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A preliminary study of the influence of room mode structure on sound absorption

E.W. Taylor, M.A.(Cantab.), C.Eng., M.I.E.E.

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A PRELIMINARY STUDY OF THE INFLUENCE OF ROOM MODE STRUCTURE ON SOUND ABSORPTION

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Summary

An account is given of the structure of room modes in rectangular rooms, leading to the derivation of relationships giving the numbers of each of the three types of mode which will be present in a rectangular room of given dimensions and for a given range of excitation frequencies. The decay of sound in an enclosure is discussed in terms of the acoustic impedances of the surfaces and the room mode structure, particularly for the case in which the room has uniform surface impedance. The effect of the proportion of each of the three types of room mode on the apparent absorption coefficient of the surface is considered. Comparisons are given between results predicted by this work and those obtained in practice, and also with other theoretical work.

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LIST OF SYMBOLS

This Report discusses the relationship between quantities (for example, absorption coefficients) which are essentially similar but which involve differing parameters in their evaluation. Subscripts to the symbols representing such quantities are therefore used extensively to identify these parameters. For greater clarity the following list shows all the subscripted versions of each symbol explicitly.

$a_{p_z x}\ a_{p_z y}\ b_{p_z x}\ b_{p_z y}$	Coordinates defining boundaries of $p_z{}^{\text{th}}$ patch of absorbing material ($a < b$). (Analogous coordinates used for $p_x{}^{\text{th}}$ and $p_y{}^{\text{th}}$ patches.)
C_n	Component of D'_n dependent on room mode structure.
$C_{ax\,x}\ C_{ax\,y}\ C_{ax\,z}$	C_n values for axial modes in which n_x, n_y or n_z respectively are non-zero.
$C_{ta\,x}\ C_{ta\,y}\ C_{ta\,z}$	C_n values for tangential modes in which n_x, n_y or n_z respectively are zero.
C_{ob}	C_n value for all oblique modes.
c	Velocity of sound.
D_n	Damping coefficient (analogous to absorption coefficient) for n^{th} room mode.
$D'_n\ (= D_n/V)$	Damping coefficient per unit volume of room.
f	Frequency (general).
f_L	(Upper) limit of frequency range (lower limit is zero).
Δf	Frequency band.
f_C	Centre frequency of band.
$f_{n_x n_y n_z}$	Frequency associated with room mode $n_x n_y n_z$.
$I_{n p_x}\ I_{n p_y}\ I_{n p_z}$	Integration of effect of $p_x{}^{\text{th}}$, $p_y{}^{\text{th}}$ or $p_z{}^{\text{th}}$ absorber on n^{th} room mode.
k	Abbreviation for $k_{n_x n_y n_z}$.
$(k_{n_x}, k_{n_y}, k_{n_z})$	Point on surface of k-space sphere ($k_{n_x} = \pi n_x/L_x$ etc.).
$k_{n_x n_y n_z}$	Radius of k-space sphere ($= 2\pi f_{n_x n_y n_z}/c$).
$L_x L_y L_z$	Room dimensions along x, y and z axes respectively.
L	Total edge length of room.
$m_{ax}\ m_{ta}\ m_{ob}$	Proportions of axial, tangential and oblique room modes respectively.
$m_{ax(S)}$ etc.	Proportions of each type of room mode in studio.
$m_{ax(R)}$ etc.	Proportions of each type of room mode in reverberation room.
$N_{ax\,x\,f_L}\ N_{ax\,y\,f_L}\ N_{ax\,z\,f_L}$	Number of axial modes, in frequency range 0–f_L, for which n_x, n_y or n_z respectively are non-zero.
$N_{ax\,f_L}$	Total number of axial modes in frequency range 0–f_L.
$\Delta N_{ax\,f}$	Number of axial modes in frequency band Δf.
$N_{ta\,x\,f_L}\ N_{ta\,y\,f_L}\ N_{ta\,z\,f_L}$	Number of tangential modes, in frequency range 0–f_L, for which n_x, n_y or n_z respectively are zero.
$N_{ta\,f_L}$	Total number of tangential modes in frequency range 0–f_L.
$\Delta N_{ta\,f\,f_C}$	Number of tangential modes in frequency band Δf centred on frequency f_C.
$N_{ob\,f_L}$	Number of oblique modes in frequency range 0–f_L.
$\Delta N_{ob\,f\,f_C}$	Number of oblique modes in frequency band Δf centred on frequency f_C.
$\Delta N_{ax\,1/3\,f_C}\ \Delta N_{ta\,1/3\,f_C}\ \Delta N_{ob\,1/3\,f_C}$	Numbers of axial, tangential and oblique modes, respectively, in one-third octave bands centred on frequency f_C.
$n_x n_y n_z$	Numbers of nodal planes normal to x, y and z room axes respectively.
n	Particular combinations of n_x, n_y and n_z, used to identify a room mode.
$p_x p_y p_z$	Integers used to identify patches of absorbing material on surfaces of room normal to x, y and z axes respectively.
p	Integer used to identify patch of absorbing material, irrespective of the surface of the room upon which it is placed.
$\bar{p}^2$	Mean square sound pressure.
q	Integer used in the definition of the behaviour of ε.
R_T	Ratio of "overall average" absorption coefficient in studio to "overall average" absorption coefficient in reverberation room

$S_x S_y S_z$	Total surface areas of pairs of surfaces normal to x, y and z room axes respectively.
S	Total surface area of room.
T_n	Reverberation time for n^{th} room mode.
T	"Conventional" reverberation time (Sabine formula).
V	Volume of room.
$\bar{\alpha}_{ax\,x}\ \bar{\alpha}_{ax\,y}\ \bar{\alpha}_{ax\,z}$	Effective absorption coefficients for axial room modes for which n_x, n_y or n_z respectively are non-zero.
$\bar{\alpha}_{ax}$	Overall effective absorption coefficient for all axial room modes.
$\bar{\alpha}_{ta\,x}\ \bar{\alpha}_{ta\,y}\ \bar{\alpha}_{ta\,z}$	Effective absorption coefficients for tangential room modes for which n_x, n_y or n_z respectively are zero.
$\bar{\alpha}_{ta}$	Overall effective absorption coefficient for all tangential room modes.
$\bar{\alpha}_{ob}$	Overall effective absorption coefficient for all oblique room modes.
$\bar{\alpha}_o$	Overall average absorption coefficient for all room modes.
$\bar{\alpha}$	Mean absorption coefficient of room surface (Sabine formula).
α_M	Measured absorption coefficient } in context of Waterhouse correction.
α_T	True absorption coefficient } in context of Waterhouse correction.
α_θ	Absorption coefficient for sound arriving at angle of incidence θ.
β	General angular variable.
$\varepsilon_{nx}\ \varepsilon_{ny}\ \varepsilon_{nz}$	Multiplying factors (1 if $n = 0$; $\frac{1}{2}$ if $n \neq 0$).
ζ	Specific acoustic impedance (general).
ζ_C	Measured complex value of specific acoustic impedance of surface.
ζ_R	Real part of ζ_C.
$\lvert\zeta_C\rvert$	Modulus of ζ_C.
ζ_{real}	Real value of specific acoustic impedance of surface, giving same sound absorption as measured complex value ζ_C.
$\zeta_x\ \zeta_y\ \zeta_z$	(Real) specific acoustic impedances of pairs of room surfaces normal to x, y and z room axes respectively.
ζ_p	(Real) specific acoustic impedance of surface of p^{th} patch of absorbing material.
θ	Angle of incidence of sound on to surface.

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1. Introduction

The dependence of the absorption of sound on the environment in which the absorber is placed is well known. An extensive literature has developed on the subject, in which a number of different mechanisms are examined to account for the effect. Because of the extreme complexity of the propagation of sound in an enclosure and of the absorption process itself, it has so far not been found possible to derive a theoretical foundation for the process of sound absorption in rooms which is at the same time both mathematically rigorous and directly applicable to practical problems. Consequently, any attempt to apply theoretical considerations to practical problems involves a degree of approximation which immediately brings into question the accuracy of the results obtained from such a process, or indeed the wisdom of even attempting to make a theoretical prediction at all. The latter point is particularly true when a major project is under consideration, since the financial and administrative implications of a serious design error would become items of prime importance. For this reason, the theoretical design work in such cases is reinforced by an extensive series of practical tests. Although it is unlikely that this situation will be altered in the forseeable future, it is nevertheless interesting and instructive to compare the results of purely theoretical predictions with the results that are obtained in practice, as an insight can be obtained into the causes of the dependence of sound absorption on environment, even though the magnitude of an effect may not be amenable to precise calculations from purely theoretical considerations. This Report examines the effect of room modes (eigentones) on sound absorption from this point of view.

2. Room modes

The term "room modes" refers to the existence of a standing sound wave within the room. Many such modes are possible, each being characterised by the structure of the standing wave pattern. In a rectangular room this structure may be described in terms of the numbers (n_x, n_y and n_z) of sound pressure nodal planes perpendicular respectively to the x, y and z co-ordinate axes (Fig. 1). The theory underlying the formation of room modes was first described by Lord Rayleigh[1] in 1896 and this

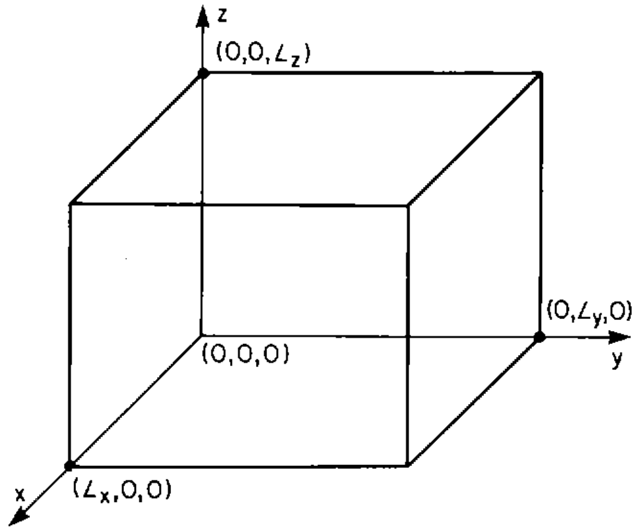

Fig. 1 – Coordinate axes for a room.

showed that, for rooms having perfectly reflecting surfaces, the frequency ($f_{n_x n_y n_z}$) associated with the room mode was given by:

$$f_{n_x n_y n_z} = \frac{c}{2}\left[\left(\frac{n_x}{L_x}\right)^2 + \left(\frac{n_y}{L_y}\right)^2 + \left(\frac{n_z}{L_z}\right)^2\right]^{1/2} \qquad (1)$$

where c is the velocity of sound, L_x, L_y and L_z are the room dimensions along the x, y and z axes respectively, and n_x, n_y and n_z are integers.

