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Cumulative effects of distributed interferers

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Abstract

Systems which re-use mains or phone wiring for communications purposes are currently of interest. As well as their obvious benefits they have the potential to cause interference to radio systems. Interference to receivers in the immediate vicinity is fairly readily measured and legislated for. However, with the potential for mass-market deployment of such systems it is also important to consider whether more-distant receivers may also be adversely affected by the cumulative effect of all the installations.

Key words: DSL, PLT, PLC, emissions, broadcasting, aircraft safety, cumulative interference

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Executive Summary

Systems which re-use mains or phone wiring for communications purposes are currently of interest. As well as their obvious benefits they have the potential to cause interference to radio systems. Interference to receivers in the immediate vicinity is fairly readily measured and legislated for. However, with the potential for mass-market deployment of such systems it is also important to consider whether more-distant receivers may also be adversely affected by the *cumulative effect* of all the installations.

The level of cumulative interference caused by distributed interference sources (such as *x*DSL, PLT or home-networking systems) has been analysed, following the method used by the author in a previous paper, Ref. [1]. However, the results have been presented more extensively and also slightly differently from the previous paper, in a system-independent way. The idea is that for any particular system under examination an appropriate scaling factor in dB (in fact the *EIRP density* of the sources) needs simply to be added to the generalised results presented here. In this way, the present results are in principle applicable to any type of distributed interferer.

Results have been presented for two classes of interference scenario, namely:

- to aircraft from sources on the ground
- to ground-based receivers from sources on the ground elsewhere, via sky-wave propagation

(The more specialised case of protecting a ‘sensitive site’ from ground-wave interference using an ‘exclusion zone’ has not been re-visited; the reader is referred back to the previous paper Ref. [1])

Aircraft use both communications and radio-navigation receivers, and are believed to operate with relatively low signal levels. The level of interference caused by distributed sources on the ground depends of course on the EIRP density, for which the author awaits information on representative values for a range of *x*DSL/PLT/etc. systems. However, using those values applicable to the PLT proposal (now withdrawn) considered in Ref. [1] shows a substantial interference contribution would be received by an aircraft flying over a well-populated area where such PLT systems were deployed.

… continued overleaf

The interference received by an aircraft when the entire visible earth is populated with systems is shown to be nearly independent of height. Limiting the area containing interferers, from the visible earth to a smaller area representative of a major conurbation, does not decrease the interference very greatly, unless the aircraft is very high.

A greater sophistication of the interference model was also tried, incorporating a vertical radiation pattern for the interference sources. In the examples examined, the radiation upwards was reduced (as would be expected if the dominant radiators in the system were vertical). Interestingly, this only marginally reduces the interference received by an aircraft.

The implications of the possibility of interference to aircraft communications and navigations systems should be rigorously studied by the relevant competent authorities.

The threat to ground-based receivers from interference propagated by sky-wave propagation has also been presented in the system-independent way, without making any specific assumptions about the ionospheric loss. This means that results for a particular scenario may be inferred from the curves presented by:

- taking account of the relevant *skip distance* in reading the curves
- *adding* the EIRP-density correction (applicable to the interferer type), and
- *subtracting* the appropriate ionospheric loss.

The results show that sky-wave interference from widespread *x*DSL/PLT/etc. systems to ground-based receivers may indeed not always be negligible, even though it is less than that shown to be suffered by aircraft.

Detailed derivations, with analytical results for many of the integrals, are presented in an Appendix.

It is hoped that the calculations presented here will guide regulators in setting limits to the emissions from potentially-widespread *x*DSL/PLT/etc. systems so that radio users may be assured adequate protection from interference.

□ 1. Introduction

■ 1.1. New communication systems — a new source of radio interference

Various new forms of communication systems are being proposed which exploit the existence of cables originally provided for some other purpose, such as mains or telephone wiring into or within the home or business premises, by superimposing an additional signal to convey data. Obviously the use of such a facility is a convenient way to provide new services, and in particular gives a commercial opportunity for the owners of such wiring infrastructure which enters homes or businesses.

Systems exploiting telephone wiring to access the home or business belong to a family called *x*DSL (for Digital Subscriber Loop), each with different capacity, range and spectral occupancy. For example ADSL (A for Asymmetric) uses frequencies up to 1.1 MHz and connects the home to the telephone exchange; another system, VDSL, uses frequencies up into the HF range, offering higher capacity but shorter range — from the home to the street cabinet.

Systems using the mains wiring to access the home or business are variously described as Power-Line Transmission or Communications, i.e. PLT/PLC. In this case communication is usually from the home or business to the nearest substation transformer.

A further category of systems uses the existing phone or mains wiring *within* the home for internal networking. There is some indication of co-operation between the proponents of such systems and of the above-mentioned access systems so that the internal-networking systems will use the upper HF band while the access systems use the frequencies below this.

However, there is a downside to all this use of existing cables for data transmission. The cables were not designed as communication cables for this new purpose, and as a result there is the undesirable side-effect that the data signals ‘leak’ and have the potential to cause interference to radio systems. In effect, the cable acts as a transmitting antenna.

Various interference scenarios can be considered.

■ 1.2. Nearby interference

An obvious interference scenario is where radio reception takes place close to the cable carrying the additional data signal. In this case the majority of the interference comes from just this single nearby cable. The interference can be regulated by imposing a limit on the permitted (interfering) field strength, as measured at some specific distance from the cable in question. Such an approach has been taken:

- in the UK, where a regulation named MPT 1570 has been drafted (at the time of writing, limits for frequencies below 1.6 MHz have been decided while those for higher frequencies are still under consideration)
- in Germany, where a regulation named NB30 is now in force, covering all relevant frequencies
- an initiative is also under way to try to achieve a consistent state of regulation throughout Europe

Where the measurement distance and type of measurement antenna specified in the regulations correspond to those likely to be used by affected radio receivers, the measured *interference* can be straightforwardly related to the strength of the *wanted* received signal in order to quantify the effect of the interference as a signal-to-interference ratio. For the case of long- and medium-wave broadcast reception (where ferrite-rod antennas responsive to the magnetic field are normally used in the home) the use of loop measurement antennas at a distance of 1 m, as specified in the UK MPT 1570, is highly appropriate. Unfortunately the signal-to-interference ratios MPT 1570 permits correspond to substantially degraded audio quality. At frequencies in the HF band it remains convenient to use loop antennas responsive to the magnetic field for measurement, but these may not represent the antennas likely to be in use for radio reception.

▪ 1.3. Cumulative interference

The interference effect is not limited to the immediate environment of the cable. The interference detectable from one such system will of course decay with distance, and so interference might appear to be less of a problem for more-remote receivers. However, if systems of this type are installed to feed many homes and businesses, each will make its own contribution to the interference. A remote receiver will thus pick up the sum of a large number of interfering sources, each somewhat attenuated but in total still having the potential to cause difficulty.

A previous paper by the present author [1] considered the impact of this *cumulative effect* of many such interferers. The original focus of that paper was to assess the impact of one particular PLT-system proposal on so-called ‘sensitive sites’ that are required to intercept weak radio signals, whether for reasons of aeronautical/marine safety, monitoring, surveillance or indeed radio astronomy. A BBC example is the World Service Monitoring reception site near Caversham. Discussions in UK regulatory bodies came up with the idea of defining an ‘exclusion zone’ around designated important sites of this nature, within which communications systems of this type could not be used. An important question was then to choose the size of these zones. Clearly the size would depend on the parameters of cable-communications system in question.

The previous paper initially considered such ground-based receiving sites suffering interference as a result of ground-wave propagation. A further step considered sky-wave propagation of interference to the same receiver, showing that sky-wave interference could be of significance once the ground-wave interference had been sufficiently reduced by the application of a large-enough exclusion zone. Once the latter calculation had been performed, it became obvious that another scenario — that of interference to receivers onboard aircraft — had very similar geometry and could be assessed by a similar method. When this was done, it was clear that the case of interference to aircraft was in many respects the most critical one.

The previous paper gave explicit results for the interference levels in the various scenarios, assuming that the interference came from the particular type of PLT system considered, with parameter values believed to be representative of it (the available data was very sparse). It was also clear that the results could be corrected to make them applicable for systems (PLT, *x*DSL or whatever) with different parameters (different emission levels, different densities of installation) by *scaling* — a simple matter of adding or subtracting correction factors in dB.

▪ 1.4. Purpose of the present paper

The particular PLT proposal on which the previous paper was based has been withdrawn; however, since then there has been an explosion of interest in PLT, *x*DSL and home networking products, all with distinct parameters. Many are already being rolled out. It would be convenient to have a ready method of assessing them all for their cumulative effects.

The purpose of the present paper is to give the results of the previous analysis in a new system-independent form, assuming a standard ‘EIRP density’ for the interfering sources. The appropriate correction must then be determined and applied in order to predict the actual interference level resulting from any practical system under consideration.

This generalised approach is given for the case of interference to aircraft (likely to be the most critical scenario in most cases) and for sky-wave interference. For ground-wave interference the reader is for the time being referred back to Ref. [1], from which the necessary procedure can still be deduced. Where possible, analytical results are quoted so that cases other than those for which plots are provided can be considered. A greater range of plots is provided than before, and discussion of some topics amplified.

The paper also includes some quite new analytical results, attempting to indicate the impact of interference sources having a non-uniform vertical radiation pattern.

What this paper does *not* address is the actual provision of relevant data for the system parameters. The density of system installations is a matter of commercial ambitions and their successful achievement or otherwise (usually secret). Of course, there is a physical upper limit, assuming every household (or electricity sub-station, if more relevant) is equipped! In principle, the per-system-installation EIRP can be measured for a particular type of system — but in practice this is very difficult to do. A promising *indirect* method is described in [2]. In any case, where regulations have to be set in advance of deployment, they will probably be based on setting permissible limits on the magnetic field, measured in relatively close proximity to the relevant cable. Relating this limit to the average per-system-installation EIRP that could result remains an open issue.

☐ 2. Key steps in the analysis

The calculations involve two key steps:

- determining the interference caused by a single interfering source at some distance

  This involves knowing how much interfering signal is radiated, and how it propagates over a distance, i.e. how much it is attenuated as a function of distance.

- accounting for the summation of interference from the many similar sources that will be present once systems of this type are fully deployed

  This includes knowing the physical distribution of the interference sources, and the manner and geometry of the propagation path(s) by which the interference reaches the victim receiver.

We can reasonably assume that the signals from the many interference sources are uncorrelated (as long as all the links are not used in some kind of broadcast mode carrying the same data!), and so their total effect on one receiver can be assessed by *power addition*. Furthermore, we may note that by the Central Limit Theorem, the more independent interference contributions there are, the more their combination will tend to have a normal amplitude distribution, like Gaussian noise.

Ideally we should consider the particular location of each interference source, determine the propagation from each source to the victim receiver, and perform a power summation. This is probably impractical for potentially-widespread systems, and is certainly not possible when the system is only a proposal so that the locations are unknown in detail. What we can do instead is to estimate the *density* of potential installations, treat the sources as being uniformly spread over an area and replace the summation of a finite number of sources by an integral over an area.

This integration in place of summation is a reasonable procedure as long as the distance travelled by the interfering signal is always large compared with the distance between sources. For the ground-wave propagation considered in Ref [1], this was only a reasonable assumption provided the calculation included at least a small exclusion zone. For sky-wave propagation, the signal always has to travel to the ionosphere and back, so the estimation of the sky-wave interference can be performed without any exclusion zone. (Of course, without any exclusion zone, the sky-wave interference may be expected to be wholly swamped by the *direct* interference from the nearest interferer, but the sky-wave contribution should nevertheless be correctly estimated). Finally, for interference to aircraft we would require the aircraft height to be large compared with the distance between sources — likely to be true except during take-off and landing. In any scenario where the distance the interference travels from the nearest source to the receiver is small compared with the distance between sources, the nearest source would clearly dominate in practice.

□ 3. Assumptions and general method

■ 3.1. General assumptions, and naming of variables

- each of the (discrete) interference sources is treated as radiating interference isotropically

  If we consider one such source in isolation then this is clearly not the case — the particular cable will have some arbitrary frequency-dependent radiation pattern. But when we sum the influence of many sources (none of which is allowed to be dominant, as previously explained) the peaks and nulls of individual sources will tend to average out.

  One exception is also considered later, where the sources are treated as radiating equally in all azimuth directions but account is taken of possible variation in radiation with elevation angle.

- each system is thus, *on the average*, equivalent to a transmitter (of power equal to that deliberately injected into the cable) coupled to an antenna which is isotropic in directivity. However, because it is a lossy 'antenna' (we hope that most of the data signal is either transmitted through the cable to its desired destination, or dissipated in cable losses — only a part is *radiated*) the effective antenna gain takes some value less than 0 dBi.

Let the antenna effective gain just described be g_{TX} in linear units or $G_{\mathrm{TX}} = 10\,\mathrm{Log}[10, g_{\mathrm{TX}}]$ dBi.

