)(\pi\,\delta f_d\,T_u)^2\}\,\{2b^2/(1+b)^2\}\,,$$

where b is the relative echo amplitude, δf_d is the Doppler shift of the echo, and T_u is the useful symbol period[10]. A full derivation of this expression is presented below.

Suppose that the frequencies of the COFDM carriers are ω_m, ω_{m+1}, ω_{m+2} and so forth. In effect, the demodulation process multiplies each carrier by a synchronous local oscillator signal, then integrates the result over the symbol period. Consider the demodulation of the first of these carriers if the frequency is in error by $\delta\omega_m$. Before integration, the demodulator output is

$$V_{d1} = \cos\omega_m t\,.\,\cos(\omega_m + \delta\omega_m)\,t\,,$$

$$= (1/2)\,\{\cos(2\omega_m + \delta\omega_m)\,t + \cos\delta\omega_m t\}\,.$$

Suppose now that V_{d1} is integrated over the symbol period T_u, and that the output of the integrator is given by

$$V_{i1} = \int k\,V_{d1}\,dt\,.$$

Then

$$V_{i1} = \int_{-Tu/2}^{-Tu/2} (k/2)\,\{\cos(2\omega_m + \delta\omega_m)t + \cos\delta\omega_m t\}\,,$$

$$= (k/2)\left[\{\sin(2\omega_m + \delta\omega_m)\,t\}/(2\omega_m + \delta\omega_m) + \{\sin\delta\omega_m t\}/\delta\omega_m\right]_{-Tu/2}^{+Tu/2}\,.$$

Since $(2\omega_m + \delta\omega_m)$ is much greater than $\delta\omega_m$, the first term of the equation can be ignored, and the output of the integrator becomes

$$V_{i1} = \{k\sin\delta\omega_m T_u/2\}/\delta\omega_m\,.$$

Thus, if the frequency of the first carrier is correct, V_{i1} equals $kT_u/2$.

Suppose the angular carrier spacing is ω_s. If the above demodulator is truly orthogonal, the only carrier to contribute to the output will be ω_m; there will be no contributions from carriers ω_{m+1}, ω_{m+2} and so forth. Hence

$$\{k\sin\omega_{m+1} T_u/2\} = \{k\sin\omega_{m+2} T_u/2\} = \{k\sin\omega_{m+....} T_u/2\} = 0\,;$$

and so

$$\omega_s T_u/2 = \pi,\ \text{since}\ \omega_s = (\omega_{m+1} - \omega_m) = (\omega_{m+2} - \omega_{m+1}) = ...$$

In other words, if $\omega_s/2\pi$ is written as f_s, T_u equals $1/f_s$. This is the well-known result that the active symbol period is the reciprocal of the carrier spacing.

It is now straightforward to calculate the ICI. If the Doppler angular frequency $\delta\omega_m$ is written as $2\pi\,\delta f_m$, the expression for the wanted output from the demodulator becomes

$$V_{i1} = (\sin\pi\,\delta f_m T_u)/(\pi\,\delta f_m T_u)\,.$$ [11]

The power associated with V_{i1} is the square of V_{i1}. This is less than unity except when δf_m is zero, and the implication is that the difference represents power falling on the other carriers. Hence the ICI is given by

$$P_{ICI} = 1 - (\sin\pi\,\delta f_m T_u)^2/(\pi\delta f_m T_u)^2\,,$$

or

$$P_{ICI} = 1 - \text{sinc}^2\,\pi\,\delta f_m T_u\,.$$

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[10] That is, T_u equals the overall symbol period T_s minus the guard interval.

[11] For convenience, the expression has been normalised to unity at $\delta f_m = 0$.

Provided that $\pi\,\delta f_m T_u$ is small, the sinc function can be expanded as a power series:

$$\text{sinc } x = \sin x / x = \{x - x^3/6 + \dots\}/ x = 1 - x^2/6 + \dots .$$

and $$\text{sinc}^2 x = 1 - x^2/3 + \dots \text{ (terms in } x^4 \text{ and above).}$$

Hence the intercarrier interference approximates to

$$P_{ICI} = (1/3)\,(\pi\delta f_m / f_s)^2 .$$

The derivation of this formula has assumed that the frequency of the direct signal is unaffected by the Doppler shift. In practice, the automatic frequency control (AFC) in the receiver is likely to take some sort of average of the direct and Doppler shifted frequencies. For instance, if the direct and echo signals are of equal level, half the Doppler shift will transfer to the direct signal and half will remain on the echo.[12] Suppose that the Doppler shift of the direct signal is δf_{md}, and that of the echo is δf_{me}. The ICI resulting from the two signals becomes

$$\underset{\text{(direct)}}{(1/3)\,(\pi\,\delta f_{md} T_u)^2} + \underset{\text{(echo)}}{b^2 (1/3)\,(\pi\,\delta f_{me} T_u)^2} .$$

The two individual Doppler shifts must now be related to the total Dopper shift δf_d. Obviously, δf_{md} and δf_{me} must sum to δf_d, and it is reasonable to take δf_{md} as equal to $b\,\delta f_{me}$; that is, the shift of the direct signal is proportional to the amplitude of the echo. Hence,

$$\delta f_{md} = b\,\delta f_d / (1+b) \text{ and } \delta f_{me} = \delta f_d / (1+b) .$$

Where a 0 dB echo is present, $\delta f_{md} = \delta f_{me} = (1/2)\,\delta f_d$; and where the echo is very small, $\delta f_{md} = 0$ and $\delta f_{me} = \delta f_d$.

The expression for the total ICI becomes

$$\begin{aligned} P_{ICI} &= \{(1/3)\,(\pi/f_s)^2\}\,\{(\delta f_{md})^2 + (\delta f_{me})^2\}, \\ &= \{(1/3)\,(\pi\,\delta f_d T_u)^2\}\,\{b^2/(1+b)^2 + (b^2/(1+b)^2\}, \\ &= \{(1/3)\,(\pi\,\delta f_d T_u)^2\}\,\{2b^2/(1+b)^2\} . \end{aligned}$$

It is interesting that the ICI contributions from the direct and echo signals are now equal. The AFC action is helpful for large echoes, since the sum of the two contributions is less than that of an echo carrying the full Doppler shift. However, where the echo is small, the reverse is true: the echo contribution is unchanged, but the direct signal now adds an equal contribution.

12 This result is intuitively reasonable. If a particular carrier of the direct signal is represented by $\sin \omega_1 t$, and the same carrier of the echo is represented by $\sin \omega_2 t$, the sum $(\sin \omega_1 t + \sin \omega_2 t)$ equals $2 \sin \{(1/2)\,(\omega_1 + \omega_2)t\} \times \cos \{(1/2)\,(\omega_1 - \omega_2)t\}$. In other words, the carrier appears to possess a frequency of $(1/2)\,(\omega_1 + \omega_2)$, which is 'corrected' by the receiver's AFC. The 'beat' component $(1/2)\,(\omega_1 - \omega_2)$ is accommodated by the channel equaliser.