By letting

$$k_{n_x n_y n_z} = \frac{2\pi}{c} f_{n_x n_y n_z} \qquad (2)$$

and

$$k_{n_x} = \frac{\pi n_x}{L_x}, \quad k_{n_y} = \frac{\pi n_y}{L_y}, \quad k_{n_z} = \frac{\pi n_z}{L_z} \qquad (3)$$

Equation (1) reduces to

$$k_{n_x n_y n_z} = (k_{n_x}{}^2 + k_{n_y}{}^2 + k_{n_z}{}^2)^{1/2} \qquad (4)$$

(Strictly speaking,[2,3] Equations 2 and 4 are a solution of the general wave equation and the relationships in Equation 3 arise because of boundary conditions imposed by the rigid room walls; these lead to Equation 1.)

Equation 4 represents the equation of a sphere centred at the origin of "k-space", so that the radius to the point $(k_{n_x}, k_{n_y}, k_{n_z})$ on the surface of the sphere is of length $k_{n_x n_y n_z} = k$ (Fig. 2). The length of the radius vector is related to the frequency of the particular room mode under consideration, via Equation 2, and the co-ordinates of the point in k-space

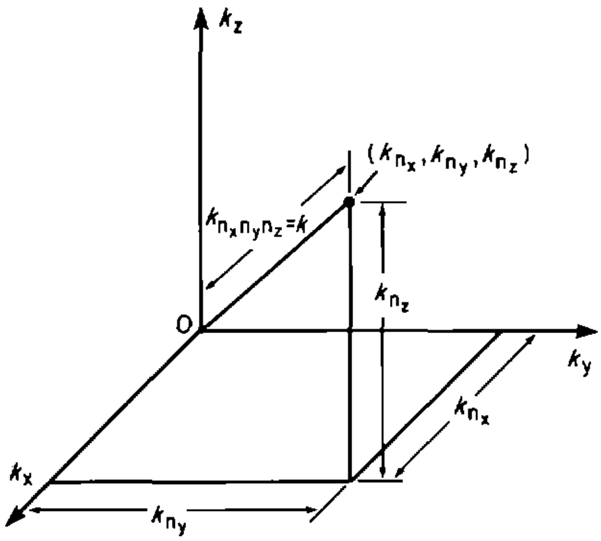

Fig. 2 – k-space.

are related to the nodal plane numbers via Equation 3. Thus there exists an orthogonal lattice of points in k-space (Fig. 3), each point representing one mode and the distance of a point from the origin representing the frequency of the mode that it represents.

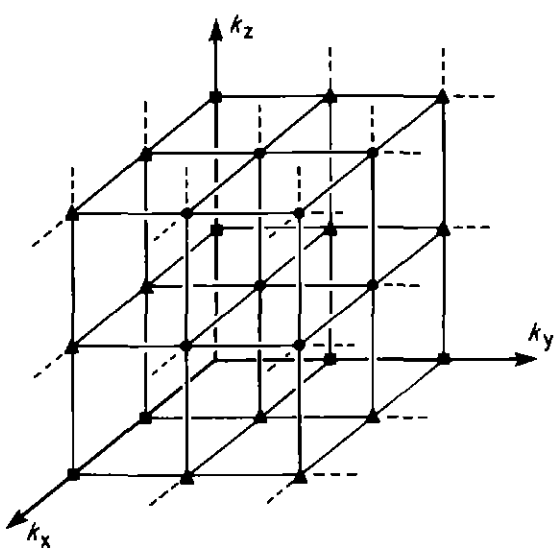

Fig. 3 – Room mode lattice in k-space.

Room modes may be classified under three headings – axial, tangential and oblique. Axial modes occur when sound is propagating parallel to one of the co-ordinate axes, or in other words, parallel to an edge between two intersecting walls. In this case, two of the nodal plane numbers, n_x, n_y, n_z are zero, and consequently two of the k-space co-ordinates k_{n_x}, k_{n_y}, k_{n_z} are also zero. Points in k-space corresponding to axial room modes therefore lie along the positive* k-space axes. In the case of tangential modes, the direction of sound propagation is perpendicular to one of the room axes. In such cases one of the nodal plane numbers is zero, and so, therefore, is one of the co-ordinates of the points in k-space corresponding to such modes. The k-space co-ordinates corresponding to tangential modes therefore occupy grids of points in the "first quadrant" (both co-ordinates positive*) portions of the three planes in k-space, each of which includes two of the axes. For oblique modes none of the nodal plane numbers is zero; consequently no k-space co-ordinate is zero either, and the k-space points corresponding to such modes occupy a lattice of points (all co-ordinates positive*), bounded by the three planes described above.

From the concept of the k-space diagram, it is possible to estimate the number of axial, tangential and oblique modes in a room of given size, both within a frequency range extending from zero to a limiting value f_L and in a band of frequencies Δf about a centre frequency f_C. Consider first the points corresponding to axial modes lying along the k_x axis. From Equations 3 it is seen that the spacing between the points as n_x takes successive integral values is π/L_x. The number of points $N_{ax\,x\,f_L}$ along the axis from the origin up to a distance k, where $k = (2\pi/c)f_L$, is therefore given by:

$$N_{ax\,x\,fL} = k \cdot \frac{L_x}{\pi} = \frac{2f_L}{c} \cdot L_x \tag{5a}$$

Similarly, for points along the k_y axis

$$N_{ax\,y\,f_L} = \frac{2f_L}{c} \cdot L_y \tag{5b}$$

and for points along the k_z axis

$$N_{ax\,z\,f_L} = \frac{2f_L}{c} \cdot L_z \tag{5c}$$

*Because of the relationship shown in Equation 4, the sign of a k-space co-ordinate is immaterial and all possible cases are included if only positive values are considered.

The total number of points $N_{\mathrm{ax}\,f_L}$ which corresponds to the total number of axial modes in the frequency range zero to f_L is therefore given by

$$N_{\mathrm{ax}\,f_L} = \frac{2f_L}{c}(L_x + L_y + L_z)$$

$$= \frac{f_L \cdot L}{2c} \tag{6}$$

where $L = 4(L_x + L_y + L_z)$ is the total "edge length" of the room.

By differentiating Equation 6 the total number of axial modes $\Delta N_{\mathrm{ax}\,\Delta f}$ in a frequency band Δf can be estimated. Thus using the symbols defined earlier

$$\Delta N_{\mathrm{ax}\,\Delta f} = \frac{L}{2c} \cdot \Delta f \tag{7}$$

Turning to tangential modes, consider first those modes whose k-space points lie in the plane containing the k_x and k_y axes and is therefore perpendicular to the k_z axis. The area of the rectangular element between neighbouring lattice points, from Equations 3, is $\pi^2/L_x L_y$. The area of a quadrant of a circle of radius k is $\pi k^2/4$, where, as before, $k = (2\pi/c)f_L$. Thus, the number of elementary areas in the quadrant, $N_{\mathrm{ta}\,z\,f_L}$, is given by

$$N_{\mathrm{ta}\,z\,f_L} = \frac{k^2 L_x L_y}{4\pi} = \left(\frac{f_L}{c}\right)^2 \cdot \pi L_x L_y \tag{8a}$$

Similarly, in the cases of the other two planes containing grids of tangential-mode k-space points

$$N_{\mathrm{ta}\,x\,f_L} = \left(\frac{f_L}{c}\right)^2 \pi L_y L_z \tag{8b}$$

and

$$N_{\mathrm{ta}\,y\,f_L} = \left(\frac{f_L}{c}\right)^2 \pi L_x L_z \tag{8c}$$

Thus the total number of points $N_{\mathrm{ta}\,f_L}$ corresponding to the total number of tangential modes in the frequency range zero to f_L is given by

$$N_{\mathrm{ta}\,f_L} = \left(\frac{f_L}{c}\right)^2 \pi(L_x L_y + L_y L_z + L_x L_z)$$

$$= \pi\left(\frac{f_L}{c}\right)^2 \cdot \frac{S}{2} \tag{9}$$

where $S = 2(L_x L_y + L_y L_z + L_x L_z)$ is the total surface area of the room.

Furthermore, by differentiating Equation 9

$$\Delta N_{\mathrm{ta}\,\Delta f\,f_C} = \frac{\pi S}{c^2} f_C \Delta f \tag{10}$$

where $\Delta N_{\mathrm{ta}\,\Delta f\,f_C}$ is the number of tangential modes in a frequency band Δf centred on a frequency f_C.

The volume of the element formed by neighbouring points in k-space corresponding to oblique modes is, from Equations 3, $\pi^3/(L_x L_y L_z) = \pi^3/V$, where V is the volume of the room. The volume of the octant of a sphere of radius k ($k = (2\pi/c) \cdot f_L$ as before) is $\pi/(6k^3)$. Thus the number of elementary volumes in the octant, or in other words the number $N_{\mathrm{ob}\,f_L}$ of oblique modes in the frequency range zero to f_L is given by

$$N_{\mathrm{ob}\,f_L} = \frac{Vk^3}{6\pi^2} = \frac{4\pi}{3}\left(\frac{f_L}{c}\right)^3 V \tag{11}$$

Differentiating Equation 11 with respect to f gives

$$\Delta N_{\mathrm{ob}\,\Delta f\,f_C} = \frac{4\pi V}{c^3} f_C^2 \Delta f \tag{12}$$

where $\Delta N_{\mathrm{ob}\,\Delta f\,f_C}$ is the number of oblique modes in a frequency band Δf centred on f_C.

Practical work on sound absorption usually involves measurement in one-third octave frequency bands, for which $\Delta f/f_C = 0.232$. It is therefore helpful to express Equations 7, 10 and 12 in terms of this quantity, viz.

$$\Delta N_{\mathrm{ax}\,1/3\,f_C} = \left(\frac{L}{2c}\right)\left(\frac{\Delta f}{f_C}\right) \cdot f_C$$

$$= (3.378 \times 10^{-4}) \cdot L f_C \tag{13}$$

$$\Delta N_{\mathrm{ta}\,1/3\,f_C} = \left(\frac{\pi S}{c^2}\right)\left(\frac{\Delta f}{f_C}\right) \cdot f_C^2$$

$$= (6.181 \times 10^{-6}) \cdot S f_C^2 \tag{14}$$

$$\Delta N_{\mathrm{ob}\,1/3\,f_C} = \left(\frac{4\pi V}{c^3}\right)\left(\frac{\Delta f}{f_C}\right) \cdot f_C^3$$

$$= (7.199 \times 10^{-8}) \cdot V f_C^3 \tag{15}$$

where room dimensions are in metres and frequency is expressed in Hz ($c = 343.4\ \mathrm{ms}^{-1}$).