Let the power injected into the cable (within the relevant bandwidth — frequently 10 kHz is used, to match measuring-receiver technique) be p_{TX} watts or $P_{\mathrm{TX}} = 10\,\mathrm{Log}[10,\ p_{\mathrm{TX}}]$ dBW. (Strictly the units of measurement are thus W/10kHz — for brevity, this will not always be spelt out).

Each system in operation thus behaves as a transmitter with EIRP (Equivalent Isotropically Radiated Power) equal to $p_{\mathrm{TX}}\, g_{\mathrm{TX}}$ W.

Let the density of systems in operation be D systems / m^2. An area dA containing systems in operation is thus equivalent to a transmitter of EIRP $p_{\mathrm{TX}}\, g_{\mathrm{TX}}\, D\, dA$ watts.

Let the propagation from a source to a receiver over distance x m (by whatever mechanism may apply) be represented by some function $f[x]$ so that the power-flux density at the receiver is given by the product of $f[x]$ and the source EIRP.

■ 3.2. General method

It follows that the interference power-flux density encountered at a specific receiver site can be evaluated as:

$$\mathrm{PFD} = \int_A p_{\mathrm{TX}}\, g_{\mathrm{TX}}\, D\, f[x]\, dA$$

where A is the area containing the interference sources. Note that for ground-wave interference no part of A may be too close to the receiver otherwise the use of integration, instead of summation of discrete sources, will not be correct, as previously discussed. (The ground-wave case is not considered further in this paper).

Note that the PFD we calculate sums all the incoming contributions without any regard to the direction from which they arrive. If we multiply this PFD by the effective area of an isotropic antenna at the relevant frequency we would correctly obtain the available power at the output of an isotropic antenna at the assumed receiver location. Where the receiving antenna takes a more practical form, we may have to interpret the results carefully — see the discussion in § 6.

If the Earth were flat, or we were considering only very nearby interferers, then we could choose an annulus of radius x, thickness dx and thus area $dA = 2\pi x\,dx$ which we could substitute in the above equation, while $f[x]$ would be whatever is necessary to account for propagation over a distance x by the mode under consideration.

Once we have to consider larger distances we have to take account of the curvature of the Earth. The area of the annulus which is distance x (measured over the curved surface) from the receiving point (or the point on the ground directly below an aircraft) is now smaller, see Appendix A 1.1. In principle the total area A, containing interferers, over which the integral is performed could be any arbitrary shape. However, in this paper we assume for simplicity of integration a spherical ‘pseudo-annulus’, beginning at curved-earth distance x_1 from the point on the ground which is underneath the aircraft or where a ground-based receiver is placed, and ending at distance x_2. In many cases x_1 will be zero, so that A is a spherical cap. Note that it is possible, with considerable caution concerning receiver directivity, to extract from this analysis the results for the case where interferers only exist in a particular sector.

The geometry of propagation to an aircraft is described in Appendix A 1.2, The interfering signal travels in a straight line from ground to aircraft by free-space propagation, so that the distance travelled (and thus the attenuation $f[x]$) is a function of x, determined by geometry. There is a limit, depending on the aircraft height, to the distance at which interferers can be ‘seen’ by the aircraft.

For sky-wave propagation we can still use the same annulus, but in this case the signal can be considered to travel in a straight line to the ionosphere, whence it is ‘reflected’† back towards the reception point. In this case the distance travelled (and thus the attenuation) is a different function of x, see Appendix A 1.3.

† Note that strictly speaking the process by which radio signals are returned to Earth is not *reflection*, but rather *refraction*. The wave therefore follows a curved path as it is turned round. However, the process can for most purposes be treated as equivalent to a simple reflection at a nominal reflection height.

For sky-wave propagation over the curved Earth, there is a maximum distance (π times the radius of the Earth, R_E) that the interferer can be distant from the receiver. It is also possible for signals to travel the ‘long way round’, as well as by the shortest direct Great Circle route. For the purpose of this paper such long-path propagation is neglected.

In order to provide a standardised presentation independent of system parameters, the *EIRP density* $p_{\mathrm{TX}}\, g_{\mathrm{TX}}\, D$ is taken out of the integration, with the intention that this system-specific value be applied afterwards on a case-by-case basis. The integral which has to be performed is thus:

$$\mathrm{PFD}_{\text{for unit EIRP density}} = \int_A f[x]\, dA$$

the result being a dimensionless quantity (pure ratio) we can for convenience express in dB.

There is one exception to this, considered in § 4.2 and § 5.2, where the elevation dependence of the effective gain g_{TX} is taken into account, so that $g_{\mathrm{TX}} = g_{\mathrm{TX}}[\theta] = g_{\mathrm{TXmax}}\, V[\theta]$, where $V[\theta]$ is the *power* vertical radiation pattern (VRP), with maximum value 1. In this case it is $p_{\mathrm{TX}}\, g_{\mathrm{TXmax}}\, D$ which is applied afterwards, but of course the calculation in this case has already taken a specific VRP into account. The integral becomes:

$$\mathrm{PFD}_{\text{for unit EIRP density}} = \int_A V[\theta]\, f[x]\, dA$$

A final step converts the PFD to the familiar electric-field-strength units:

$$\text{electric field strength in dB}\mu\text{V/m} = 145.76 + (\text{PFD, in dBW/m}^2)$$

This last step needs to be interpreted with care in the event that the receiving antenna actually used is directional, see § 6.

Note that all field-strength results presented in this paper can be considered to be RMS values — the value indicated on a standard measuring receiver (as proposed in the various regulations) would be *greater*, by roughly 5 dB for a quasi-peak detector or 10 dB for a peak detector.

■ 3.3. Assumptions particular to the PLT system considered in the previous paper

As already discussed, this particular system is of no continuing interest, nevertheless it is useful to determine its EIRP density so that the results presented here may be compared with those in the previous paper. For this particular system, we assumed $p_{\mathrm{TX}} = 0.5\,\mathrm{mW}/10\,\mathrm{kHz}$, $g_{\mathrm{TX}} = 0.01$, and $D = 1/(90000\,\pi)$ systems / m^2 (based on representative sub-station spacing), so that the EIRP density was $1.76839 \times 10^{-11}\ W/\mathrm{m}^2$ or $-107.524\,\mathrm{dBW}/\mathrm{m}^2$, in 10 kHz.

■ 3.4. Appropriate data for a potentially-interfering system

When estimating the *density*, we have to be careful that we count ‘system installations’ appropriately. Many pitfalls exist!

A first concern is to consider whether every household equipped with the system *can* simultaneously cause emissions; a further step might consider whether it *will*.

The PLT access system considered in Ref. [1] used the same spectrum, on the same section of mains distribution wiring, to transmit data between the sub-station (transformer) and all the houses connected to it. It used time-division multiplexing so that at any one time either the sub-station end or one of the connected houses could be transmitting within the same spectrum. Clearly, in cases like this the appropriate density is the *density of equipped sub-stations*. The appropriate EIRP is a suitable average to reflect the fact that sometimes it is houses that are transmitting and sometimes the sub-station, and the characteristics (p_{TX} and g_{TX}) of houses and sub-station might differ. Note that in this shared-resource situation, the EIRP density does not really depend strongly on the degree of take-up of the service: two households per sub-station using the service cause the same emissions as 100 households per sub-station. As more households use the system simultaneously, each gets a smaller share of the available capacity but the emissions are unchanged as long as the system is always busy. Admittedly, if very few households per sub-station are equipped it becomes possible that the system will not be fully loaded and, depending on the system design, this might reduce the emissions on average. However, allowing for this should be done very cautiously indeed. As broadband connections become more common, it seems inevitable that ‘demand inflation’ will occur — e.g. web pages will become adorned with more graphics and so on, so that the act of choosing a new page when web browsing initiates the delivery of a large amount of data. Thus, where the total capacity is fairly small, as in Ref.[1], just one or two users per sub-station browsing the web may be enough to keep the system fully busy — but they will enjoy a fairly quick response. As more users browse simultaneously, each will see a slower response. We may also observe that a system with few users per sub-station is unlikely to repay the investment required in equipping every sub-station with modem equipment and a back-bone connection.

In contrast, *x*DSL systems using the telephone wiring for access do not share resource between households in the same way, nor would in-house networks using telephone wiring. (In-house networks using mains wiring are perhaps an intermediate case — adjacent households might not be able to use the same spectrum simultaneously because of cross-talk). So with these types of system, the density we should apply is the *density of equipped households*. Note that there is much talk of using *x*DSL to provide TV services: where this happens it seems reasonable to assume that 100% utilisation will occur at peak viewing times.

A further point concerns accounting for the variation in emissions between systems. The analysis assumes that there are enough interference sources that an average EIRP density can reasonably be taken. If some installations are known to have a higher EIRP because of fault conditions (e.g. wiring faults) that are likely to occur, we must include this knowledge in the average that we take. (Mathematically, there will be a probability distribution for the EIRP. We have to take its mean).

Consider an arbitrary simple example. Let the EIRP of a ‘normal’ system be 1 unit. Suppose that 1 in 10 systems has a fault which makes the EIRP 1000 units (30 dB greater). The total EIRP of 10 systems (1 faulty, 9 normal) is 1009 units or an average per-system EIRP of 100.9 units. We could, perfectly correctly, take the density of all installations, with their average EIRP of 100.9 units. However, in a case as extreme as this it would be more instructive to say that faulty systems dominate the problem. We could take the density of faulty systems together with their EIRP of 1000 units.

(Strictly correctly, we should take 1009). What this would usefully highlight is that the density of systems we are interested in — the density of faulty systems — is smaller, and we should take care to check whether integration (rather than summation) remains reasonable, as discussed at the end of § 2. Whichever way we calculate it, in this example the faults increase the cumulative effect significantly — by about 20 dB.

In contrast, consider another example, say only 1 in 100 systems has a fault which causes an EIRP of 10 units. The average per-system EIRP is now 1.09 units. The faults cause only a small increase in the cumulative emissions. Of course, a receiver close to one of these faults may be badly affected, but the faults are otherwise insignificant.

■ 3.5. Appropriate data for a mix of systems

Radio reception will be affected by whatever mix of potentially-interfering systems becomes established in use. Regulators charged with protecting radio reception may therefore have to take this mix into account. Suppose two types of system (e.g. PLT and *x*DSL) both operate in the same part of the spectrum. To obtain the cumulative effects of all the installations of both types we could apply a combined EIRP density:

$$\text{combined EIRP density} = p_{\mathrm{TX1}}\, g_{\mathrm{TX1}}\, D_1 + p_{\mathrm{TX2}}\, g_{\mathrm{TX2}}\, D_2$$

where the subscripts 1 and 2 refer to the two systems. Where more than 2 systems co-exist in the same spectrum (e.g. the two above plus home-phone networking) the summation is extended in an obvious way.

What is slightly hidden is that each system density D_i above will vary depending on the mix of systems that is deployed and their relative success. D_i (the density of deployed installations using system i) is the product of the *maximum density physically possible* (in the case of *x*DSL, the density of households equipped with a suitable phone connection; for PLT it may be the density of sub-stations, depending on the system configuration) and a *fractional implementation* measure. Note that these fractional implementations are an indirect measure of market success, and need not add up to 1. E.g. if every sub-station were equipped with PLT of the type in Ref. [1], with at least 1 user, its fractional implementation would be 1; this would remain the case although many households did not use it and used *x*DSL instead. In this situation the fractional implementations would add up to more than 1; equally plausible examples, where some sub-stations were uneconomic for PLT, and some households never took up either system, might sum to less than 1.

Regulators assessing the risk of cumulative interference affecting a radio service should consider a range of scenarios in which the possible, reasonable and likely combinations of values of fractional implementation were explored. Having competing access methods (which should in principle exert downward pressure on costs charged to the consumer) may well cause more cumulative emissions than where one access method has the monopoly.

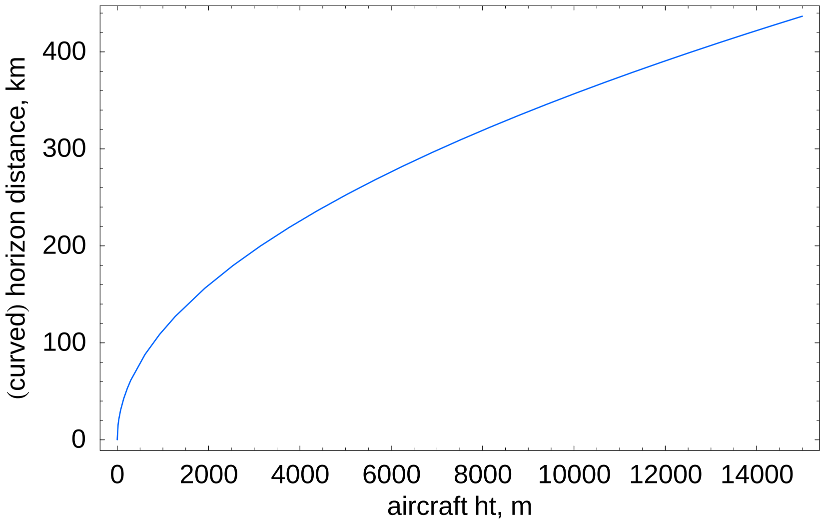
The simplification of using a combined EIRP density will not be appropriate if different systems are deployed in different areas, e.g. if country A uses system X and country B uses system Y. In this case the PFDs at a particular receiver position caused by each system should be separately calculated and the PFDs then summed.

