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CEEFAX:
computer based time-domain and frequency-domain analysis of system responses to test pulses

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Summary

In the course of CEEFAX tests conducted in Germany, the 1T pulse responses of systems and sub-systems were photographed.

This report describes techniques developed to compute amplitude and phase responses in the frequency-domain derived from 1T waveforms, and to enable the responses of sub-systems to be calculated from the responses at different points within a system. A method of computing the CEEFAX performance of systems is also described.

The techniques are only applicable to the analysis of the responses of linear systems. The accuracy of these results is adequate with one or two intermediate multiplications or divisions of responses.

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1. Introduction

The response of a linear system can be completely defined as either an impulse response in the time-domain or an amplitude and phase response in the frequency-domain. In some cases it is convenient to define a system response in the frequency-domain, in other cases the response in the time-domain must be defined. In television specifications both the frequency-domain (amplitude and phase responses at colour subcarrier frequency) and time-domain (2T pulse response) are used.

A result obtained in one domain can be converted to the equivalent response in the other domain using Fourier and inverse Fourier transforms. Such transforms involve manipulating large amounts of data and can be most conveniently performed using a computer.

The analysis techniques described here were developed for use in connection with CEEFAX tests in Germany.[1] For these tests the most convenient method of rapidly measuring and calibrating systems was to photograph their output waveforms when a signal from a calibrated pulse source was applied at their inputs. After the tests, the photographed waveforms were digitised and the corresponding frequency-domain responses were computed.* From these results the responses of component parts of systems were derived. The analysis work was also extended to allow the CEEFAX performance of systems to be computed and their CEEFAX eye patterns to be plotted.

The input data for the computer program is a digitised time response of a reference pulse which has been passed through the network. Such a pulse response can be Fourier analysed to provide the group delay and amplitude spectrum of the pulse and the network combined. If the original reference pulse, direct from the generator, is also digitised, the group delay and amplitude spectral response of the undistorted pulse can be obtained. If the complex spectrum at the network output is divided by the complex spectrum at the pulse generator output then the result gives the relative amplitude and group delay response of the network alone. From this an inverse Fourier analysis can be used to produce the impulse response of the network. This process for producing the impulse response of a network is referred to as 'convolution division', following from the multiplication of spectra which corresponds to 'convolution' of two time waveforms.

2. The test pulse

For the method to be valid the energy in the pulse used as the reference must be significant right up to the edge of the band of frequencies of interest. Hence a 1T pulse must be used in preference to a 2T pulse when a bandwidth up to 5 or 5·5 MHz is involved since the 2T pulse has no energy at 5 MHz. A unit impulse might seem preferable to a 1T pulse but generators providing these do not exist, whereas 1T pulses are normally readily obtainable at all points from which a transmission network might be tested.

It should be noted that it is not generally possible to predict the performance of a network by subjectively inspecting the 1T pulse waveform since its 10 MHz spectrum will almost always be severely band limited. The band limiting causes rings which mask any other echo effects. On the other hand when using a computer analysis the required information is obtained in spite of the rings.

3. Digitising a waveform photograph

3.1. General

As stated in Section 1, the techniques described in this report were developed for analysis of results obtained during CEEFAX tests in Germany. A typical waveform photograph obtained during these tests is shown in Fig. 1. It shows three separately exposed traces giving:

(a) Positive 1T pulse.
(b) Negative 1T pulse.
(c) Timing and tilt reference waveform.

The 1T pulse waveforms are digitised by securing the pulse photograph to the table of a trace digitiser. A cursor is then used to trace the waveform digitising points along its length. This process introduces small errors, some of which can be corrected in the subsequent analysis of the waveform. Moreover, the photograph might not contain all necessary information about some frequencies because of the limited length of the recorded waveform.

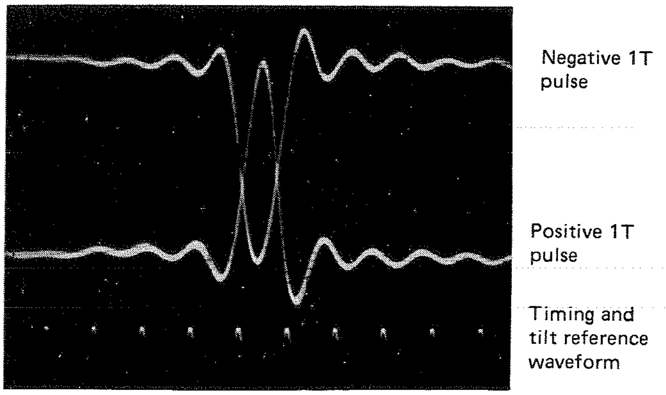

Fig. 1 - A typical waveform photograph obtained during tests in Germany

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* The computer programmes for the work described in this report were devised by R.E. Davies.