It can be seen that as f_c increases, the proportion of axial, tangential and oblique modes in the band centred on f_c will also change due to their dependence on the first, second and third powers of frequency respectively and that the number of oblique modes will rise faster than the number of tangential modes, which, in turn, rises faster than the number of axial modes. The same effect occurs if, for a given centre frequency, rooms of different size are compared. The numbers of axial, tangential and oblique modes depend on the linear dimensions, surface area and volume of the room respectively and thus on the first, second and third powers of the room scale factors.

The theory outlined above only gives an approximately correct result, especially when the number of modes is small (i.e. at low frequencies and for small rooms). In such cases the spacing between the k-space points becomes comparable with the length of the frequency-dependent vector k (Fig. 2) with the result that the numbers of points are no longer given accurately by simple ratios of volumes, areas or lengths. Furthermore, a third-octave band of frequencies cannot be considered of infinitesimal magnitude and the use of differential calculus to derive Equations 7, 10 and 12 is of questionable validity. Nevertheless, these equations (or their rearrangements in Equation 13, 14 and 15) illustrate the dependence of the proportions of the different types of room modes on the frequency band under consideration.

A computer program has been prepared which counts the number of modes of each type in a third-octave band, given the band centre frequency and the dimensions of the (rectangular) room. Figs. 4, 5 and 6 show the number of modes in a "rectangular approximation"* to the large reverberation room at BBC Research Department as calculated using this program, for third-octave bands between 50 Hz and 2 kHz. Also shown in Figs. 4 to 6 are the number of modes calculated using Equations 13, 14 and 15 respectively. It can be seen that these equations predict the numbers of modes with a fair degree of accuracy for frequencies of about 1 kHz and above, where the sound wavelength is less than about one-tenth of the smallest room dimension. At low frequencies, however, the formulae fail to give accurate predictions for the reasons discussed above.

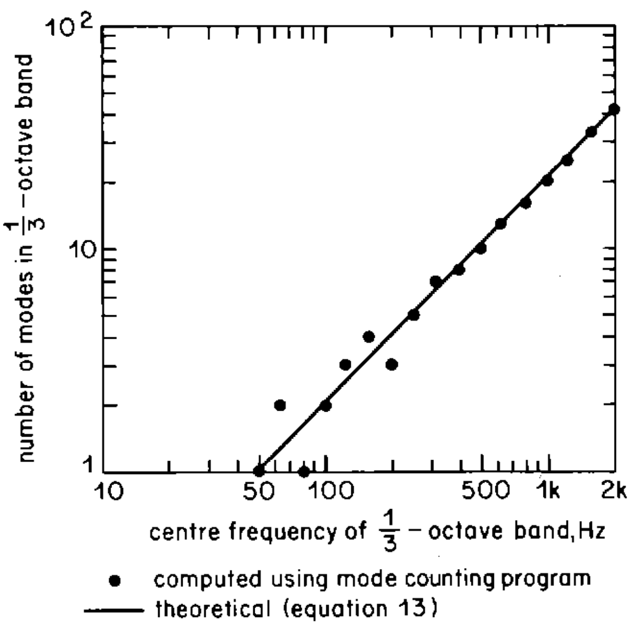

Fig. 4 – Number of axial modes in $\frac{1}{3}$-octave bands for room of dimensions 7 m × 5 m × 3 m.

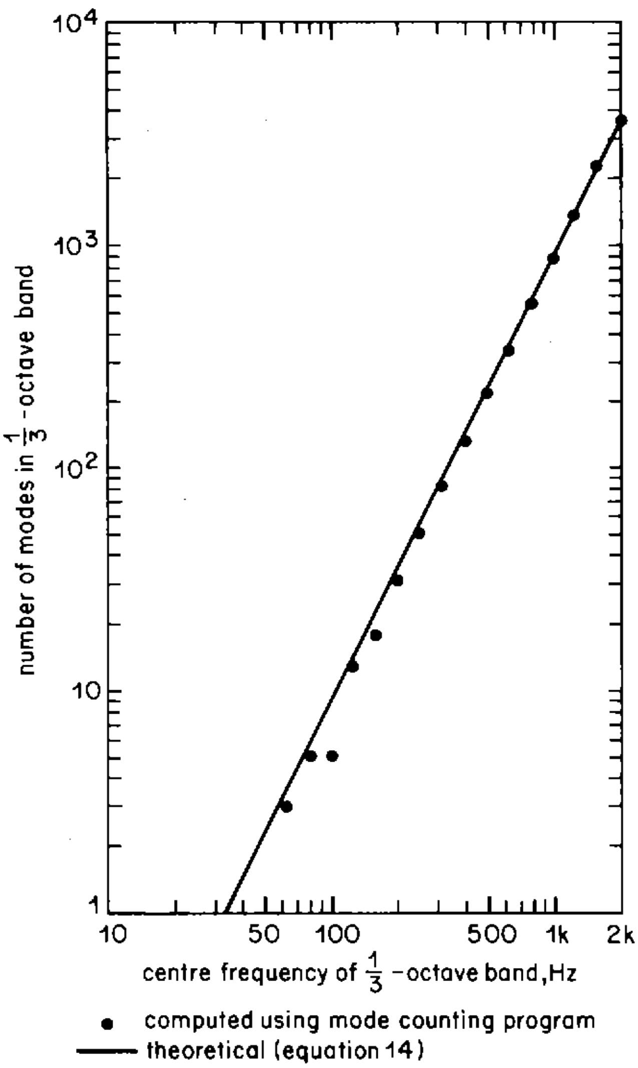

Fig. 5 – Number of tangential modes in $\frac{1}{3}$-octave bands for room of dimensions 7 m × 5 m × 3 m.

*In reality both pairs of walls are non-parallel.

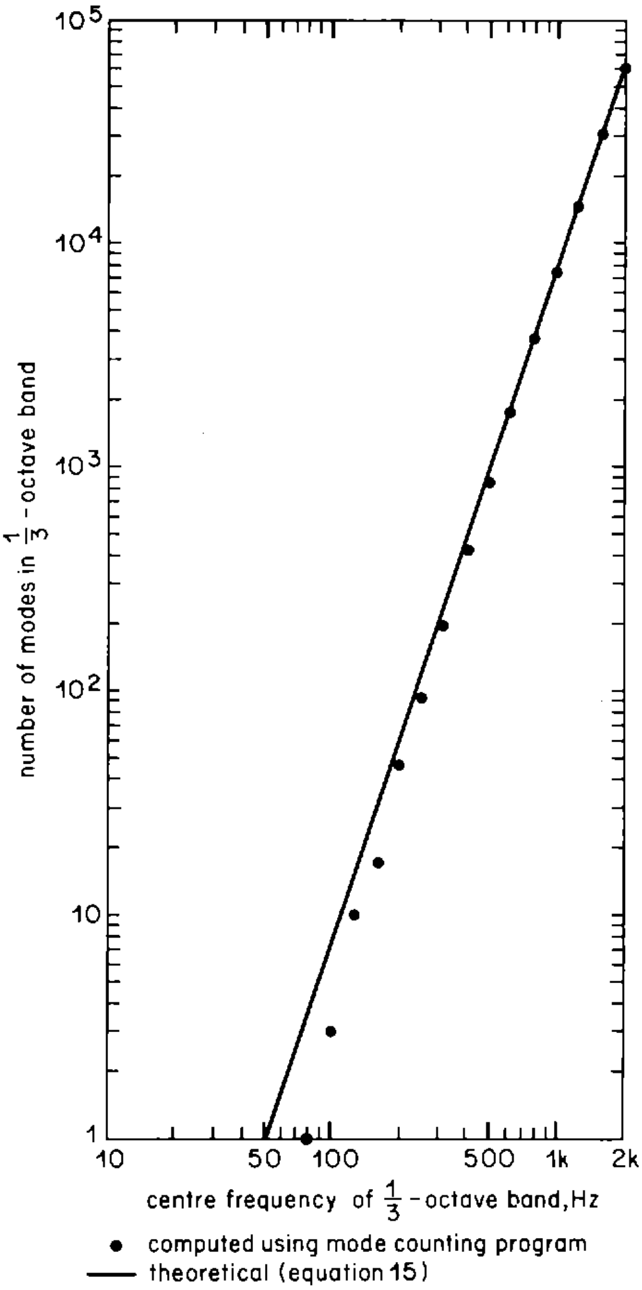

Fig. 6 – Number of oblique modes in $\frac{1}{3}$-octave bands for room of dimensions 7 m × 5 m × 3 m.