□ 4. Interference to aircraft

■ 4.1. *x*DSL/PLT/etc. systems assumed to be isotropic radiators, as in the previous paper

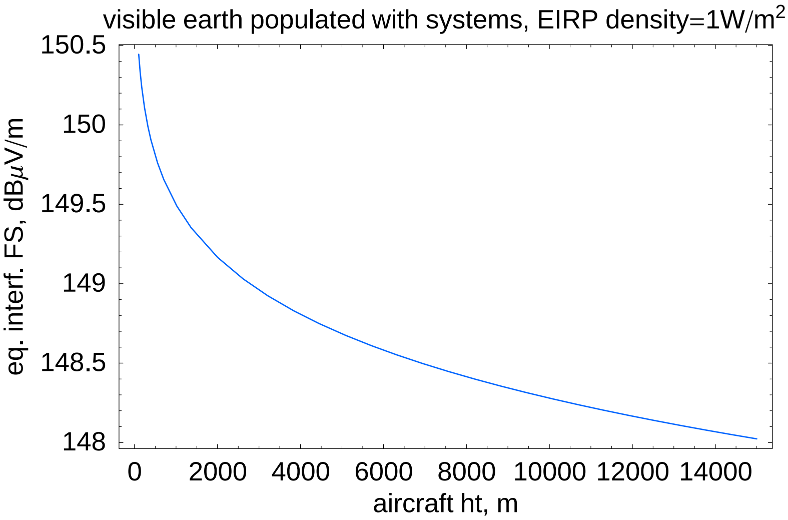
Aircraft flying over areas which are populated with *x*DSL/PLT/etc. systems may see an increase in the apparent noise floor. The geometry of the problem is derived in Appendix A 1.2, whereupon the interfering PFD at the aircraft can be calculated assuming free-space propagation to the aircraft from the interfering systems 'visible' to it. See Appendix A 2.1 for the derivation, including the analytical result by means of which different scenarios from those shown below may be plotted if needed. Note that the results are independent of frequency.

The region visible to the aircraft depends on the height at which it is flying. The Figure below shows the distance (measured around the curved Earth) from the point below the aircraft to the horizon, as seen from the aircraft:



It follows that the number of *x*DSL/PLT/etc. systems able to interfere with the aircraft increases substantially with height.

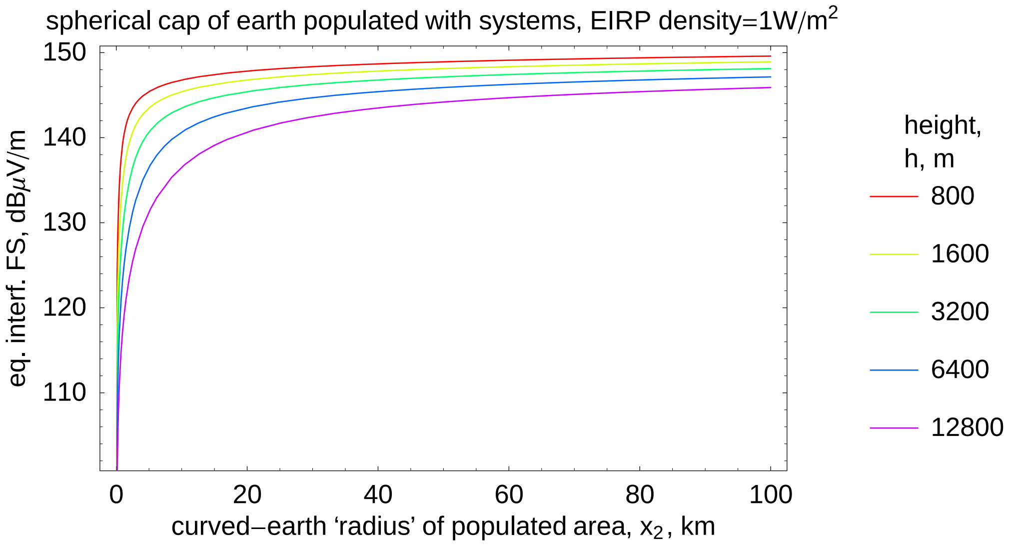
Suppose that the *x*DSL/PLT/etc. systems are present with unit EIRP density over the *whole visible area*. In this case we can calculate the interfering PFD at the aircraft (expressed as an equivalent electric field strength, FS), as shown in the following Figure:



Interestingly, under the assumption that all the visible Earth is populated with *x*DSL/PLT/etc. systems at the same density, there is relatively little variation in the interfering FS with aircraft height. In effect, as height increases, the strength of the contribution from any one interferer decreases, but the number of visible interferers increases nearly as quickly. To put this normalised result into perspective, we add the correction factor for the actual EIRP density (which is of course much less than $1\,W/\mathrm{m}^2$). For the particular PLT system considered in the previous paper [1], the relevant correction is $-107.5\,\mathrm{dBW/m^2}$, giving equivalent field strengths of the order of 40 to 43 $\mathrm{dB}\mu\mathrm{V/m}$ in $10\,\mathrm{kHz}$.

Now, the assumption of constant *x*DSL/PLT/etc. system density can perfectly reasonably be challenged for aircraft flying at great height, as in this case even while flying over a major conurbation there will be areas of countryside also visible. Nevertheless, it appears that there is scope for problems which would require more detailed examination. Aircraft often fly over major conurbations (especially London) while on approach to airports, and in some such cases the height is sufficiently low that all of the visible Earth *is* densely populated. Furthermore, the level of interference suggested by the paragraph above is sufficiently high that it would appear that a very significant amelioration will be necessary.

The following Figure looks at the problem a different way. We take various heights, and then plot the interference field strength received at the aircraft as a function of x_2, the curved-earth 'pseudo-radius' of the area populated with systems (thus modelling a range of conurbation sizes). The range of x_2 has been carefully chosen so that it just does not exceed x_{Max} (the limit of the visible earth) for the lowest height shown, so that for this height the interference reaches the level shown in the previous Figure. For greater heights, x_2 never approaches the corresponding x_{Max} — the interference at these heights therefore does not reach the levels shown in the previous Figure, as the visible earth is never filled with interfering systems. Nevertheless, the level of interference remains potentially significant:



At 800 m height, a populated area of 20 km 'radius' is sufficient to cause interference only 2 dB less than when the entire visible earth is filled.

None of this should be surprising. The existing level of man-made noise 'seen' by aircraft over cities reaches them by exactly the same free-space propagation mechanism that we have assumed for *x*DSL/PLT/etc. systems. It therefore follows that if a *x*DSL/PLT/etc. system raises the noise level in its immediate environment, then it must also increase the noise level for aircraft.

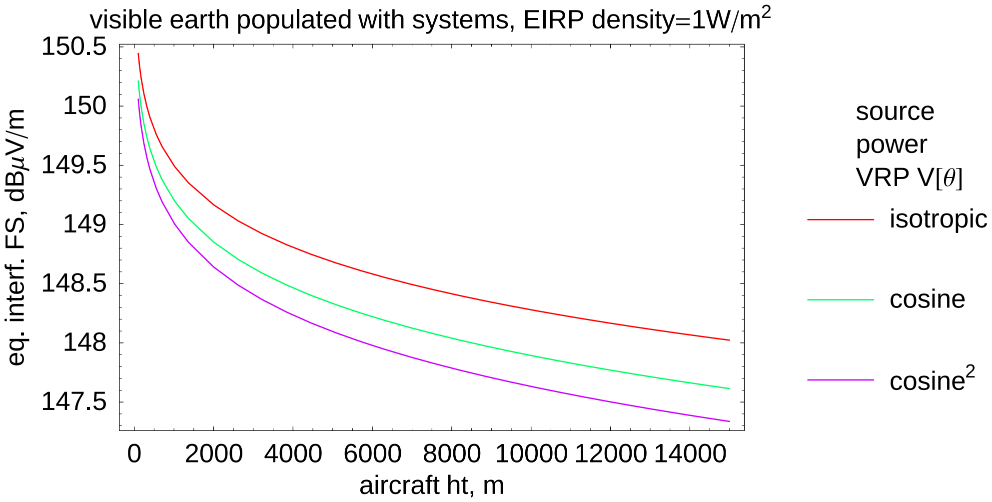
Clearly, those more familiar with the requirements for aircraft communications should study this topic closely, in view of the possible safety implications of disrupting aircraft communications or radio-navigation systems.

■ 4.2. *x*DSL/PLT/etc. systems assumed to have elevation-dependent radiation

Some proponents of *x*DSL/PLT/etc. systems have insisted that it is wrong to assume that the interference sources are, taken on the average, isotropic in behaviour. While the assumption appears well-founded in relation to *azimuth* (unless road, house and wiring orientation are *very* closely regulated indeed!) there is some justification to suppose that the average vertical radiation pattern may be non-uniform. One of the studies presented within CEPT SE 35, Ref. [2], suggests that for the particular PLT system it studied in Switzerland there was a variation with elevation angle, the variation depending on the injection point. For injection points within the home (using the normal mains sockets) the radiation decreases for high elevations. This possibly suggests that in this situation, with Swiss wiring practice, the vertical parts of the house wiring are the dominant radiators. Somewhat different radiation patterns were observed when considering the injection point at the sub-station transformer or the point where mains entered the home (normally the basement in Switzerland).

In principle, the interference can be calculated for any power VRP $V[\theta]$. In order to keep the problem mathematically tractable, we consider here two very simple VRPs, namely $V[\theta] = \mathrm{Cos}[\theta]$ or $\mathrm{Cos}[\theta]^2$. In fact the latter is not an unreasonable assumption, as it corresponds to the *power* VRP of a short dipole (or short monopole over a ground plane). The necessary integration is presented in Appendix A 2.3.

We plot the results assuming once again that *x*DSL/PLT/etc. systems having *unit EIRP density* (and isotropic HRP) populate the entire visible Earth, but with three different *power* VRPs — uniform (isotropic sources), $\mathrm{Cos}[\theta]$ and $\mathrm{Cos}[\theta]^2$. (The unit EIRP density now refers to the EIRP in the max direction, horizontal in these cases). We plot the equivalent interfering field strength as a function of aircraft height, with the following results:



We may note that, as we might expect, the VRPs which radiate less at high elevations cause less interference. However, despite the radiation being least from the points on the ground nearest the aircraft, the interference is not very greatly reduced — less than 1 dB.

□ 5. Interference to ground-based receivers via sky-wave

■ 5.1. *x*DSL/PLT/etc. systems assumed to be isotropic radiators

Sky-wave interference can be calculated in a similar way to the aircraft case — the main difference is the calculation of attenuation as a function of (curved-Earth) distance x from the source. We can approximate the very complicated behaviour of ionospheric propagation as being equivalent to the attenuation arising from free-space propagation, over a distance equivalent to that travelled by the wave on its one or more hops, plus an allowance for ionospheric absorption in each hop, and a further allowance for the loss in reflection from the Earth, where there is more than one hop.

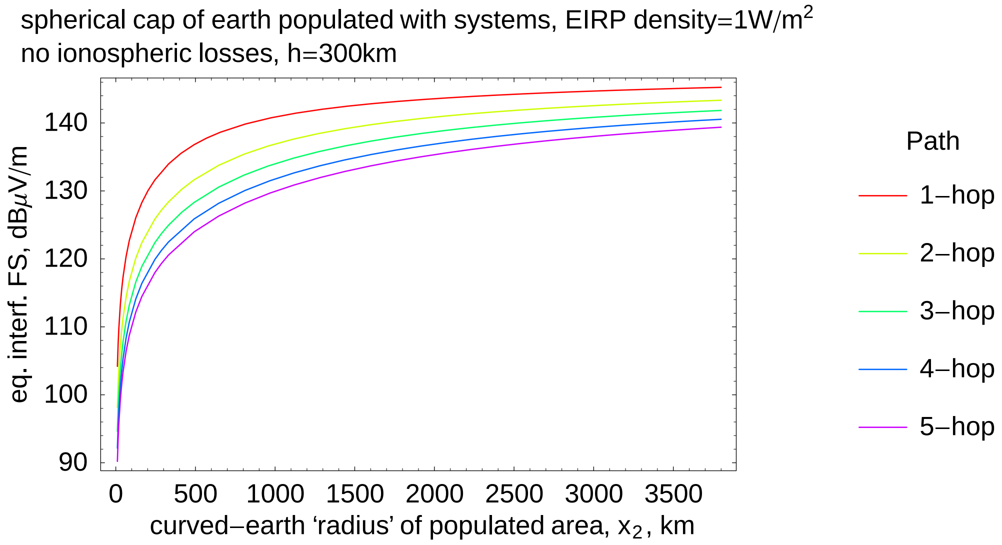
Details of the calculations are given in Appendix A 3. We assume that potentially-interfering *x*DSL/PLT/etc. systems uniformly populate an area of the Earth. We take the area as a pseudo-annular region, centred on the reception site, with outer curved-earth radius x_2 and possibly having a hole in the middle of curved-earth radius x_1. Thus x_2 defines the extent of the populated region, while x_1 can be used to allow for an *exclusion zone* around the receiver, or, if the frequency is above the critical frequency, to account for the effects of *skip distance*. The following plots assume an ionospheric height of 300 km.

Note that the results presented are *independent of frequency*. Ionospheric propagation is, of course, frequency dependent, but this is taken account of when *applying* the curves: the appropriate skip distance must be used, and the allowance for ionospheric losses chosen.

We can separately plot the contributions made by propagation via 1, 2 or more hops, always assuming uniform EIRP density and zero ionospheric losses — the appropriate corrections should be applied for any practical situation when interpreting the curves.

It is of interest to look at the problem two ways: we can vary x_1, to see the value of any exclusion distance or effect of skip, and we can vary the outer limit x_2 — in effect limiting the area over which systems are assumed. Note that x_2 must not be taken greater than the longest physically-possible value appropriate to the number of hops.

First we look at the effect of varying x_2; the plot range is chosen so that x_2 never exceeds the maximum distance for which 1-hop propagation is possible. x_1 is taken as zero — there is no exclusion zone and no limitation is imposed by skip (only possible if the frequency is below the critical frequency for vertical incidence). This case is shown in the Figure below, where the contributions from paths having different numbers of hops are shown separately, *assuming no ionospheric loss*.



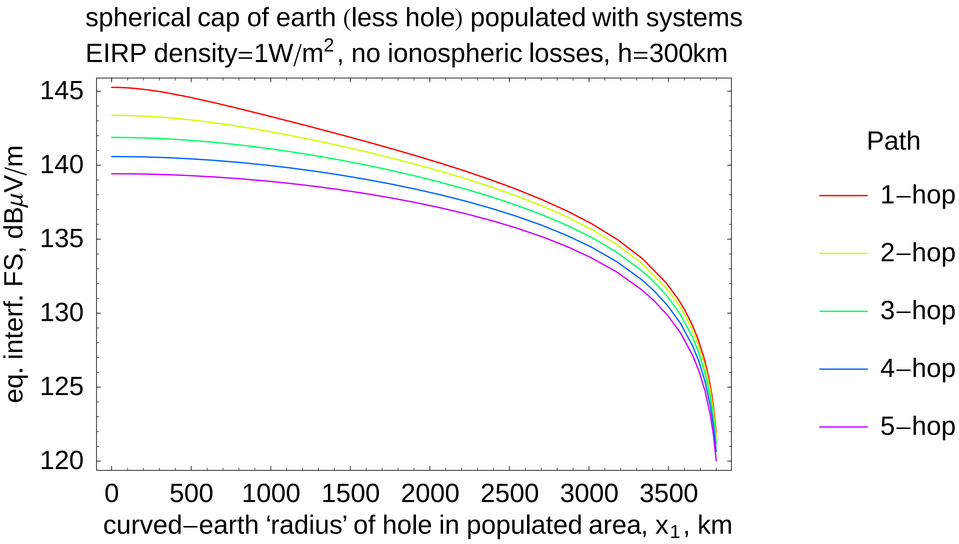
To obtain the *total* interference PFD, the contributions of each path should be determined, allowing for the loss applicable to the number of hops in each case†, and power-summed. Finally the total should be scaled to account for the appropriate EIRP density. However, it is clear that when ionospheric losses are significant the 1-hop path will dominate the calculation.

At first there is a rapid increase in interference as the outer limit is increased — the number of interferers present increases rapidly, while the attenuation of the added outermost ones is scarcely less than for the nearest ones. For larger distances, the added area is relatively less important and the attenuation of its contributions greater. Once a fairly large area around a receiver (say 500 km radius) is densely populated with systems, then the influence of more distant-interferers does not greatly increase the interference further.

If we consider the parameters applicable to the corresponding example presented in Ref. [1], namely EIRP density of -107.5 dBW / m^2 and 1-hop ionospheric loss of 10 dB, then the equivalent field strength in the 1-hop case (red curve above) reaches a value of 28.5 dBμV / m at 3500 km, in agreement with the single curve presented in [1].

† To make this clear, suppose we assume that each hop incurs an ionospheric loss of I dB while the ground reflection between hops causes a further reflection loss of R dB. We have to subtract I dB from the 1-hop curve, $(2I + R)$ dB from the 2-hop curve, and $nI + (n-1)R$ dB from the n-hop curve. (If we expect different losses to be incurred on successive hops, then we could write e.g. $I_1 + I_2$ in place of $2I$, and so on).

Secondly, in the Figure below we look at the effect of varying x_1 while x_2 is set to the maximum distance for which 1-hop propagation is possible (namely 3836 km). This means we can account for skip effects or the introduction of any exclusion zone.



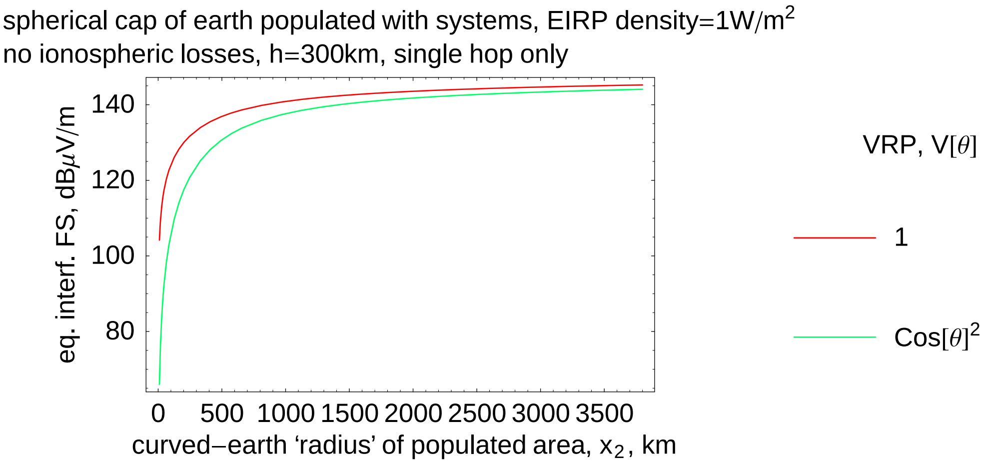
It can be seen that the interference decreases only slowly until a very large exclusion distance is reached. This is easily explained in combining two concepts. The rate of increase of attenuation with distance from the receiver is not dramatic, as previously explained, while the nearby part of the Earth has a relatively small area compared with the whole area included in the calculation. So only when the exclusion distance is large is the number of interferers greatly reduced, and their value further diminished by distance, whence the shape of the curve follows.

These results show that relatively-sensitive sites, which are relatively free from direct (e.g. ground-wave) interference because there are no nearby interferers (e.g. the site is isolated, there is an exclusion zone, or the receiver is in a country where the systems are not deployed) are indeed potentially vulnerable to the effects of cumulative interference from far-off interferers. Although the curves presented only show the effects of varying either x_1 or x_2 while the other takes a single assumed value, other scenarios can be assessed as desired, using the exact analytical results provided in Appendix A 3.

■ 5.2. *x*DSL/PLT/etc. systems assumed to have elevation-dependent radiation

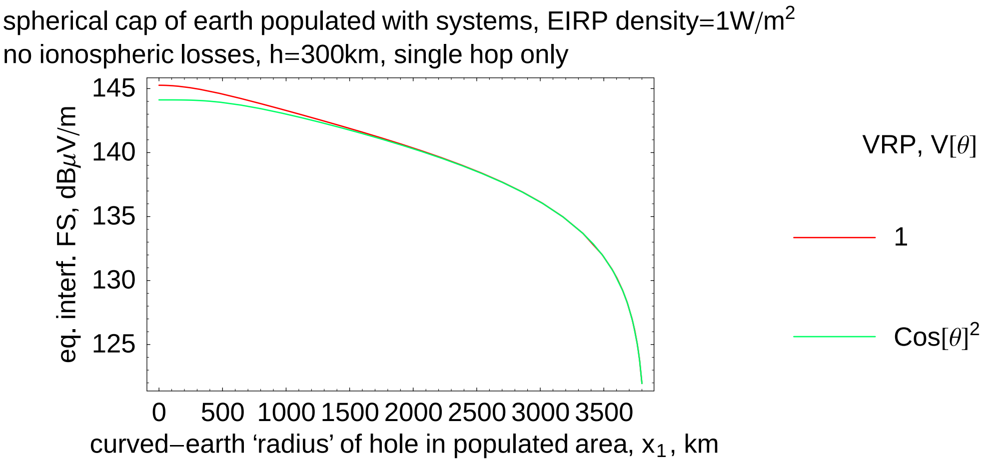
As for the aircraft case discussed in § 4.2, we can also study how the effect of interference might be changed if the vertical radiation pattern of the interference sources is non-uniform. We consider one example, where the sources' *power* VRP $V[\theta] = \mathrm{Cos}[\theta]^2$. Details of the necessary integral are given in Appendix A 3.3. We can plot the results so as to show how the two plots of the previous Section change when the VRP is introduced. In both cases we show only the 1-hop case, for simplicity and because it can be expected normally to represent the dominant component.

In the Figure below we look at the effect of varying x_2; as before, the plot range is chosen so that x_2 never exceeds the maximum distance for which 1-hop propagation is possible, while x_1 is taken as zero:



Only when x_2 is small is the interference reduced significantly by the presence of the $\text{Cos}[\theta]^2$ power VRP. This is reasonable, since when the area containing interferers is small, the elevation angle is always high and thus the VRP has a significant effect.

Secondly, in the Figure below we look at the effect of varying x_1, while x_2 is set to the maximum distance for which 1-hop propagation is possible (namely 3836 km). This means we can account for skip effects or the introduction of any exclusion zone.



Introducing the VRP has less effect in this case. When x_1 is large, all the interferers are far away and have a low elevation, so the particular VRP chosen has negligible effect. Even when x_1 is small, the effect of introducing the VRP is small as x_2 is large (corresponding to the RHS of the *previous* figure) — the majority of the sources are further away, with low elevation, while the nearest sources having high elevation are less numerous.

□ 6. Discussion of receiving-antenna pattern

The analysis presented so far has assumed that all the interference contributions reaching the receiver, from whatever direction, are added up on an equal basis. It might appear that this assumes that an omnidirectional receiving antenna is being used — and we know that purely isotropic antennas are very rare, albeit a useful theoretical concept. Do we have to worry about this, and if so when?

6.1. Can we even talk about field strength?

To start in a rather abstract way, suppose that a noise-like signal impinges on a receiver equally from all directions in 3-D space — the noise is isotropic in nature. Suppose that it is of such an amplitude that a true isotropic (and lossless) antenna gives an available output of 1 power unit. Now replace the isotropic antenna by one having gain g (and which is also lossless). The available output remains unchanged — still 1 power unit.

This briefly counter-intuitive result follows from what is perhaps better understood in relation to using the same antennas for *transmitting*. If we feed 1 W *into* an isotropic antenna it radiates equally in all directions, with an EIRP (Equivalent Isotropically Radiated Power) which is by definition 1 W. If we now feed the same power into a directional antenna, the radiation is concentrated so that more of it goes in the direction of the main lobe of the antenna. As far as a distant receiver is concerned, it is as if more power had been radiated. If it seems as though the EIRP in the direction of that distant receiver has some value g W, then we say the (power) gain of the (transmitting) antenna in that direction is g. But the (transmitting) antenna has not added any power; we said it was lossless, and it radiates exactly as much power as is fed into it. It follows that if $g > 1$ in some directions, then it must also be true that $g < 1$ in others. In fact, if we integrate g over all solid angle, it must average to unity:

$$\frac{1}{4\pi} \int_{\text{all }\Omega} g \, d\Omega = 1.$$

An analogous result applies when we receive the isotropic noise. If the receiving antenna has gain $g > 1$ in some directions (causing it to receive more noise from those directions), then it will have $g < 1$ in others (which therefore will contribute less to the total output). The available output power is the same, regardless of the directivity pattern of the antenna.

Now we are used to the idea that we can calibrate a receiving antenna for its (boresight) gain, so that when we receive a signal from a distant transmitter, if we measure the voltage output of the antenna, we can apply the calibration factor and thereby determine the field strength of the received field as some value $E\,\mu\text{V}/\text{m}$. If we use another antenna of different gain, then we expect to measure a *different* output voltage, to which we apply the appropriate, *different* calibration factor — and expect to get back to the conclusion that the field strength is the *same* value $E\,\mu\text{V}/\text{m}$. If we use the same pair of antennas to measure the isotropic noise field, then (assuming they are lossless) the output they deliver is the same — also the same as that from an isotropic antenna. When we apply the relevant calibration factors we conclude from the measurements made with the two different antennas that the noise field strength has two different values.

This makes it potentially misleading to talk about field strengths!