3. Relationships between the properties and distribution of absorbing material, the room modes and the sound decay.

The relationships outlined in this Section are based on the work of Dowell.[4] This work consists of a theoretical study of sound decay in an enclosure in terms of the decay of each room mode, the modes being treated as harmonic oscillators with damping provided by the finite wall impedance. The complete theory is capable of taking into account (at least in principle) enclosures of any shape (not necessarily rectangular) and absorptive material distributed in patches over the room surfaces. In this Report only rectangular enclosures are considered, since (in Dowell's words) "for simple geometries the natural acoustic modes of the room are known analytical functions." In the treatment that follows, Dowell's nomenclature has been simplified so as to present the theory in as concise a manner as possible. Room modes are characterised by the number of nodal planes (n_x, n_y and n_z) perpendicular respectively to the x, y and z co-ordinate axes (see Section 2), but for descriptive convenience each mode will be identified by a single symbol (n) where different values of n signify different combinations of the three values, n_x, n_y and n_z. For the nth mode, then, and for the p^{th} rectangular patch of absorbing material on one of the two surfaces of the enclosure normal to the z co-ordinate axis (Fig. 7), a quantity I_{n, p_z} may be defined, where

$$\begin{aligned} I_{n\,p_z} &= I(a_{p_z x}, b_{p_z x}, a_{p_z y}, b_{p_z y}, n_x, n_y) \\ &= \left[\frac{b_{p_z x} - a_{p_z x}}{2} + \frac{L_x}{4\pi n_x}\left(\sin\frac{2\pi n_x b_{p_z x}}{L_x} - \sin\frac{2\pi n_x a_{p_z x}}{L_x}\right)\right] \\ &\quad \times \left[\frac{b_{p_z y} - a_{p_z y}}{2} + \frac{L_y}{4\pi n_y}\left(\sin\frac{2\pi n_y b_{p_z y}}{L_y} - \sin\frac{2\pi n_y a_{p_z y}}{L_y}\right)\right] \qquad (16) \end{aligned}$$

$I_{n\,p_z}$ represents an integration of the effect of the p_z^{th} absorber on the nth room mode, over the surface of the absorber. Two other sets of this integration

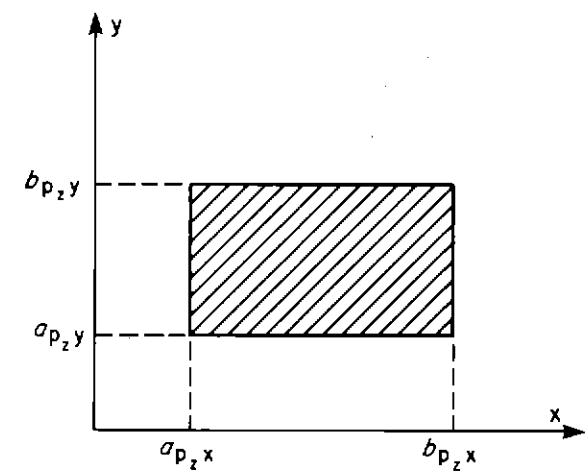

Fig. 7 – Coordinates of patch of absorbing material.

parameter $I_{n p_x}$ and $I_{n p_y}$ also exist, referring to patches of absorbing material on the pairs of the surfaces normal to the x- and y-axes respectively, and therefore involving (y, z) and (x, z) co-ordinates. These parameters still refer to the same (nth) room mode.

A further quantity D_n may now be defined, such that

$$D_n = \frac{1}{\varepsilon_{n_x}\varepsilon_{n_y}\varepsilon_{n_z}}\left(\sum_{\text{all } p} \frac{I_p}{\zeta_p}\right) \qquad (17)$$

In Equation 17, ζ_p is the specific acoustic impedance of the surface of the p^{th} patch of absorbing material ($\zeta = 1$ is the impedance of "free air" or in other words, of a plane sound wave propagating in the absence of any obstructions). In Dowell's analysis, the acoustic impedance is assumed to be real. The quantities ε are defined by the relationship

$$\varepsilon_n \begin{cases} = 1 \text{ when } n = 0 \\ = \frac{1}{2} \text{ when } n \geqslant 1 \end{cases} \qquad (18)$$

The reverberation time for the n^{th} room mode is then given* by

$$T_n = \frac{6V \cdot \ln 10}{cD_n} \qquad (19)$$

At this stage it is interesting to compare the result of Dowell's analysis with earlier work on the subject. The Sabine formula for reverberation time (ignoring air absorption) may be written as

$$T = \frac{24V \cdot \ln 10}{CS\bar{\alpha}} \qquad (20)$$

where $\bar{\alpha}$ is the mean absorption coefficient of the surfaces of the room. Comparison of Equations 19 and 20 immediately shows that, if T_n is taken as equivalent to T,

$$S\bar{\alpha} = 4D_n \qquad (21)$$

Equation 21 shows that Dowell's relationships are of the same form as are at present used in acoustical design work. They differ in that they take into account the size and disposition of the patches of absorbing material, and the frequency of excitation and the room geometry insofar as these factors influence the formation of room modes. Thus a value of D_n only refers to specific conditions of absorber disposition and room geometry and it would not, strictly speaking, be valid to apply this value to conditions other than those used to calculate it in the first place. Conventional methods of room acoustic design using "absorption coefficients" do, however, involve precisely this process since the conditions under which the properties of acoustic absorbers are at present measured seldom correspond with the conditions under which they are used and measurements are, in any case, made using a rather wide band of excitation frequencies. In principle, therefore, Dowell's method should provide a more rigorous and accurate method of predicting the behaviour of absorbing materials in given environments.

It is unfortunate that the practical difficulties of using Dowell's work for undertaking acoustic design appear to be rather formidable. In the first place, the number of room modes in even a small enclosure is high, for all but the lowest frequencies (see Equations 13, 14 and 15) but this should not present too great a difficulty using present day computational methods. The greatest difficulty would be in deducing appropriate values of specific acoustic impedance ζ (Equation 17) for use in calculating the D_n values. In Equation 17 the value of ζ is assumed to be real, whereas in practice measurements of acoustic impedance often show a strong reactive component. One possibility in such cases could be to use the measured complex value of acoustic impedance (ζ_C) to calculate a nominal absorption coefficient for the material in question and then derive a corresponding real impedance (ζ_{real}) giving the same absorption coefficient. It can be shown[5] that for sound arriving at an angle of incidence θ to the surface, the absorption coefficient α_θ is given by the expression

$$\alpha_\theta = \frac{4\zeta_R \cos\theta}{(|\zeta_C|\cos\theta)^2 + 2\zeta_R\cos\theta + 1} \qquad (22)$$

In Equation 22, ζ_C is the modulus of the measured complex acoustic impedance and ζ_R is its real part. It can also be shown that in the complex ζ-plane the locus of constant α_θ is a circle (Fig. 8) with centre $((2-\alpha_\theta)/(\alpha_\theta\cos\theta), 0)$ and radius $2\sqrt{1-\alpha_\theta}/\cos\theta$. Thus there are two values of real acoustic impedance (ζ_{real}) given by the ζ_R-co-ordinate of the intersection of this circle with the real axis, one of these values being greater and one less than unity. It can be seen from Fig. 8 that the magnitude of ζ_{real} is given by

$$\zeta_{real} = \frac{2-\alpha_\theta}{\alpha_\theta\cos\theta} \pm \frac{2}{\cos\theta}\sqrt{1-\alpha_\theta} \qquad (23)$$

*ln 10 is the natural logarithm of 10 (2.3026).

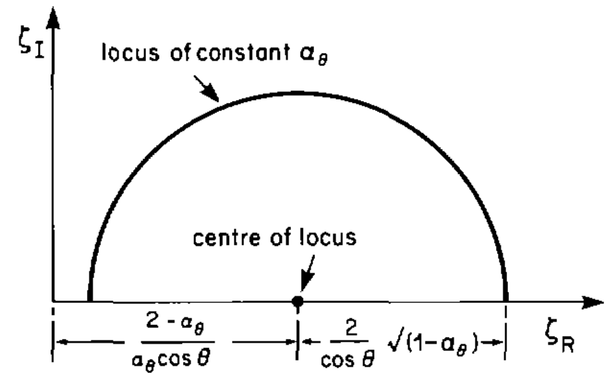

Fig. 8 – Locus of constant α_θ in the complex ζ-plane.

Since the relationships giving the room mode structure (Equations 1–4, Section 2) are based on the assumption that the room surfaces are perfectly reflecting, or in other words $\zeta = \infty$, the positive sign in Equation 23 is chosen so that the value of ζ_{real} approaches this condition as nearly as possible. It must, however, be remembered that this procedure introduces an approximation, since the presence of reactive components of acoustic impedance affect the behaviour of the room modes themselves and therefore make Equations 1–4 and Equation 16 inexact. In practice, however, this approximation would probably be no worse than the initial assumption that the room under consideration is precisely rectangular: irregularities in the surfaces will in practice also affect the room mode structure. A further difficulty is that the angle of incidence (θ) is not known with any precision: for example, it may differ between each of the three pairs of surfaces in a rectangular room, for a given room mode, as well as changing between one room mode and another. It is also necessary to measure (or calculate, making assumptions about the properties of the absorbing surface[5]) the acoustic impedance as a function of angle of incidence: work in this field has been described by Davies and Mulholland.[6]

Equation 16 does not take into account the effect of diffraction caused by the absorbing surface dimensions being commensurate with the wavelength of the sound excitation. This effect serves to increase the effectiveness of the sound absorption[7] and also effectively introduces surface irregularities, thus affecting the room mode structure as described above. The presence of diffraction effects therefore represents another reason why Equation 16 may be inexact.

Despite the difficulties outlined above, it is thought that Dowell's work could provide a method of obtaining greater insight into the apparently anomalous behaviour of low-frequency absorbers in reverberation room measurements and studio use. In the reverberation room, empty room measurements of reverberation time, for a particular room mode, can lead directly to a value of D_N (Equation 19) and thence to a nominal value of specific acoustic impedance for the bare walls, using Equation 17. All surfaces are assumed to be identical: the form that Equation 16 takes under these conditions is discussed later in this Report. Absorbent material is then introduced and an appropriate value (or values) of I_n calculated using Equation 16, knowing the arrangement of the sample (or samples) of material in the reverberation room. After the new reverberation time has been measured it is possible to use Equations 19 and 17 to calculate the "effective" specific acoustic impedance of the sample, since only one unknown value of impedance is involved. The process is repeated using different reverberation room modes, giving a relationship between effective specific acoustic impedance and frequency. This relationship would take the place of the conventional "absorption coefficient" data at present specified. When calculating the effect of the absorbing material in a practical studio, values of I_n are derived taking into account the disposition of the material in the studio (again for each of the relevant modes of the studio) and the reverberation time for each mode then calculated using Equations 17 and 19. The overall effect of the modes in, say, a third-octave band is then found by summing the contributions of individual modes as a function of time, to obtain the final sound-decay curve.

In general, because of the different room sizes in the "measurement" and "practical use" stages in the absorber assessment described above, the mode pattern will be different in the two cases, for a particular excitation frequency (or nearest possible approach to frequency equality when the room modes are rather widely spaced). This means that the angle of incidence of the sound on the absorbers may also be different in the two cases. The method of assessment outlined above is therefore inexact, because of the known dependence of the acoustic impedance on angle of incidence (see Equations 22 and 23). The "effective" values of impedance will in the "measurement" case only at best be averages over the different angles of incidence of the sound onto the various areas of the material under test in the reverberation room, and furthermore will not necessarily be valid in the "practical use" conditions. Hopefully this source of imprecision will not be so great as to invalidate the use of this relatively simple alternative method of acoustic design.