This appears to be indirectly confirmed if we consult the ITU-R Recommendation on various types of noise [3]. It presents values for the noise due to all manner of causes in terms of the noise figure or noise temperature of the antenna. It then explains how to calculate the equivalent field strengths for two example antennas, giving *different* results. (Unfortunately it quotes the formulae, with numerical values for the constants, without explanation).

6.2. Effect of receiving-antenna directivity, with sources evenly distributed in azimuth

Now, our interference ‘noise’ is not truly isotropic. By the definitions of the situations we have chosen, we can expect the noise to come uniformly from all *azimuths*, but not from all *elevations*. Nevertheless, the calculations we have performed correctly give the PFD (or the equivalent electric field strength derived from it) *as would be measured using an isotropic antenna*. If we substitute different antennas we will not necessarily get the same available output, as we would if the noise field were truly isotropic. (Again, Ref. [3] seems to encounter the same limitation in relation to atmospheric noise, which, quite obviously, comes preferentially from the regions where the storms are).

We may usefully distinguish the effects of the elevation and azimuth patterns of the receiving antenna. If the *elevation* pattern has a main lobe covering the range of elevations from which propagation is physically possible, and its regions of reduced response correspond to those elevations at which no ‘noise’ arrives anyway, then the antenna can exhibit increased output (without cheating the rules on directivity!). And similarly, an antenna whose elevation-pattern minima discriminate against all incoming ‘noise’ rays will deliver less output than our isotropic reference.

Concerning azimuth, our examples have ensured, by choice of rotationally symmetric populations of interference sources, that the ‘noise’ comes equally from all azimuth directions. So if our receiving antenna were to have directivity in azimuth, but not elevation, then its output would be the same as for an isotropic one. Such an antenna is probably physically impossible; however, many receiving antennas in the HF range at least have gains which are fairly small (high gain implies sizes that are physically inconvenient for most users), suggesting that the error will not be so great in practice.

So, for many practical situations (with rotationally-symmetric source populations) it would seem that the output of the receiving antenna will be roughly the same as the output of an isotropic antenna, as predicted by the calculations presented in this paper.

If we had a receiving antenna which was omni-directional in azimuth, but had a vertical radiation pattern varying with elevation (a vertical monopole is an example), then it should be possible to extend the integration method to accommodate it. Instead of calculating the PFD, we would include the elevation dependence of the gain (and hence of the effective area) inside the integrand so that we calculate the received available power.

It is interesting to consider the case of sky-wave interference, as in § 5.2, as, resulting from the assumption of a constant-height ionosphere, the elevation angle at the receiver is the same as that at the source. It follows that in this case the effect of introducing a VRP at the receiver will be just the same as introducing the same VRP at the source. As we have seen, a $\text{Cos}[\theta]^2$ power VRP (corresponding to a short monopole) would cause the available ‘noise’ power at receiver to reduce, but not by a significant amount unless the scenario involves only nearby interferers so that all rays arrive at near-vertical incidence.

■ 6.3. What if the interferers only come from one azimuth?

We have used rotationally-symmetric scenarios in order to keep the integrations simple. Can we make inferences about other situations?

Suppose that interferers are confined to some sector occupying α radians of azimuth. If we take $x_1 = 0$, the interferers will lie within a spherical triangle. With $x_1 > 0$, the interferers will lie on a spherical triangle with a spherical-triangle piece cut off (is this a *spherical rectangle*?).

Clearly, if the receiving-antenna is omni-directional in azimuth, the available power will be $\frac{\alpha}{2\pi}$ of that predicted for the rotationally-symmetric case.

If the receiving-antenna azimuthal pattern has a main lobe matching the α-wide sector exactly (while having negligible response in all other directions), or its main lobe is even narrower than α, then the available power will be same as for an omni-directional antenna in the rotationally-symmetric case.

If the main lobe is broader than α, then the available power will be somewhere in between the two cases.

☐ 7. Conclusions

Systems which re-use mains or phone wiring for communications purposes are currently of interest. As well as their obvious benefits they have the potential to cause interference to radio systems. Interference to receivers in the immediate vicinity is fairly readily measured and legislated for. However, with the potential for mass-market deployment of such systems it is also important to consider whether more-distant receivers may also be adversely affected by the *cumulative effect* of all the installations.

The level of cumulative interference caused by distributed interference sources (such as *x*DSL, PLT or home-networking systems) has been analysed, following the method used by the author in a previous paper, Ref. [1]. The sources are treated as if they were a continuum spread over the area populated with systems, so that their cumulative effect can be obtained by integration rather than summation. However, the results have been presented slightly differently from the previous paper, in a system-independent way. The idea is that for any particular system under examination a representative *EIRP density* is determined, giving a correction factor in dB which is added to the generalised system-independent results presented here. In this way, the present results are in principle applicable to any type of distributed interferer.

Results have been presented for two classes of interference scenario, namely:

- to aircraft from sources on the ground
- to ground-based receivers from sources on the ground elsewhere, via sky-wave propagation

(The more specialised case of protecting a ‘sensitive site’ from ground-wave interference using an ‘exclusion zone’ has not been re-visited; the reader is referred back to Ref. [1], from which the necessary procedure can still be deduced).

Aircraft use both communications and radio-navigation receivers, and are believed to operate with relatively low signal levels. The level of interference caused by distributed sources on the ground depends of course on the EIRP density, for which the author awaits information on representative values for a range of systems. However, using those values applicable to the PLT proposal (now withdrawn) considered in Ref. [1] shows that a substantial interference contribution would be received on an aircraft flying over a well-populated area where such PLT systems were deployed.

The interference received by an aircraft when the entire visible earth is populated with systems at a uniform density is nearly independent of height — the increase in attenuation with height being very nearly balanced by an increase in the number of visible interferers. Limiting the area containing interferers, from the visible earth to a smaller area representative of a major conurbation (in effect assuming that outlying areas around a city contain no interferers), does not decrease the interference very greatly, unless the aircraft is very high.

A greater sophistication of the interference model was also tried, incorporating a vertical radiation pattern for the interference sources. In the examples examined, the radiation upwards was reduced (as would be expected if the dominant radiators in the system were vertical). Interestingly, this only marginally reduces the interference received by an aircraft.

The implications of the possibility of interference to aircraft communications and navigations systems should be rigorously studied by the relevant competent authorities.

The threat to ground-based receivers from interference propagated by sky-wave propagation has also been presented in the system-independent way, without making any specific assumptions about the ionospheric loss. This means that results for a particular scenario may be inferred from the curves presented by:

- taking account of the relevant *skip distance* in reading the curves
- *adding* the EIRP-density correction (applicable to the interferer type), and
- *subtracting* the appropriate ionospheric loss.

The results show that sky-wave interference from widespread *x*DSL/PLT/etc. systems to ground-based receivers may indeed not always be negligible, even though it is less than that shown to be suffered by aircraft. Furthermore, incorporating a vertical radiation pattern for the sources into the model did not significantly reduce the interference, just as for the aircraft case.

Detailed derivations, with analytical results for many of the integrals, are presented in the Appendix so that other workers may try the scenarios of interest to them.

☐ 8. References

1. STOTT, J H, 1999. Protection of ‘sensitive’ receiving sites. BBC R&D Technical Note No. 1282C(99).

Note: this document was originally submitted to a UK group, the RA Working Group on HF mains signalling. It was subsequently input to various bodies, including the CEPT SE 35. It is openly available on the BBC R&D website, http://www.bbc.co.uk/rd.

2. OFCOM & ASCOM, 2001. Measured skywave radiation characteristics of low voltage distribution networks excited by PLC systems transmitting in the HF-band (FAFIRA Report No 2)

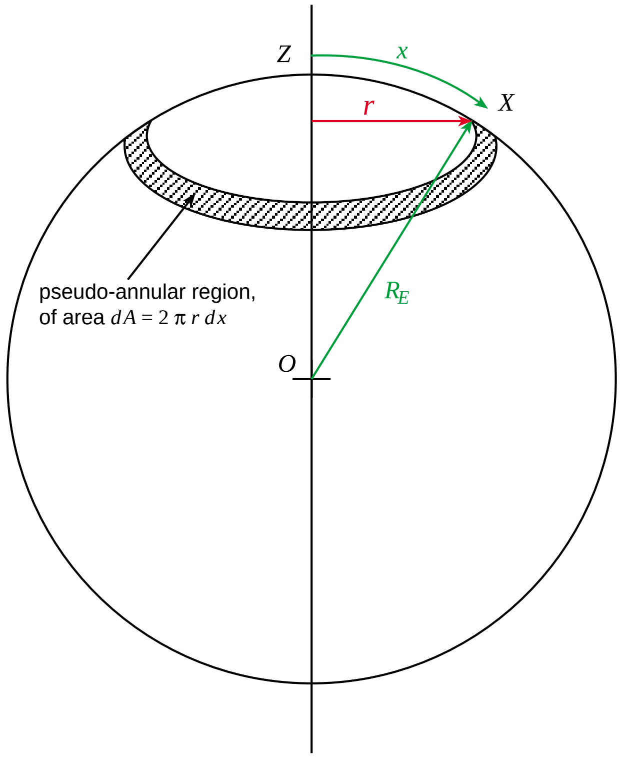
Note: this document was input to CEPT SE 35, gaining the reference number SE 35(01) 22; it may not be publicly available.

3. ITU-R Rec. P.372. Radio Noise

Appendices

◻ A 1. Geometry

▪ A 1.1. Annulus on curved Earth



The Figure above shows an annulus-like infinitesimal ring of area dA on the curved surface of the Earth, whose radius is R_E and whose centre is at O. The ring is at distance x from the receiving point Z (as measured round the curved surface), and has a radius, measured from OZ, of r. The angle $\angle$ ZOX, i.e. half that subtended at O by the ring, is $\frac{x}{R_E}$ radians and so r is given by:

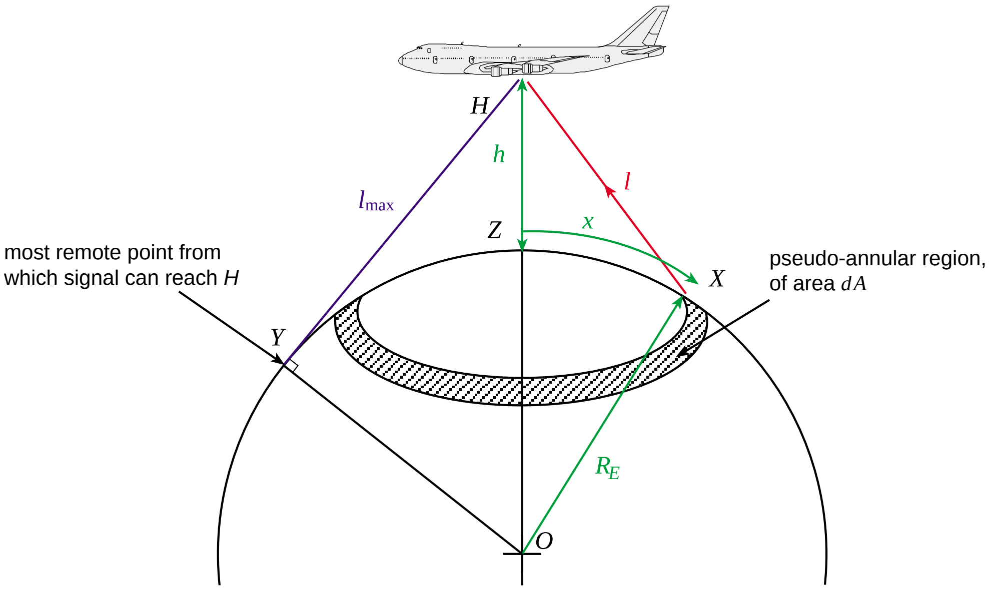
$$r = R_E \operatorname{Sin}\left[\tfrac{x}{R_E}\right]$$

and so the area of the ring is given by:

$$dA = 2\,\pi\,r\,dx = 2\,\pi\,R_E \operatorname{Sin}\left[\tfrac{x}{R_E}\right] dx$$

■ A 1.2. Geometry of propagation to aircraft

Interference from PLT and other distributed-source systems can reach aircraft. The geometry of the problem is shown in the Figure below:



The aircraft is flying at height *h* above the Earth, and is at point *H* vertically above *Z*. Signals from point *X* travel to the aircraft along the slant path *XH*, whose length *l* is given by:

$$l = \sqrt{R_E{}^2 - 2\,\mathrm{Cos}\left[\tfrac{x}{R_E}\right] R_E\,(h + R_E) + (h + R_E)^2}$$