The fact that the effects of individual room modes have to be summed to obtain an overall result over a band of frequencies also introduces a possible ambiguity, since there is no "a priori" means of defining the relative magnitudes of the individual components that make up this overall result. One reasonable assumption is that the sound pressure levels in each component are the same at the start of the sound decay while another equally valid assumption is that the sound energy contained in each mode is the same (i.e. the area contained between the plot of the square of the instantaneous sound pressure against time and the co-ordinate axes is the same). For modes showing nearly equal reverberation times the difference introduced by these two assumptions will not, however, be great. It is also possible that modes will be unequally excited and unequally detected, due to the positioning of the sound source and the receiver respectively, relative to the room surfaces. If this factor is ignored it may be hoped that the result will correspond with normal experimental technique in which an average effect over many source and receiver positions is taken. The result of combining decay curves of differing time constants will, in general, result in a curve which cannot be described by a single exponential function: this corresponds with the practical observation (especially at low frequencies) of "non-linear" (on a logarithmic ordinate scale) decays.

4. Overall effects in a rectangular room with uniform surface impedance

Dowell states a relationship derived from Equation 16 for the case of a rectangular room having uniform surface impedance without, however, giving any indication of his method of derivation. A brief account of this process* may, however, be of interest. If the surfaces considered when defining Equation 16 are assumed to absorb uniformly over their entire area, then the effect of one of the pair of surfaces normal to the z-axis may be obtained by setting $a_{p_z x} = a_{p_z y} = 0$, $b_{p_z x} = L_x$ and $b_{p_z y} = L_y$ in this Equation. Under these conditions

$$I_{nz} = \left[\frac{L_y}{2}\left(1 + \frac{\sin 2\pi n_y}{2\pi n_y}\right)\right]\left[\frac{L_x}{2}\left(1 + \frac{\sin 2\pi n_x}{2\pi n_x}\right)\right]$$

$$= \left[\frac{L_y}{2}(1 + \operatorname{sinc} 2\pi n_y)\right]\left[\frac{L_x}{2}(1 + \operatorname{sinc} 2\pi n_x)\right] \quad (24)$$

where, as before, the "n^{th}" room mode under consideration has nodal plane numbers n_x, n_y, n_z. Now, in general the $\sin \beta/\beta$ or "sinc" function is zero if its argument is an integral multiple of π, but unity if its argument is zero. Thus if n_y (or n_x) is zero the first (or second) quantity in square brackets (Equation 24) becomes L_y (or L_x), while if n_y or n_x is non-zero these terms become $L_y/2$ or $L_x/2$ respectively. Thus Equation 24 may be written

$$I_{nz} = (\varepsilon_{n_y} L_y)(\varepsilon_{n_x} L_x)$$

$$= \tfrac{1}{2}(\varepsilon_{n_x} \varepsilon_{n_y} S_z) \quad (25)$$

where ε_{n_x} and ε_{n_y} are as given by Equation 18 and $S_z = 2L_x L_y$. (Note also that $S_x = 2L_y L_z$ and $S_y = 2L_x L_z$). An identical value of I_{nz} will also be found for the second wall of the pair normal to the z-axis. Four other values of I may similarly be derived, two referring to the pair of walls normal to the x-axis and two for the pair normal to the y axis. Equation 17 then becomes

$$D_n = \frac{1}{\varepsilon_{n_x}\varepsilon_{n_y}\varepsilon_{n_z}} \left(\frac{\varepsilon_{n_y}\varepsilon_{n_z} S_x}{\zeta_x} + \frac{\varepsilon_{n_x}\varepsilon_{n_z} S_y}{\zeta_y} + \frac{\varepsilon_{n_x}\varepsilon_{n_y} S_z}{\zeta_z}\right) \quad (26)$$

where ζ_x, ζ_y and ζ_z are the (real) specific acoustic impedances of the three pairs of surfaces. This is a simplified form of the result quoted by Dowell.

It is convenient in the following discussion to define a parameter D' where

$$D'_n = D_n/V$$

where V is the volume of the room. Then, if the impedance of all three pairs of surfaces is the same ($\zeta_x = \zeta_y = \zeta_z = \zeta$)

$$D'_n = \frac{1}{\zeta V \varepsilon_{n_x}\varepsilon_{n_y}\varepsilon_{n_z}}(\varepsilon_{n_y}\varepsilon_{n_z} S_x + \varepsilon_{n_x}\varepsilon_{n_z} S_y + \varepsilon_{n_x}\varepsilon_{n_y} S_z)$$

$$= \frac{2}{\zeta}\left(\frac{1}{\varepsilon_{n_x} L_x} + \frac{1}{\varepsilon_{n_y} L_y} + \frac{1}{\varepsilon_{n_z} L_z}\right)$$

$$= \frac{2}{\zeta}\cdot C_n \quad (27)$$

where

$$C_n = \frac{1}{\varepsilon_{n_x} L_x} + \frac{1}{\varepsilon_{n_y} L_y} + \frac{1}{\varepsilon_{n_z} L_z} \quad (28)$$

and from Equation 19

$$T_n = \frac{3 \cdot \ln 10}{c} \cdot \frac{\zeta}{C_n} \quad (29)$$

The effective "absorption coefficient" of the surface

*The author is grateful for the assistance given by T. A. Moore of Mathematics Unit, Research Department.

can then be seen from Equation 21 to be

$$\bar{\alpha} = \frac{8VC_n}{S\zeta} \tag{30}$$

Now the value of C_n depends on the ε-values (Equation 28) and these depend in turn on whether the nodal plane numbers defining the particular room mode under consideration are zero or non-zero. Seven categories of room mode can be identified (if $L_x \neq L_y \neq L_z$) as follows

(1) Three axial modes: either n_x, n_y or n_z non-zero.

(2) Three tangential modes: either n_x, n_y, or n_z zero.

(3) All oblique modes: n_x, n_y and n_z all non-zero.

Thus, in general, one room having all surfaces of the same acoustic impedance may, in principle, exhibit seven different "absorption coefficients" according to which of the above categories of room mode is excited. In practice it would be difficult to excite room modes of each category separately and an average absorption coefficient would be obtained, together possibly with some curvature of the logarithmic display of sound pressure level decay. In the discussion which follows, this curvature is ignored and the average absorption coefficient is (for a given room size and shape) taken as being proportional to the C_n value.

The effects of the presence of the different categories of room mode can now be discussed. In the case of the oblique modes, all the ε_n values are equal to $\frac{1}{2}$ (Equation 18): thus from Equation 28

$$C_{ob} = 2\left(\frac{1}{L_x} + \frac{1}{L_y} + \frac{1}{L_z}\right)$$
$$= \frac{S}{V} \tag{31}$$

where C_{ob} is the C_n value for all oblique modes. From Equation 30

$$\bar{\alpha}_{ob} = \frac{8}{\zeta} \tag{32}$$

Thus the effective absorption coefficient ($\bar{\alpha}_{ob}$) obtained for oblique modes is a function only of the specific acoustic impedance of the surface of the room and is independent of the room dimensions. Equation 32 apparently indicates that the value of $\bar{\alpha}_{ob}$ will exceed unity if ζ is less than eight; this anomaly is discussed in Appendix 1.

Dealing next with the three categories of tangential modes, consider first the case in which n_x is zero so that $\varepsilon_{n_x} = 1$ while $\varepsilon_{n_y} = \varepsilon_{n_z} = \frac{1}{2}$. Denoting the relevant value of C_n as $C_{ta\,x}$

$$C_{ta\,x} = \frac{1}{L_x} + \frac{2}{L_y} + \frac{2}{L_z}$$
$$= \frac{S_x/2 + S_y + S_z}{V}$$
$$= \frac{S - S_x/2}{V} \tag{33}$$

Inserting this value of C_n into Equation 30 gives

$$\bar{\alpha}_{ta\,x} = \frac{8}{\zeta}\left(1 - \frac{S_x}{2S}\right)$$
$$= \bar{\alpha}_{ob}\left(1 - \frac{S_x}{2S}\right) \tag{34a}$$

Thus the effective absorption coefficient for this particular category of tangential room mode is lower than that obtained in the case of oblique modes by a factor $(1 - S_x/2S)$ which depends on the proportions of the room (but not on the scale factor between two rooms having the same proportions, since $S_x/2S$ is dimensionless).

It may, similarly, be shown for the other two categories of tangential room mode, having n_y and n_z equal to zero, that respectively

$$\bar{\alpha}_{ta\,y} = \frac{8}{\zeta}\left(1 - \frac{S_y}{2S}\right) \tag{34b}$$

and

$$\bar{\alpha}_{ta\,z} = \frac{8}{\zeta}\left(1 - \frac{S_z}{2S}\right) \tag{34c}$$

Equations 34a, 34b and 34c may become important individually for very low excitation frequencies, when the number of room modes is small and individual modes may be excited. Consider, however, the case in which an equal number of tangential modes in each category are excited. In this case, an average value of the effective absorption coefficient ($\bar{\alpha}_{ta}$) may be found: thus

$$\bar{\alpha}_{ta} = \tfrac{1}{3}(\bar{\alpha}_{ta\,x} + \bar{\alpha}_{ta\,y} + \bar{\alpha}_{ta\,z})$$
$$= \frac{\bar{\alpha}_{ob}}{3}\left[3 - \left(\frac{S_x + S_y + S_z}{2S}\right)\right]$$
$$= \tfrac{5}{6}\bar{\alpha}_{ob} \tag{35}$$

This averaging process ignores the fact that modes may be unequally excited, and avoids the issue of whether the modes contain equal energy or have equal sound pressure level at the start of the decay by ignoring the non-linearity of the resulting decay as seen using a logarithmic ordinate scale. It can be seen that this average effective absorption coefficient is again independent of the room dimensions but is some 17% lower than in the case of the coefficient obtained when only oblique room modes are present. Averages involving unequal numbers of the three categories of room modes can also be found, to take account of the actual modes present in a room in a certain frequency band. Such averages would no longer be entirely independent of room proportion (see discussion following Equation 34a).

Sound absorption in the presence of the three categories of axial mode may be treated by a similar argument to that given above for tangential modes. For the case in which n is non-zero, so that $\varepsilon_{n_x} = \frac{1}{2}$ and $\varepsilon_{n_y} = \varepsilon_{n_z} = 1$, the relevant value ($C_{ax\,x}$) of C_n becomes (compare with Equation 33)

$$C_{ax\,x} = \frac{2}{L_x} + \frac{1}{L_y} + \frac{1}{L_z}$$

$$= \frac{S - \frac{1}{2}(S_y + S_z)}{V} \qquad (36)$$

Thus the appropriate mean absorption coefficient $\bar{\alpha}_{ax\,x}$ becomes

$$\bar{\alpha}_{ax\,x} = \bar{\alpha}_{ob}\left(1 - \frac{S_y + S_z}{2S}\right) \qquad (37a)$$

and the effective absorption coefficient for the other two categories of axial mode (n_y and n_z non-zero) become respectively

$$\bar{\alpha}_{ax\,y} = \bar{\alpha}_{ob}\left(1 - \frac{S_x + S_z}{2S}\right) \qquad (37b)$$

and

$$\bar{\alpha}_{ax\,z} = \bar{\alpha}_{ob}\left(1 - \frac{S_x + S_y}{2S}\right) \qquad (37c)$$

Again, taking an average effective absorption coefficient over an equal number of each of the three categories of axial room mode gives

$$\bar{\alpha}_{ax} = \frac{\bar{\alpha}_{ob}}{3}\left(3 - \frac{S_x + S_y + S_z}{S}\right)$$

$$= \tfrac{2}{3}\bar{\alpha}_{ob} \qquad (38)$$

This average effective absorption coefficient is as before independent of the room dimensions but is some 33% lower than in the case of the coefficient obtained when only oblique room modes are present. Again as before, the averaging process ignores the production of "non-linear" sound decay characteristics and it can be seen that dependence on room proportion exists if the numbers of the three categories of axial mode are not equal.

It was shown in Section 2 that the proportion of room modes of each type (axial, tangential and oblique) depended on the excitation frequency and upon the room dimensions. Suppose that the proportion of axial modes is m_{ax}, of tangential modes is m_{ta} and of oblique modes m_{ob}, so that

$$m_{ax} + m_{ta} + m_{ob} = 1$$

Then, with the same approximations as discussed for averaging categories of axial or tangential modes, an "overall average" ($\bar{\alpha}_o$) may be calculated, so that

$$\bar{\alpha}_o = m_{ax}\bar{\alpha}_{ax} + m_{ta}\bar{\alpha}_{ta} + m_{ob}\bar{\alpha}_{ob}$$

$$= \bar{\alpha}_{ob}\left(\frac{2m_{ax}}{3} + \frac{5m_{ta}}{6} + m_{ob}\right)$$

$$= \frac{8}{\zeta}\left(\frac{2m_{ax}}{3} + \frac{5m_{ta}}{6} + m_{ob}\right) \qquad (39)$$

The values of m_{ax}, m_{ta} and m_{ob} corresponding to $\frac{1}{3}$-octave frequency bands may be obtained from Equations 13, 14 and 15, since for example, in a $\frac{1}{3}$-octave frequency band having centre frequency f_C

$$m_{ax} = \frac{\Delta N_{ax\,1/3\,f_C}}{\Delta N_{ax\,1/3\,f_C} + \Delta N_{ta\,1/3\,f_C} + \Delta N_{ob\,1/3\,f_C}} \qquad (40)$$

with similar expressions for m_{ta} and m_{ob}. An increase in room dimensions leads to a higher value of m_{ob} relative to m_{ta}, and a higher value of m_{ta} relative to m_{ax} (see discussion following Equation 15). Equation 39 shows that this trend will give rise to an increase in the value of $\bar{\alpha}_o$ as the room dimensions increase, for rooms with surfaces having the same (uniform) acoustic impedance. The greatest change in $\bar{\alpha}_o$ would be 33% where only axial modes ($m_{ax} = 1$, $m_{ta} = m_{ob} = 0$) were present in one case and oblique modes ($m_{ax} = m_{ta} = 0$, $m_{ob} = 1$) in the other case. Frequently the change to be expected in $\bar{\alpha}_o$ would be considerably lower than this extreme value.

5. Comparisons with other practical and theoretical work

In the course of the design of a large music studio (Studio 7, New Broadcasting House, Manchester) a one-eighth scale model was used to simulate the acoustic properties of the full-size studio.[8] Absorbers for use in this model studio were tested using a model reverberation room[9] built to the same scale factor. Using sound at eight times the baseband frequency, the acoustic room mode structure was then the same in the models as would have been obtained in the full-size rooms at baseband frequency. For descriptive convenience dimensions and frequencies relating to the full-sized rooms are used in the following discussion.

It was found that the predictions of absorption coefficient made using the reverberation room (dimensions 6.7 m × 5.3 m × 5.7 m) did not agree with the effective values obtained when the same absorbers were placed in the studio (dimensions 27 m × 22 m × 14 m). In the case of the measurements in the studio, results were obtained under three conditions with the absorbers mounted on the floor, the ceiling and all four walls. The last of these conditions is of interest in the present context, since it comes closest to the assumption of uniform surface absorption adopted in the foregoing discussion in this Report. Indeed, Kuttruff[10] suggests that it is valid to take an average value of surface absorption in cases where the dimensions of the absorbers and the spaces between them are less than the quantity $4V/S$ (i.e. the "mean free path" between reflections). This quantity takes the value of 4 m for the reverberation room and 13 m for the studio. The absorbers themselves have dimensions 1.2 m × 0.5 m and thus satisfy Kuttruff's condition: furthermore, the spacing between absorbers in the studio also satisfies this condition[11] (see, in particular, Fig. 4 of Reference 11). Thus the walls of the studio may be regarded as having uniform surface absorption, using this criterion. However, it must be remembered that the floor and ceiling remained untreated and therefore of lower surface absorption. Fig. 9 shows (solid circles) the ratios of the absorption coefficients obtained[11] in the studio under these conditions, as compared with the absorption coefficients found for the same $\frac{1}{3}$-octave frequency band using the reverberation room. Also shown in Fig. 9 are estimated confidence limits associated with each ratio value, which take into account the spread in reverberation time values usually met with in this type of measurement, and also the possible error involved in reading off the absorption coefficient values from Figs. 2 and 3 of Reference 11.

From Equation 39 it is possible to predict a theoretical ratio of the "overall average" absorption coefficients that apply to the two rooms. If R_T is this ratio, and denoting the parameters associated with the studio and the reverberation room by the subscripts (S) and (R) respectively, then

$$R_T = \frac{4m_{ax(S)} + 5m_{ta(S)} + 6m_{ob(S)}}{4m_{ax(R)} + 5m_{ta(R)} + 6m_{ob(R)}} \qquad (41)$$

The m_{ax} values are derived using Equation 40 and the m_{ta} and m_{ob} values derived in a similar way as noted in the discussion following this equation. The values of $\Delta N_{ax\,1/3f_C}$, $\Delta N_{ta\,1/3f_C}$ and $\Delta N_{ob\,1/3f_C}$ for each room can be obtained from the mode-counting computer program referred to in Section 2 (the frequencies involved are too low for Equations 13–15 to be regarded as accurate). Columns 2–7 of Table I show the m-values obtained in this way,

TABLE I

Proportions of the three types of room mode (for conditions see text)

Frequency (Hz)	Proportions of room modes for conditions heading columns						Ratio of theoretical "overall average" absorption coefficients (R_T)
	Studio			Reverberation room			
	Axial ($m_{ax(S)}$)	Tangential ($m_{ta(S)}$)	Oblique ($m_{ob(S)}$)	Axial ($m_{ax(R)}$)	Tangential ($m_{ta(R)}$)	Oblique ($m_{ob(R)}$)	
50	0.048	0.369	0.583	0.500	0	0.500	1.107
63	0.028	0.313	0.659	0.250	0.625	0.125	1.155
80	0.017	0.268	0.715	0.083	0.500	0.417	1.068
100	0.013	0.220	0.767	0.136	0.500	0.364	1.101
125	0.008	0.179	0.813	0.077	0.410	0.513	1.068
160	0.005	0.147	0.848	0.053	0.368	0.579	1.058
200	0.003	0.120	0.877	0.033	0.338	0.629	1.050
250	0.002	0.097	0.901	0.022	0.263	0.715	1.036
315	0.001	0.078	0.921	0.013	0.226	0.761	1.030

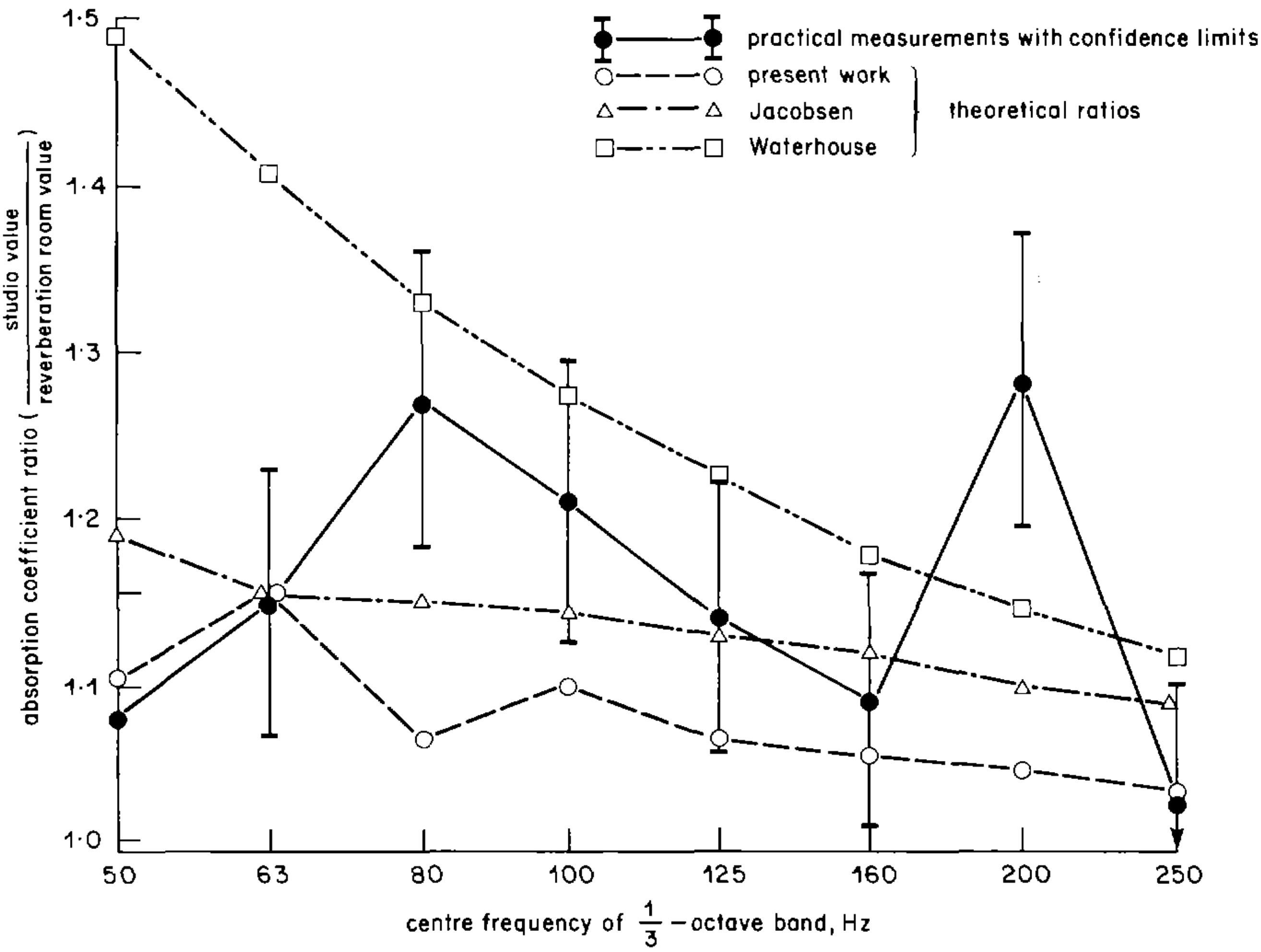

Fig. 9 – Practical and theoretical ratios of the absorption coefficient of a particular type of absorber, when placed in two rooms of different sizes.

while Column 8 of this Table shows the corresponding values of R_T. These values of R_T are also shown in Fig. 9 (open circles).