Note that there is a limit to the distance from which signals can reach *H*, since the aircraft can only ‘see’ a limited area of the Earth. The furthest position from which signals can directly reach *H* is *Y*, where *HY* is tangential to the Earth's surface. The distance (over the curved surface) from *Z* to *Y* is then given by:

$$x_{\mathrm{Max}} = R_E\ \mathrm{ArcCos}\left[\tfrac{R_E}{R_E + h}\right]$$

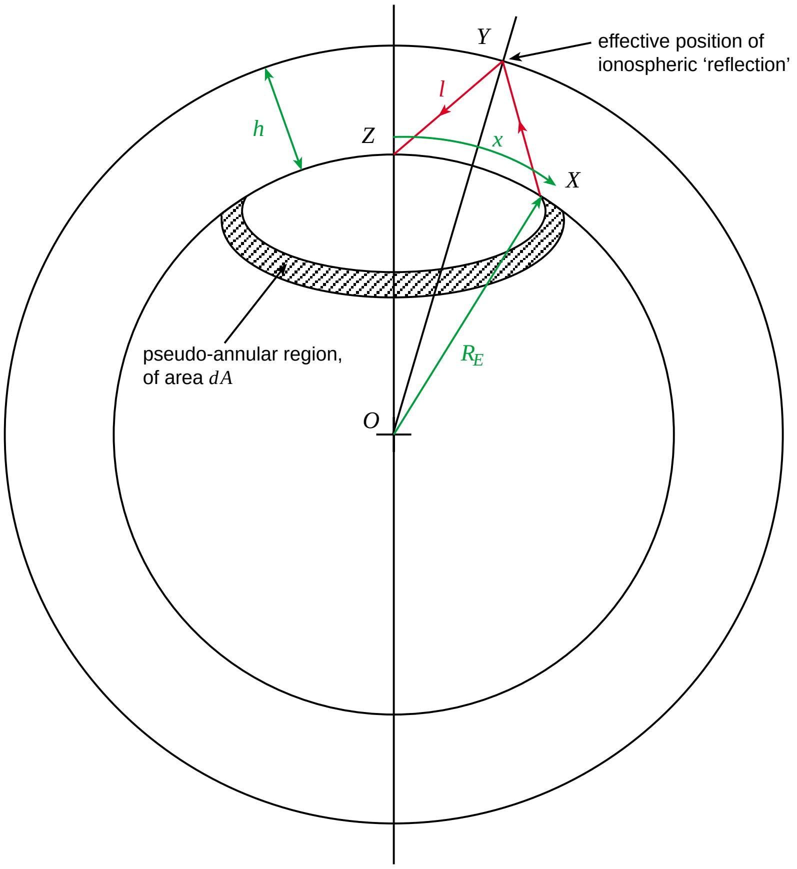
The distance $l_{\max}$ from aircraft to *Y* is simply $\sqrt{h(h + 2\,R_E)}$.

The elevation angle θ of the aircraft as seen from point X is of interest, i.e. the angle that XH makes with the tangent to the Earth’s surface at X. Using the Sine rule in triangle OXH we get:

$$\mathrm{Sin}[\theta + \pi/2] = \mathrm{Cos}[\theta] = \tfrac{(R_E + h)}{l}\ \mathrm{Sin}\left[\tfrac{x}{R_E}\right]$$

$$= \frac{(R_E + h)\,\mathrm{Sin}\left[\frac{x}{R_E}\right]}{\sqrt{R_E{}^2 - 2\,\mathrm{Cos}\left[\frac{x}{R_E}\right] R_E\ (h + R_E) + (h + R_E)^2}}$$

■ A 1.3. Geometry of sky-wave propagation



Consider sky-wave propagation from a point X to a receiver at Z. The curved-surface distance ZX is x. The angle $\angle$ ZOX is thus $\frac{x}{R_E}$ radians. Suppose that the signal makes n hops in general (each from earth to ionosphere to earth), and that reflection takes place at an effective ionospheric height of h. (The Figure above shows a single hop for clarity, $n = 1$). Assuming constant effective ionospheric height h, each half-hop involves a slant-path (e.g. ZY) of identical length l, which can be computed using the Cosine rule for the triangle ZYO, noting first that angle $\angle$ ZOY is, in general, angle $\frac{\angle \mathrm{ZOX}}{2\,n}$. The total sky-wave path length s is thus given by:

$$s \;=\; 2\,n\,l \;=\; 2\,n\sqrt{{R_E}^2 - 2\,\mathrm{Cos}\left[\tfrac{x}{2\,n\,R_E}\right] R_E\,(h + R_E) + (h + R_E)^2}$$

Note that using n hops there is a *maximum* distance $x_{\mathrm{Max},n}$ which can be reached, whereupon rays leave the Earth tangentially, given by:

$$x_{\mathrm{Max},n} = 2\,n\,R_E\;\; \mathrm{ArcCos}\left[\tfrac{R_E}{R_E + h}\right]$$

If the operating frequency $f > f_C$, where f_C is the critical frequency above which a wave normally incident on the ionosphere is not reflected, then there is also a *minimum* distance (the *skip distance*) at which a single-hop path can be detected. The skip distance depends on the ratio f / f_C. Similarly, each possible multi-hop mode will also have a corresponding distance. However, some systems we are concerned with use relatively-low 'high frequencies' so that even vertical rays will be reflected and no skip-distance effect occurs.

The elevation angle θ of the radiation is of interest, i.e. the angle that XY makes with the tangent to the Earth's surface at X. Using the Sine rule in triangle OXY, in a similar way to the aircraft case, we get:

$$\mathrm{Sin}[\theta + \pi / 2] = \mathrm{Cos}[\theta] = \frac{(R_E + h)}{l}\ \mathrm{Sin}\left[\frac{x}{2\,n\,R_E}\right] = \frac{\mathrm{Sin}\left[\frac{x}{2\,n\,R_E}\right](h + R_E)}{\sqrt{R_E^2 - 2\,\mathrm{Cos}\left[\frac{x}{2\,n\,R_E}\right] R_E\,(h + R_E) + (h + R_E)^2}}$$

□ A 2. Calculation of interference to aircraft

■ A 2.1. Derivation for isotropic sources

Taking the geometry from Appendix A 1.2, we have a slant-path length l given by:

$$l \;=\; \sqrt{R_E{}^2 - 2\,\mathrm{Cos}\left[\tfrac{x}{R_E}\right] R_E\,(h + R_E) + (h + R_E)^2}$$

while the maximum distance around the Earth from the point below the aircraft to the horizon (as seen from the aircraft) is given by:

$$x_{\mathrm{Max}} \;=\; R_E\;\;\mathrm{ArcCos}\left[\tfrac{R_E}{R_E+h}\right].$$

We may assume that simple free-space propagation applies, so:

$$\begin{aligned} f[x] \;&=\; \frac{1}{4\,\pi\,l^2} \\ &=\; \frac{1}{4\,\pi\,(R_E{}^2 - 2\,\mathrm{Cos}[\tfrac{x}{R_E}]\,R_E\,(h+R_E) + (h+R_E)^2)} \end{aligned}$$

and the integrated interference, assuming that the interference sources average to isotropic behaviour, becomes:

$$\begin{aligned} \mathrm{PFD} = \int_A p_{\mathrm{TX}}\,g_{\mathrm{TX}}\,D\,f[x]\,dA \;&=\; \int_{x_1}^{x_2} p_{\mathrm{TX}}\,g_{\mathrm{TX}}\,D\,\frac{1}{4\,\pi\;(R_E{}^2 - 2\,\mathrm{Cos}[\tfrac{x}{R_E}]\,R_E\,(h+R_E) + (h+R_E)^2)}\;2\,\pi\,R_E\,\mathrm{Sin}\left[\tfrac{x}{R_E}\right]dx \\ &=\; \frac{p_{\mathrm{TX}}\,g_{\mathrm{TX}}\,D\,R_E}{2}\int_{x_1}^{x_2}\frac{\mathrm{Sin}[\tfrac{x}{R_E}]}{(R_E{}^2 - 2\,\mathrm{Cos}[\tfrac{x}{R_E}]\,R_E\,(h+R_E) + (h+R_E)^2)}\;dx \end{aligned}$$

where x_1 and x_2 are the curved-earth distances from the point Z below the aircraft to the inner and outer boundaries of the region containing the interferers (a curved-surface pseudo-annular region like that illustrated in Appendix A 1.2, except that in this case it is of distinctly finite width). Normally we would expect to take $x_1 \to 0$, so that the populated region is a spherical cap. x_2 can of course never exceed $x_{\mathrm{Max}} = R_E\;\mathrm{ArcCos}\left[\tfrac{R_E}{R_E+h}\right]$, as that is the limit of the visible Earth. For our purposes we need the unit-EIRP-density PFD, which is:

$$\mathrm{PFD}_{\text{for unit EIRP density}} \;=\; \frac{R_E}{2}\int_{x_1}^{x_2}\frac{\mathrm{Sin}[\tfrac{x}{R_E}]}{(R_E{}^2 - 2\,\mathrm{Cos}[\tfrac{x}{R_E}]\,R_E\,(h+R_E) + (h+R_E)^2)}\;dx$$

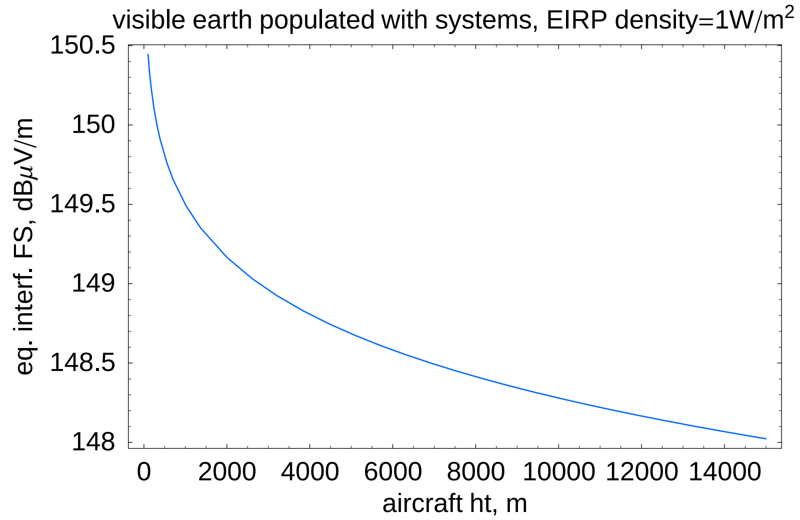
which can be shown to be:

$$\frac{R_E}{4\,(h+R_E)}\;\mathrm{Log}\left[\frac{-h^2 - 4\,h\,\mathrm{Sin}[\tfrac{x_2}{2\,R_E}]^2\,R_E - 2\,R_E^2 + 2\,\mathrm{Cos}[\tfrac{x_2}{R_E}]\,R_E^2}{-h^2 - 4\,h\,\mathrm{Sin}[\tfrac{x_1}{2\,R_E}]^2\,R_E - 2\,R_E^2 + 2\,\mathrm{Cos}[\tfrac{x_1}{R_E}]\,R_E^2}\right].$$

Note that the Log[ ] function above is a natural logarithm, sometimes written Log_e[ ] or ln[ ].

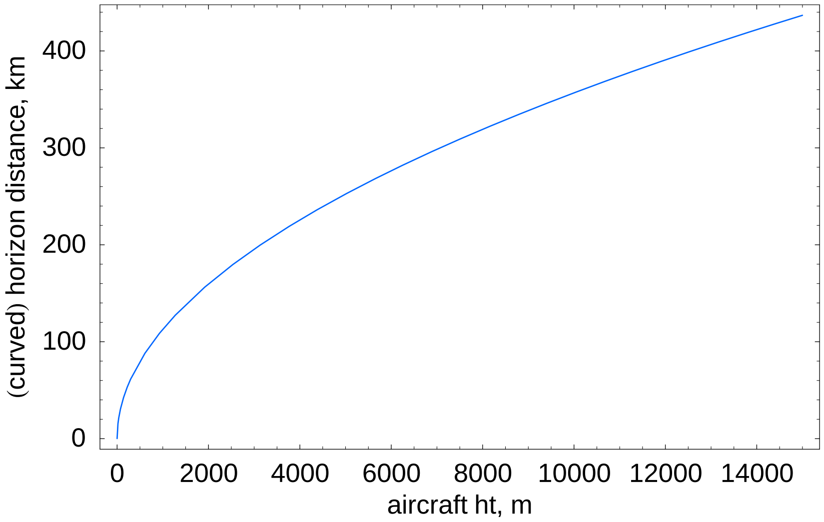
■ A 2.2. Graphical results for isotropic sources

We assume that *x*DSL/PLT systems having *unit EIRP density* (and isotropic VRP) populate the entire visible Earth. Note that this assumption might well be reasonable for moderate aircraft heights when flying over an extended conurbation. Then we can plot the equivalent interfering field strength as a function of aircraft height by setting $x_1 = 0$ and $x_2 = R_E \text{ ArcCos}\left[\frac{R_E}{R_E + h}\right]$, with the following results:

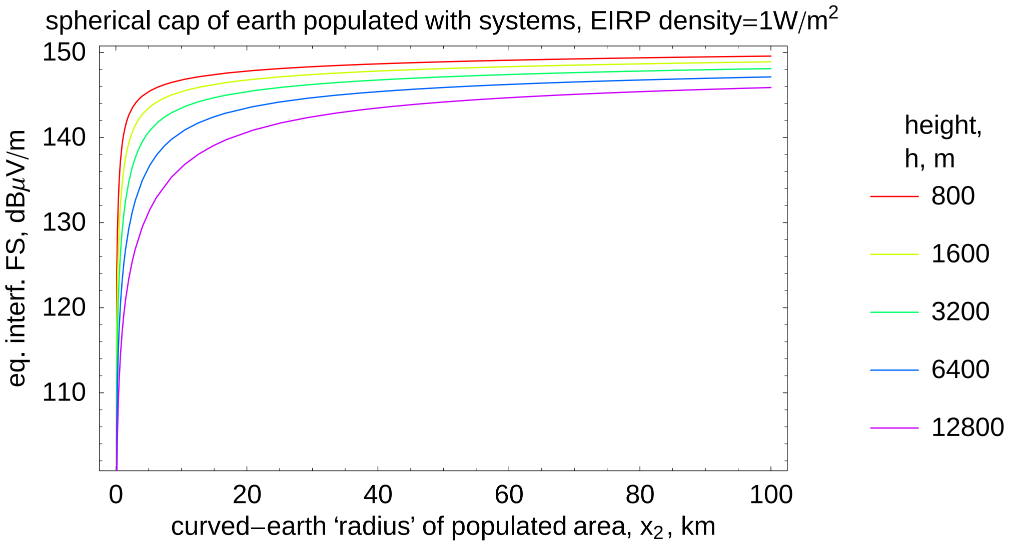


In effect, as height increases, the strength of the contribution from any one interferer decreases, but the number of visible interferers increases nearly as quickly. Of course, from *very* great heights the entire visible surface would be unlikely to be fully populated with *x*DSL/PLT systems.

We can put this into perspective by plotting the distance to the aircraft's visible horizon (measured around the Earth from the point vertically below the aircraft) as a function of height:



It may reasonably be argued that typical conurbations are never so large as would be required to fill the visible earth for an aircraft flying at heights in the upper part of the range shown above. We can check this by plotting the result another way. We take various heights, and then plot the interference FS received at the aircraft as a function of x_2, the curved-earth 'pseudo-radius' of the area populated with systems. The range of x_2 has been carefully chosen so that it just does not exceed x_{Max} for the lowest height shown. For greater heights, x_2 never approaches the corresponding x_{Max} — the interference at these heights therefore does not reach the levels shown in the first graph in this section, as the visible earth is never filled with interfering systems. Nevertheless, the level of interference remains potentially significant:



■ A 2.3. Derivation for examples where sources have a non-isotropic VRP

Some proponents of *x*DSL/PLT systems have insisted that the assumption that the interference sources are, taken on the average, isotropic in behaviour is wrong. While the assumption appears well-founded in relation to *azimuth* (unless road, house and wiring orientation are *very* closely regulated indeed!) there is some justification to suppose that the average vertical radiation pattern may be non-uniform. One of the studies presented within CEPT SE 35 [2] suggests that for the PLT system it studied in Switzerland there was a variation with elevation angle, the variation depending on the injection point. For injection points within the home (using the normal mains sockets) the radiation decreases for high elevations. This possibly suggests that in this situation, with Swiss wiring practice, the vertical parts of the house wiring are the dominant radiators. Somewhat different radiation patterns were observed when considering the injection point at the sub-station transformer or the point where mains entered the home (normally the basement in Switzerland).

In order to keep the problem mathematically tractable, we assume simple VRPs, namely $V[\theta] = \mathrm{Cos}[\theta]$ or $\mathrm{Cos}[\theta]^2$. In fact the latter is not a totally unreasonable assumption, as it corresponds to the *power* VRP of a short dipole (or monopole over a ground plane). The necessary integral is

$$\mathrm{PFD}_{\text{for unit EIRP density}} = \frac{R_E}{2} \int_{x_1}^{x_2} \frac{V[\theta]\ \mathrm{Sin}[\frac{x}{R_E}]}{(R_E{}^2 - 2\,\mathrm{Cos}[\frac{x}{R_E}]\,R_E\,(h+R_E) + (h+R_E)^2)}\,dx$$

▫ Case with $V[\theta]$ = Cos[θ]

From §A 1.2 , we get $\mathrm{Cos}[\theta] = \frac{(R_E+h)}{l}\,\mathrm{Sin}\left[\frac{x}{R_E}\right]$, so the integral becomes:

$$\mathrm{PFD}_{\text{for unit EIRP density}} = \frac{R_E}{2}\int_{x_1}^{x_2}\frac{(R_E+h)\,\mathrm{Sin}[\frac{x}{R_E}]^2}{(R_E{}^2-2\,\mathrm{Cos}[\frac{x}{R_E}]\,R_E\,(h+R_E)+(h+R_E)^2)^{3/2}}\,dx$$

which, with $x_1 \to 0$, can be shown to be a rather complicated function:

$$\begin{aligned}\mathrm{PFD}_{\text{for unit EIRP density}} = \frac{1}{2\,(h+R_E)}\\
&\left(-i\,h\left(\mathrm{EllipticE}\left[\mathrm{ArcSin}\left[\frac{h}{h+2\,R_E}\right],\frac{(h+2\,R_E)^2}{h^2}\right]-\mathrm{EllipticE}\left[\mathrm{ArcSin}\left[\frac{\sqrt{h^2+4\,h\,\mathrm{Sin}[\frac{x2}{2\,R_E}]^2\,R_E-2\,(-1+\mathrm{Cos}[\frac{x2}{R_E}])\,R_E^2}}{h+2\,R_E}\right],\frac{(h+2\,R_E)^2}{h^2}\right]\right)+\right.\\
&R_E^2\left(-\frac{2\,i\,\mathrm{EllipticF}\left[\mathrm{ArcSin}\left[\frac{h}{h+2\,R_E}\right],\frac{(h+2\,R_E)^2}{h^2}\right]}{h}+\frac{1}{h}\left(2\,i\,\mathrm{EllipticF}\left[\mathrm{ArcSin}\left[\frac{\sqrt{h^2+4\,h\,\mathrm{Sin}[\frac{x2}{2\,R_E}]^2\,R_E-2\,(-1+\mathrm{Cos}[\frac{x2}{R_E}])\,R_E^2}}{h+2\,R_E}\right],\frac{(h+2\,R_E)^2}{h^2}\right]\right)-\right.\\
&\left.\frac{\mathrm{Sin}[\frac{x2}{R_E}]}{\sqrt{h^2+4\,h\,\mathrm{Sin}[\frac{x2}{2\,R_E}]^2\,R_E-2\,(-1+\mathrm{Cos}[\frac{x2}{R_E}])\,R_E^2}}\right)+R_E\left(-2\,i\,\mathrm{EllipticF}\left[\mathrm{ArcSin}\left[\frac{h}{h+2\,R_E}\right],\frac{(h+2\,R_E)^2}{h^2}\right]+\right.\\
&\left.\left.2\,i\,\mathrm{EllipticF}\left[\mathrm{ArcSin}\left[\frac{\sqrt{h^2+4\,h\,\mathrm{Sin}[\frac{x2}{2\,R_E}]^2\,R_E-2\,(-1+\mathrm{Cos}[\frac{x2}{R_E}])\,R_E^2}}{h+2\,R_E}\right],\frac{(h+2\,R_E)^2}{h^2}\right]-\frac{h\,\mathrm{Sin}[\frac{x2}{R_E}]}{\sqrt{h^2+4\,h\,\mathrm{Sin}[\frac{x2}{2\,R_E}]^2\,R_E-2\,(-1+\mathrm{Cos}[\frac{x2}{R_E}])\,R_E^2}}\right)\right)\end{aligned}$$

▫ Case with $V[\theta]$ = Cos[θ]^2

In this case the integral is:

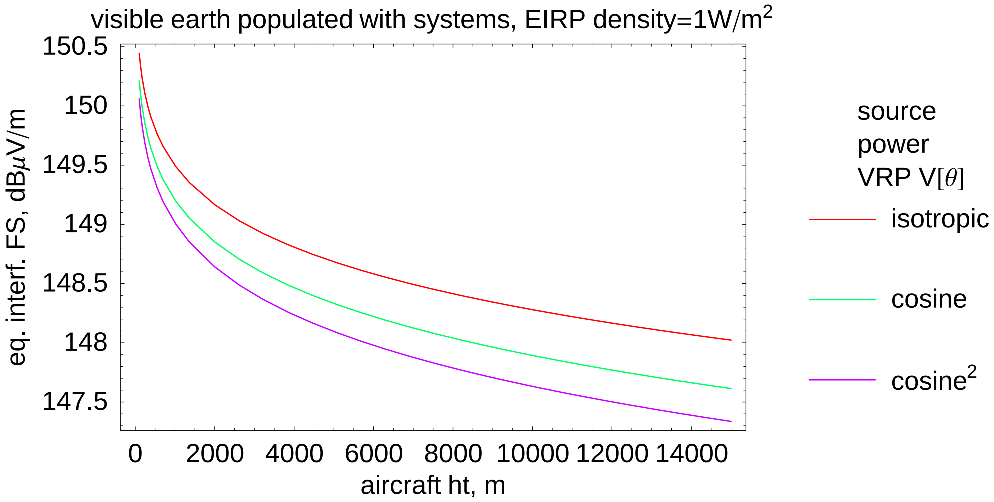
$$\mathrm{PFD}_{\text{for unit EIRP density}} = \frac{R_E}{2}\int_{x_1}^{x_2}\frac{(R_E+h)^2\,\mathrm{Sin}[\frac{x}{R_E}]^3}{(R_E{}^2-2\,\mathrm{Cos}[\frac{x}{R_E}]\,R_E\,(h+R_E)+(h+R_E)^2)^2}\,dx$$

which, with whole earth populated, i.e. $x_1 \to 0$ and $x_2 \to x_{\mathrm{Max}} = R_E\,\mathrm{ArcCos}\left[\frac{R_E}{R_E+h}\right]$, gives a simple result:

$$\mathrm{PFD}_{\text{for unit EIRP density}} = \frac{1}{8}\left(\left(\frac{h^2}{R_E\,(h+R_E)}+2\right)\mathrm{Log}\left[1+\frac{2\,R_E}{h}\right]-2\right)$$

■ A 2.4. Graphical results for examples where sources have a non-isotropic VRP

We assume once again that *x*DSL/PLT systems having *unit EIRP density* (and isotropic HRP) populate the entire visible Earth, but with three different *power* VRPs — uniform, Cos[θ] and Cos[θ]^2. (The unit EIRP density here refers to the EIRP in the max direction, horizontal in these cases). We plot the equivalent interfering field strength as a function of aircraft height by setting $x_1 = 0$ and $x_2 = R_E \text{ ArcCos}\left[\frac{R_E}{R_E+h}\right]$, with the following results:



We may note that, as we might expect, the VRPs which radiate less at high elevations cause less interference. However, despite the radiation being least from the points on the ground nearest the aircraft (directly below it), the interference is not very greatly reduced — less than 1 dB.

Interestingly, the total radiated interference power (averaged over all elevation angles) of the ‘average source’, given that we equalised the EIRP along the horizon to 1 in each case, is in any case reduced, by about 1 and 1.76 dB respectively for the Cos[θ] and Cos[θ]^2 cases. The secret lies in the geometry!

□ A 3. Sky-wave propagation calculation

■ A 3.1. Derivation for isotropic sources

Taking the geometry of the situation from Appendix A 1.3, and making a simple allowance for losses by applying an attenuation factor α to account for the power lost to ionospheric absorption in each hop, and another factor β to account for the loss involved in each ground reflection, we obtain the following form for $f[x]$:

$$\begin{aligned} f[x] &= \frac{\alpha^n\,\beta^{n-1}}{4\,\pi\,s^2} \\ &= \frac{\alpha^n\,\beta^{n-1}}{16\,\pi\,n^2\,\left({R_E}^2 - 2\,\mathrm{Cos}\left[\frac{x}{2\,n\,R_E}\right]R_E\,(h+R_E) + (h+R_E)^2\right)} \end{aligned}$$

We can then apply this in the integral we use to calculate the interference:

$$\begin{aligned} &\mathrm{PFD} = \int_A p_{\mathrm{TX}}\,g_{\mathrm{TX}}\,D\,f[x]\,dA = \\ &\int_{x_1}^{x_2} p_{\mathrm{TX}}\,g_{\mathrm{TX}}\,D\,\frac{\alpha^n\,\beta^{n-1}}{16\,\pi\,n^2\,\left({R_E}^2 - 2\,\mathrm{Cos}\left[\frac{x}{2\,n\,R_E}\right]R_E\,(h+R_E) + (h+R_E)^2\right)}\,2\,\pi\,R_E\,\mathrm{Sin}\left[\frac{x}{R_E}\right]dx \\ &= \frac{\alpha^n\,\beta^{n-1}\,p_{\mathrm{TX}}\,g_{\mathrm{TX}}\,D\,R_E}{8\,n^2}\int_{x_1}^{x_2}\frac{\mathrm{Sin}\left[\frac{x}{R_E}\right]}{\left({R_E}^2 - 2\,\mathrm{Cos}\left[\frac{x}{2\,n\,R_E}\right]R_E\,(h+R_E) + (h+R_E)^2\right)}\,dx. \end{aligned}$$

Note that the upper limit of integration x_2 that we choose may not exceed $x_{\mathrm{Max},n} = 2\,n\,R_E\ \mathrm{ArcCos}\left[\frac{R_E}{R_E+h}\right]$ (for which rays are tangential to the ground).