It can be seen in Fig. 9 that there is a measure of agreement between the results obtained theoretically and experimentally. In over half of the observations the theoretical ratio lies within the tolerance limit of the practical measurements and the overall trend of the two sets of values is the same with the ratio value tending to approach unity as frequency of excitation increases. The ratio value seems in general to be higher by a factor of about two in the case of the practical measurements, as compared with the theoretically-derived values. Nevertheless, the agreement is sufficiently good to permit the conclusion that in the case of comparisons between absorption coefficients of absorbers measured in the reverberation room and when mounted on the walls of the studio, the predominant factor in causing the observed differences in absorption coefficient was the difference in the room mode structure within the two enclosures, for a given $\frac{1}{3}$-octave band of excitation frequencies.

Jacobsen[12] has recently published work on the influence of room mode structure on sound absorption. This work relates the value of absorption coefficient to the room volume and the excitation frequency, and involves an estimate of the average proportions of axial, tangential and oblique room modes as a function of these parameters over rooms of different shapes, together with a numerical estimate of the error caused by approximating the non-linear logarithmic decay curve by a straight line drawn between the −5 dB and −35 dB crossing points on this curve. The relationship is presented graphically (Fig. 1 of Reference 12). Fig. 9 shows (open triangles) the result of applying Jacobsen's relationship to the comparison of changes in absorption coefficient with room size discussed above. It can be seen that a rather greater estimate of change in absorption coefficient is obtained, as compared with the results derived by the method discussed in this Report: nevertheless, the same number of theoretical predictions lie within the tolerance limits of the practical results in the two cases.

Jacobsen compares his results with a relationship involving the "Waterhouse Correction",[13] which may be expressed in this context as

$$\frac{\alpha_T}{\alpha_M} = 1 + \frac{cS}{8Vf} \qquad (42)$$

where α_T is the "true" absorption coefficient, and α_M is the value measured in a room of surface area S and volume V, and at a frequency f (c is the velocity of sound as before). However, it must be remembered that Waterhouse derived this relationship to express the true energy content of a room, relative to the energy calculated from measurements in the central portion of the room, where the effect of standing-wave patterns due to reflections from the walls is negligible. The relationship is not directly applicable to considerations involving the absorption of energy at the room surfaces, since the effect of such standing-wave patterns will be the same (at least to a first-order approximation, neglecting the additional effects of edges and corners) irrespective of the size of the room. The use of the Waterhouse correction in this context must therefore be regarded as empirical, giving as it does a relationship of roughly the correct order of magnitude and which varies in an appropriate way with frequency and room size. This relationship is also plotted in Fig. 9 (open squares): it appears to overestimate the changes in absorption coefficient with room size, particularly at low frequencies, and its predictions do not agree so well with the practical results as in the case of the other two relationships. Nevertheless it remains useful as a rough estimate of the magnitude of the effect, despite its empirical nature, if only because of the simplicity of the relationship (Equation 42) and the ease with which values may be calculated from it.

6. Conclusions

Theoretical work by Dowell[4] leads to a relationship applicable to rectangular rooms by which the sound decay (in terms of the reverberation time) of a particular room mode can be calculated in terms of the nodal plane numbers of the mode, the dimensions of the room, the size and arrangement of the absorbers and the specific acoustic impedance of the absorbing surfaces. This equation can be greatly simplified if it is assumed that all room surfaces have the same impedance, and all room modes are equally excited. Then, using also the Sabine relationship between reverberation time and absorption coefficient, it becomes possible to calculate the "effective" absorption coefficient in terms only of the acoustic impedance of the surface and the proportions of the three types of room mode (axial, tangential and oblique) that are excited in the frequency band under consideration. Since these proportions vary with room size for a given frequency band, the change in apparent absorption coefficient of a particular type of absorber when transferred from one room to another (e.g. from a reverberation room to a large studio) can be calculated. Comparisons of this theoretical relationship with practical measurements obtained using one-eighth scale models of a large studio and a reverberation room show encouraging similarities, as do comparisons with theoretical work recently published by Jacobsen.[12] It appears that Dowell's method holds some promise as an alternative method of assessing the performance of absorbers in different environments. In its most general version the method calculates the reverberation time for each room mode separately and these have then to be summed: a fast computer may therefore be required as the number of room modes involved can be large, especially for large rooms and at high frequencies. Its most useful field of application is however likely to be in cases involving small rooms and low frequencies, where the limited number of modes gives rise to anomalies in conventional methods of measurement. Some consideration will have to be given to the measurement of the acoustic impedance of the absorbing surface: this aspect is complicated by the fact that Dowell's method assumes the impedance to be real whereas, in practice, it often exhibits a reactive component. It may be possible to calculate an equivalent real value of impedance from the complex value actually measured for use in the calculations.

The use of a simple method of assessing the effect of room size and excitation frequency on the apparent value of the absorption coefficient (the "Waterhouse correction") has also been briefly examined. Although this method must in the present context be regarded as empirical, it gives results of the same order of magnitude as those calculated by more sophisticated methods.

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APPENDIX 1: A RELATIONSHIP BETWEEN ABSORPTION COEFFICIENT AND SPECIFIC ACOUSTIC IMPEDANCE, IN CASES WHERE THE OVERALL ABSORPTION IS NOT VERY SMALL

Equation 30 shows a relationship between mean absorption coefficient ($\bar{\alpha}$) and specific acoustic impedance (ζ) which has been derived using the Sabine formula for reverberation time (Equation 20). This formula is only accurate for relatively small values of mean absorption coefficient. The apparent anomaly shown by Equation 32 (derived in turn from Equation 30), where an absorption coefficient value greater than unity is obtained when the specific acoustic impedance adopts a value less than eight, is due to this involvement of the Sabine formula. A more accurate relationship between reverberation time and absorption coefficient, due to Eyring, may be written:

$$T = \frac{24 \cdot \ln 10}{c} \cdot \frac{V}{-S \ln (1 - \bar{\alpha})} \qquad (43)$$

Using this equation instead of Equation 20 in deriving the relationship between absorption coefficient and specific acoustic impedance leads to the result:

$$-\ln (1 - \bar{\alpha}) = \frac{8VC_n}{S\zeta}$$

which gives

$$\bar{\alpha} = 1 - \exp\left(-\frac{8VC_n}{S\zeta} \right) \qquad (44)$$

Using Equation 44 instead of Equation 30, the relationship (compare Equation 32) for oblique room modes becomes:

$$\bar{\alpha}_{ob} = 1 - \exp\left(-\frac{8}{\zeta} \right) \qquad (45)$$

With this relationship it can be seen that as the value of ζ becomes smaller, so the value of $\bar{\alpha}_{ob}$ tends towards the value of unity, thus avoiding the apparent anomaly shown in Equation 32. It must however be remembered that values of specific acoustic impedance considerably greater than unity are implicit in the existence of room modes (see footnote in Appendix 2, page 16). An impedance value of unity would imply a perfect match between the walls of the room and the sound field within it, with consequent complete sound absorption. In fact, mean absorption coefficients less than 0.1 (in other words, specific acoustic impedances greater than about 80) were involved in the comparison of absorption coefficient predictions described in Section 5: under these conditions results derived using either the Sabine or the Eyring relationship agree to within 5%.

APPENDIX 2: SOUND PRESSURE DISTRIBUTION AND OVERALL ABSORPTION IN A ROOM

An appreciation of the physical processes which give rise to the relationships between the effective absorption coefficients obtained for the three classes of room mode (Equations 35 and 38) may be obtained by considering the sound pressure distribution over the surfaces of a room having uniform surface absorption in relation to the room mode structure. Waterhouse[13] has analysed the variation of sound pressure with distance from a reflecting wall, a reflecting rectangular edge and a reflecting rectangular corner. This work is not directly applicable to the present case, since the general analysis is in terms of random sound incidence and assumes that the room mode structure is averaged out in regions of the room away from its walls. However, Waterhouse does compare the variations of mean square pressure with distance from a plane wall, for the cases of normal and random incidence. In the first case the mean square pressure varies between a maximum value at the wall and at distances which are multiples of half the sound wavelength from it, and zero at positions half-way between the maxima, according to a "raised cosine" relationship. In the second case, for a sound field of the same energy density, the mean square pressure at the surface has the same value as in the case of normal incidence. For the cases of reflections at room edges and room corners, still assuming random incidence, the mean square pressures at the wall surface are shown to be respectively twice and four times the values obtained at a plane wall, again referring to sound fields of the same energy density. It may be assumed that this relationship also applies in the presence of a single room mode, but that in this latter case the sound pressure distribution is repeated in each "anti-nodal element" of the mode, as discussed below, the distance between adjacent pressure maxima being determined by the mode parameters.