We can take the EIRP density $p_{\mathrm{TX}}\,g_{\mathrm{TX}}\,D$ out of the integral and also the ionospheric losses $\alpha^n\,\beta^{n-1}$, as they can similarly be accounted for afterwards, so we have

$$\mathrm{PFD}_{\text{for unit EIRP density, lossless }n\text{−hop propagation}} = \frac{R_E}{8\,n^2}\int_{x_1}^{x_2}\frac{\mathrm{Sin}\left[\frac{x}{R_E}\right]}{\left({R_E}^2 - 2\,\mathrm{Cos}\left[\frac{x}{2\,n\,R_E}\right]R_E\,(h+R_E) + (h+R_E)^2\right)}\,dx.$$

The author has failed to obtain a fully general analytical solution to this integral. However, it seems to be soluble for any specific value of n (the number of hops) but with the result becoming increasingly complicated as n increases. The result for $n = 1$ can be simplified sufficiently to be worth presenting here:

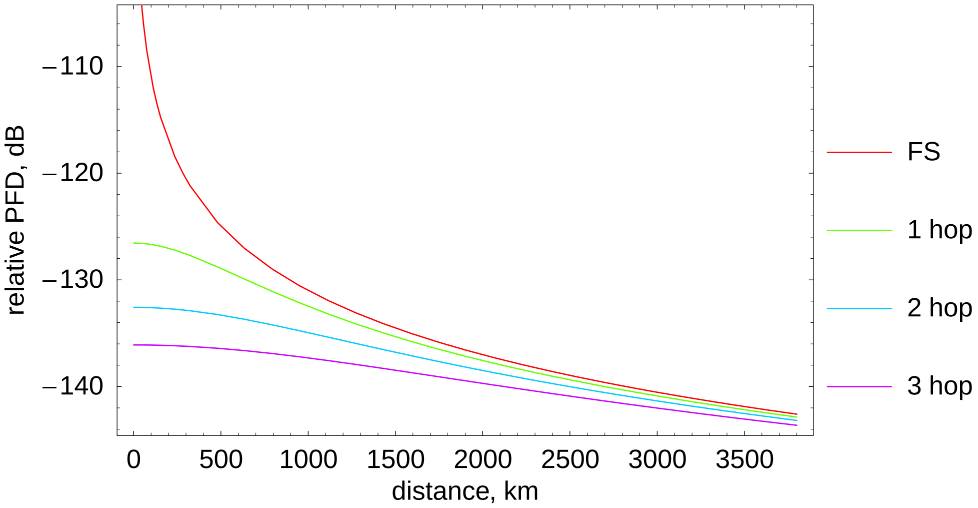
$$\begin{aligned} &\mathrm{PFD}_{\text{for unit EIRP density, lossless 1−hop propagation}} = \\ &\frac{1}{8\,(h+R_E)^2}\left(2\,R_E\,(h+R_E)\left(-\mathrm{Cos}\left[\frac{x_1}{2\,R_E}\right] + \mathrm{Cos}\left[\frac{x_2}{2\,R_E}\right]\right) - \left(h^2 + 2\,h\,R_E + 2\,{R_E}^2\right)\mathrm{Log}\left[\frac{-h^2 - 2\,h\,R_E - 2\,{R_E}^2 + 2\,R_E\,(h+R_E)\,\mathrm{Cos}\left[\frac{x_1}{2\,R_E}\right]}{-h^2 - 2\,h\,R_E - 2\,{R_E}^2 + 2\,R_E\,(h+R_E)\,\mathrm{Cos}\left[\frac{x_2}{2\,R_E}\right]}\right]\right) \end{aligned}$$

Expressions (for brevity, not shown here!) were also obtained for the cases of $n = 2$ to 5 and were used to produce the plots shown later.

■ A 3.2. Graphical results for isotropic sources

◽ Comparison with free-space propagation

It is instructive to plot $f[x]$ for various numbers of hops, and to compare these with free-space propagation over a distance equivalent to the curved-Earth distance x. This is illustrated in the following Figure, for which values of $\alpha = \beta = 1$ have been assumed (i.e. no losses in reflections), together with an ionospheric height of 300 km:



As expected, free-space attenuation (red curve, ‘FS’) decreases dramatically for small distances. In contrast, the variation with distance is less marked for the ionospheric-propagation cases. This is because the signal has to go to the ionosphere and back anyway, so the distance travelled is at least $2\,n\,h$ (the value for any nearby points). This also explains a difference of 6 dB between 1- and 2-hop cases. For more-distant locations, it makes little difference how many hops there are, as the polygonal path does not deviate so much from the Great Circle around the surface of the Earth. The x-axis range of the plot has been limited to the distance over which one-hop propagation is possible — care must be taken not to apply the formula over longer ranges, as this would be equivalent to rays passing through the Earth!

Note that our calculations will make a simple allowance for reflection losses, as a result of which the attenuation will further increase with the number of hops. The above-noted difference of 6 dB between 1- and 2-hop cases (for short distances) will therefore be greater in practice.

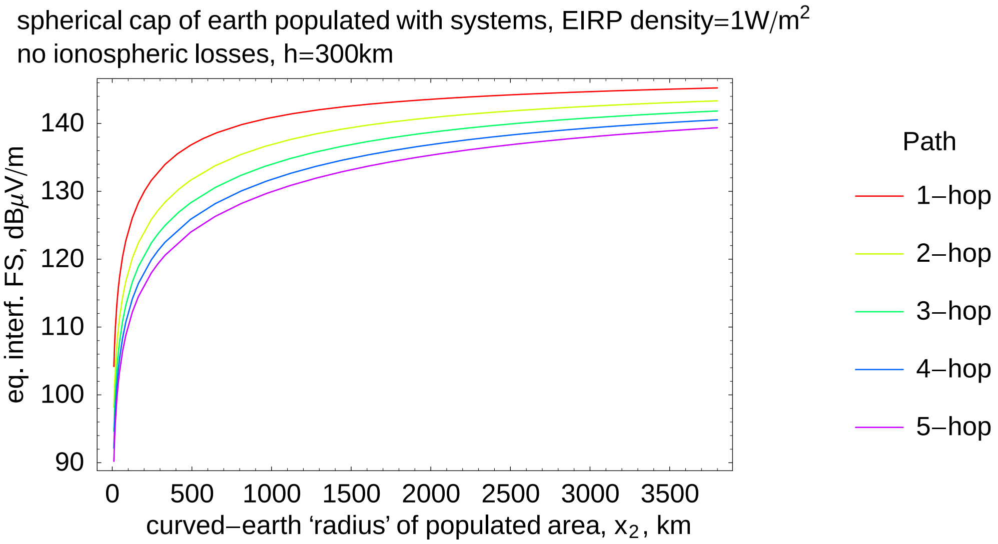
◽ Calculated interference PFD for varying scenarios

We assume that potentially-interfering systems uniformly populate an area of the Earth. We take the area as a pseudo-annular region, centred on the reception site, with outer curved-earth radius x_2 and possibly having a hole in the middle of curved-earth radius x_1. x_2 thus defines the extent of the populated region, while x_1 can be used to allow for an *exclusion zone* around the receiver, or, if the frequency is above the critical frequency, to account for the effects of *skip distance*. The following plots assume an ionospheric height of 300 km.

We can separately plot the contributions made by propagation 1, 2 or more hops, always assuming uniform EIRP density and zero ionospheric losses — the appropriate corrections should be applied for any practical situation when interpreting the curves.

It is of interest to look at the problem two ways: we can vary x_1, to see the value of any exclusion distance or effect of skip, and we can vary the outer limit x_2 — in effect limiting the area over which systems are assumed. Note that x_2 must not be taken greater than the longest possible value appropriate to the number of hops, $x_{\text{Max},n} = 2\,n\,R_E\ \text{ArcCos}\left[\frac{R_E}{R_E+h}\right]$.

First we look at the effect of varying x_2; the plot range is chosen so that x_2 never exceeds the maximum distance for which 1-hop propagation is possible. x_1 is taken as zero — there is no exclusion zone and no limitation is imposed by skip (only possible if the frequency is below the critical frequency for vertical incidence). This case is shown in the Figure below, where the contributions from paths having different numbers of hops are shown separately, *assuming no ionospheric loss*.

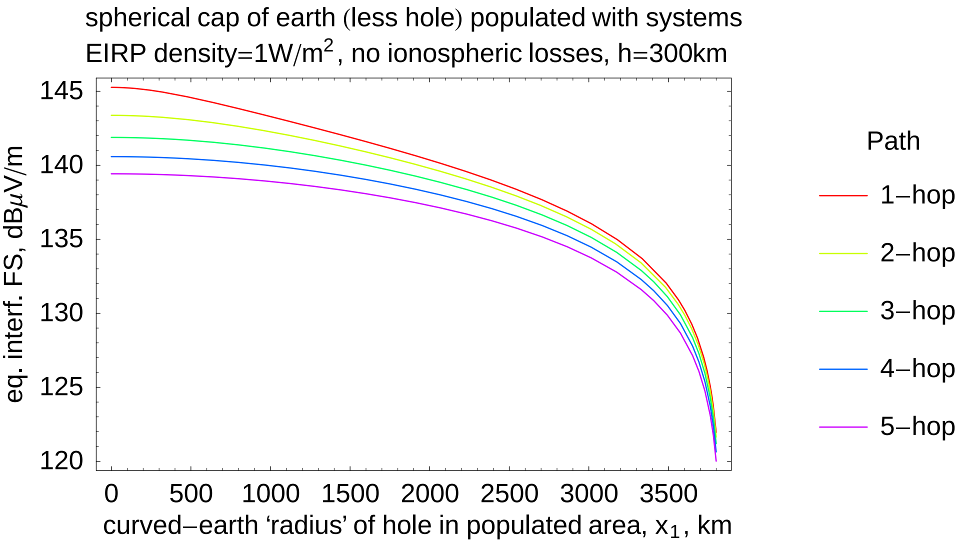


To obtain the total interference PFD, the contributions of each path should be determined, allowing for the loss applicable to the number of hops in each case, and power-summed. Finally this total should be scaled to account for the appropriate EIRP density. It is clear that when ionospheric losses are significant the first hop will dominate the calculation.

At first there is a rapid increase in interference as the outer limit is increased — the number of interferers present increases rapidly, while the attenuation of the added outermost ones is scarcely less than for the nearest ones. For larger distances, the added area is relatively less important and the attenuation of its contributions greater.

If we consider the parameters applicable to the corresponding example presented in Ref. [1], namely EIRP density of $-107.5\,\mathrm{dBW/m^2}$ and 1-hop ionospheric loss of 10 dB, then the equivalent field strength in the 1-hop case (red curve above) reaches a value of 28.5 dBμV / m at 3500 km, in agreement with the single curve presented in [1].

Secondly, in the Figure below we look at the effect of varying x_1 while x_2 is set to the maximum distance for which 1-hop propagation is possible (namely 3836 km). This means we can account for skip effects or the introduction of any exclusion zone.



It can be seen that the interference decreases only slowly until a very large exclusion distance is reached. This is easily explained in combining two concepts. The rate of attenuation with distance from the receiver is not dramatic, as previously explained, while the nearby part of the Earth has a relatively small area compared with the whole area included in the calculation. So only when the exclusion distance is large is the number of interferers greatly reduced, and their value further diminished by distance, whence the shape of the curve follows.

These results show that relatively-sensitive sites, which are relatively free from direct (e.g. ground-wave) interference because there are no nearby interferers (e.g. there is an exclusion zone) are indeed potentially vulnerable to the effects of cumulative interference from far-off interferers.

■ A 3.3. Derivation for examples where sources have a non-isotropic VRP

For simplicity, we assume one simple VRP, namely $V[\theta] = \text{Cos}[\theta]^2$. Using the formula for the elevation angle θ obtained in §A 1.3, the necessary integral becomes:

$$\text{PFD}_{\text{for unit EIRP density, lossless } n\text{–hop propagation}} = \frac{R_E}{8\,n^2} \int_{x_1}^{x_2} \frac{\text{Sin}\left[\frac{x}{2\,n\,R_E}\right]^2 (h+R_E)^2\, \text{Sin}\left[\frac{x}{R_E}\right]}{\left(R_E{}^2 - 2\,\text{Cos}\left[\frac{x}{2\,n\,R_E}\right] R_E\,(h+R_E) + (h+R_E)^2\right)^2}\, dx.$$

An analytical solution was found for the case $n = 1$, but the expression is far too long to justify reproducing here — even in an Appendix! It was nevertheless used to plot the results given in § 5.2 of the main text.