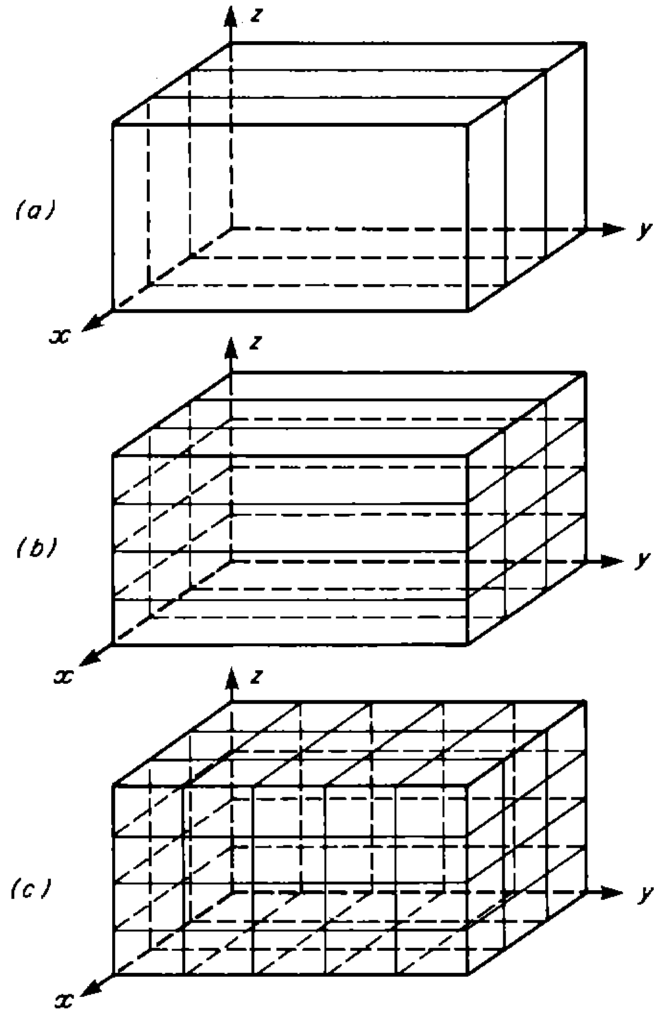

Fig. 10 – Illustrations of sets of antinodal planes in a room: (a) Axial modes: one set of planes; (b) Tangential modes: two sets of planes; (c) Oblique modes: three sets of planes. For clarity only the lines of intersection of the antinodal planes with the room surfaces are shown.

In the case of axial modes the room may be considered as divided into a number of smaller slab-shaped elements by the single set of pressure antinodal planes (Fig. 10 (a)). The distribution of mean square sound pressure ($\bar{p}^2$) over the surfaces of one such element (Fig. 11 (a)) may now be considered. In the case shown in Fig. 11 (a) the sound pressure distribution will be uniform in both the y- and the z- directions over the surfaces normal to the x-axis. In the case of the surfaces normal to the y-axis the sound pressure distribution will be uniform in the z-direction, but the mean square pressure will vary according to the raised cosine law in the x-direction, with a peak value along the edges of the surfaces parallel to the z-axis equal to the uniform sound pressure in the z-direction. The mean squared pressure averaged along the x-direction is therefore half the uniform value in the z-direction. Assuming that the sound absorption taking place at the surface is proportional to the energy contained in the sound field at the surface, or in other

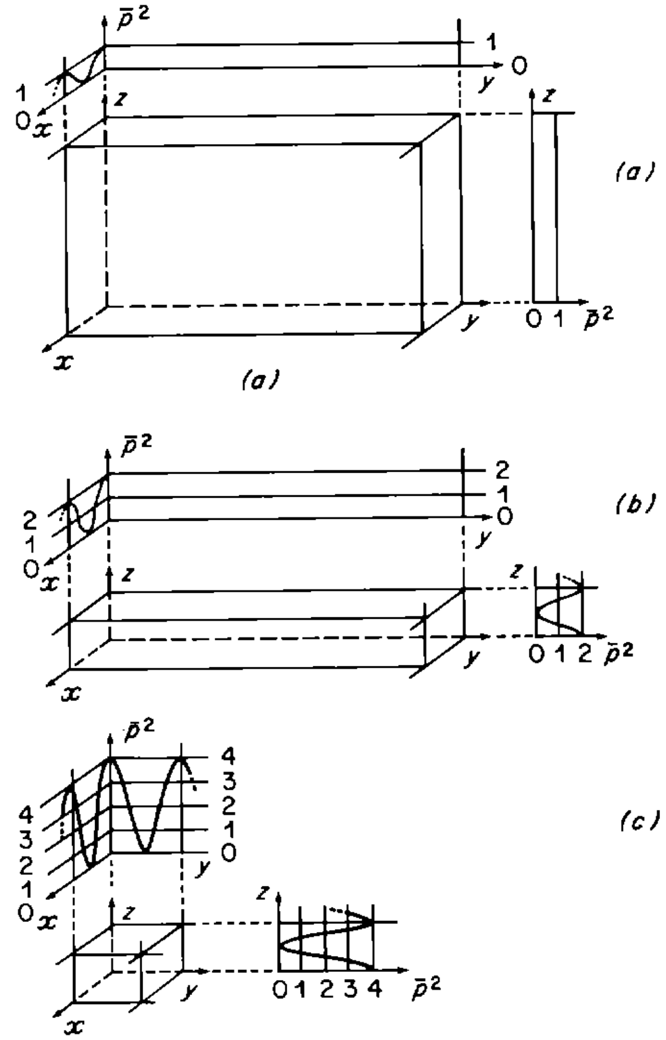

Fig. 11 – Variation of mean square sound pressure along edges of elements formed by intersection of room surfaces and antinodal planes. (a) axial room modes (slab-shaped element); (b) tangential room modes (bar-shaped element); (c) oblique room modes (box-shaped element).

words proportional to the mean squared pressure,* the absorption taking place at the walls normal to the y-axis is therefore also half the absorption taking place at the walls normal to the x-axis. The effective absorption coefficient of these walls is therefore also halved. A similar argument shows that the absorption coefficients of the walls normal to the z-axis are also half those of the walls normal to the x-axis.

Considering now the complete room (Fig. 10 (a)), and in the first instance regarding it as cubic, so that all the walls have equal area, it can be seen that two surfaces (those normal to the x-axis) will each have a certain absorption coefficient $\bar{\alpha}$, while the other four surfaces (those normal to the y- and z-axes) will have an absorption coefficient of $\bar{\alpha}/2$. Thus the average absorption ($\bar{\alpha}_{ax}$) is given by

$$\bar{\alpha}_{ax} = \tfrac{1}{6}\left(2\bar{\alpha} + 4\cdot\frac{\bar{\alpha}}{2}\right) = \tfrac{2}{3}\bar{\alpha} \qquad (46)$$

The assumption that the room is cubic is justified by the consideration that in practice the three types of axial room mode should be considered, with nodal planes normal to each room axis. If the above argument is repeated for each such type of axial mode, but taking into account the actual wall areas of a rectangular (not cubic) room, and the effects are then averaged on the assumption that each type of axial mode contains equal energy, the result shown by Equation 46 will be obtained.

Turning now to the case of tangential room modes (Fig. 10(b)), the room may be considered as divided into a number of bar-shaped elements by the two orthogonal intersecting sets of pressure antinodal planes. One such bar-shaped element is shown in Fig. 11(b). Tangential room modes involve reflections of sound from two pairs of room surfaces and the mean square sound pressure distribution therefore corresponds to the condition for reflection from room edges described by Waterhouse. Thus the maximum mean square sound pressure is twice the value obtained for axial modes. Along the surfaces normal to the x- and z-axis in Fig. 11(b) the sound pressure distribution is uniform in the y-direction but has a raised cosine distribution in the z- and x-direction respectively. For those four surfaces in the complete room (Fig. 10(b)) the total absorption, and therefore the effective absorption coefficient, will be doubled because of the increase in maximum sound pressure, but halved because of the sound pressure distribution. For the surfaces of the bar-shaped element normal to the y-direction the sound pressure distribution will be of the raised cosine form in both the x- and z-directions. Applied to the complete room, this distribution would by itself reduce the effective absorption coefficient to one-quarter of the value obtained with uniform sound pressure distribution. Combined with the doubling of maximum mean square sound pressure discussed above, the overall result is a halving of the absorption coefficient of the surfaces in the complete room normal to the y-direction.

If, for the reasons discussed above, the complete room is regarded as cubic so that all the walls have equal area, then the average absorption for tangential modes ($\bar{\alpha}_{ta}$) is given by:

$$\bar{\alpha}_{ta} = \tfrac{1}{6}(4 \times 2 \times \tfrac{1}{2} \times \bar{\alpha} + 2 \times 2 \times \tfrac{1}{4} \times \bar{\alpha}) = \tfrac{5}{6}\bar{\alpha} \qquad (47)$$

*The presence of room modes presupposes a "hard" wall surface at which the velocity component of the sound field will be small and will not contribute significantly to the energy content of the field.

When oblique room modes are present the room may be considered as divided into a number of "box-shaped" elements by the three orthogonal sets of pressure antinodal planes (Fig. 10(c). One such box-shaped element is shown in Fig. 11(c). Oblique room modes involve reflections of sound from all three pairs of room surfaces and the sound pressure distribution therefore corresponds to the condition for reflection from room corners described by Waterhouse. Thus the maximum mean square sound pressure is four times the value obtained for axial modes. Over all surfaces the mean square sound pressure distribution will be of the raised cosine form in both axis directions contained in the surface. In the complete room this distribution would by itself reduce the effective absorption coefficient to one-quarter of the value obtained with uniform sound pressure distribution. These two factors therefore compensate for each other: thus for the complete room the total absorption for oblique modes ($\bar{\alpha}_{ob}$) is given by:

$$\bar{\alpha}_{ob} = \tfrac{1}{6}(6 \times 4 \times \tfrac{1}{4} \times \bar{\alpha}) = \bar{\alpha} \qquad (48)$$

It can be seen from Equations 46 and 48 that $\bar{\alpha}_{ax}/\bar{\alpha}_{ob} = \frac{2}{3}$ and from Equations 47 and 48 that $\bar{\alpha}_{ta}/\bar{\alpha}_{ob} = \frac{5}{6}$, thus accounting in physical terms for the relationships, shown in Equations 38 and 35 respectively, that were obtained using Dowell's simplified theoretical relationship (Equation 24) which assumes uniform absorption over the surfaces of the room. Furthermore, the reason for the variation in effective absorption as a patch of absorbing material is moved from place to place on the walls of a room (Equation 16) can also be seen to be caused by its position relative to the sound pressure level distribution over the walls, for the particular room mode under consideration.

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