The errors and inaccuracies introduced in photographing and digitising a pulse waveform, and some corrections that can be applied, are discussed below.

3.2. Frequency resolution

Since all the amplitude and group delay information is derived from the time response of a 1T pulse through a network the accuracy of the method relies heavily upon how well the pulse shape is interpreted in the digitisation process. The pulse alone conveys mainly high frequency information. In order to obtain sufficient information about the response at lower frequencies, and to give a reasonable frequency resolution to the computed frequency-domain responses, it is necessary to analyse at least 2 μs of the pulse waveform. This will give a frequency scale resolution of 0·5 MHz which is just about adequate for the use for which the analysis is intended. It is unlikely that any device in the transmission chain would cause very sharp changes in the frequency response such as would be completely missed by a 0·5 MHz resolution. A relatively sharp notch for example would appear as a broader shallower notch because of the limited resolution of the frequency scale resolution.

3.3. Possible digitising errors

Given a photograph of about 2 μs of pulse waveform which also contains a time marker waveform there are four common types of error which can occur in the digitising process.

The photograph may not be placed square on the digitising table or may have been shot with the camera slightly crooked.

Each digitised point will be randomly in error, by some small amount, from the true waveform position due to trace thickness, operator error and confusion caused by double imaging of the waveform (oscilloscope triggering problems), or the quantising error of the digitiser.

The time scale of the photograph may not be measured with sufficient accuracy by the digitiser because the relevant digitised points may have errors for the reasons given above.

The base line of the waveform may not be correctly defined by the digitising process because the pulse rings continue beyond the edges of the photograph and also because of the digitising errors previously mentioned.

3.4. Effects of digitising errors

These errors, if not adequately taken into account or corrected for can cause the following effects.

3.4.1. Errors due to rotation of the base line cannot be corrected but it is relatively easy to determine the true angle of the base line using a reference included in the photograph.

If the true time and voltage axes on the photograph are rotated with respect to the X and Y axes of the digitiser then the X and Y distances measured by the digitiser will be shortened. For small angles of error the measured length will be the true length multiplied by the cosine of the error angle. It is unlikely that the error angle will exceed about 5° and on this basis the errors would not be significant, (less than 0·1%). In practice points are digitised to indicate the angle of the time axis so that an angular rotation correction can be made by the computer to the digitised data. This correction allows large rotational errors without affecting the final dimensional accuracy. Such a correction does of course rely upon correct digitisation of the points which represent the angle of the time axis. For maximum accuracy two widely spaced points are required to minimise the effect of quantising errors. In practice a cursor is used to extend these points well beyond the edges of the photograph.

The assumption is that this line is parallel to the base line of the pulse. This depends upon good geometry in the oscilloscope display and on placing the camera at the same angle relative to the display time axis for each exposure of the composite photograph. The camera used during the field tests in Germany was a hand held type and was located by aligning one edge of the mask with the edge of the graticule prespex. Small errors were thus possible between successive exposures of the composite photograph and small effects have been observed on the frequency response which could be attributed to base line tilt. The effect of such errors is discussed in Section 5.4. If the importance of camera position had been realised before the field trials, a means of accurately repositioning the camera would have been provided.

3.4.2. Random errors in the digitising of the waveform do not in general cause a problem since the points representing the pulse waveform are closely spaced (10 — 20 ns) and therefore appear in the Fourier analysis as noise up to 50 MHz bandwidth. Since we are only interested in frequencies up to 10 MHz any components above 10 MHz which appear in the Fourier analysis are deleted by the computer. This reduces the effect of random errors. For the measurement of time axis and the angle of the time scale which each use only two points, the points are spaced as widely as possible to reduce the effect of digitising errors.

3.4.3. An error in the measurement of the time scale, represented by the time markers on the photograph, will result in the frequency scale appearing stretched or compressed, but the shape of the response will be unaltered. Measuring the largest practical time interval makes this error negligible.

3.4.4. On many pulse waveforms the rings extended to the edge of the photograph. This made it very difficult to define where the base line of the pulse should be. Furthermore, accuracy is required for this parameter since a base line error of 1% can cause amplitude response errors of as much as 20%.

Initially the base line level is taken as the Y value (corrected for rotation) of the first point of the waveform (the left hand edge of the photograph). Due to a wave-

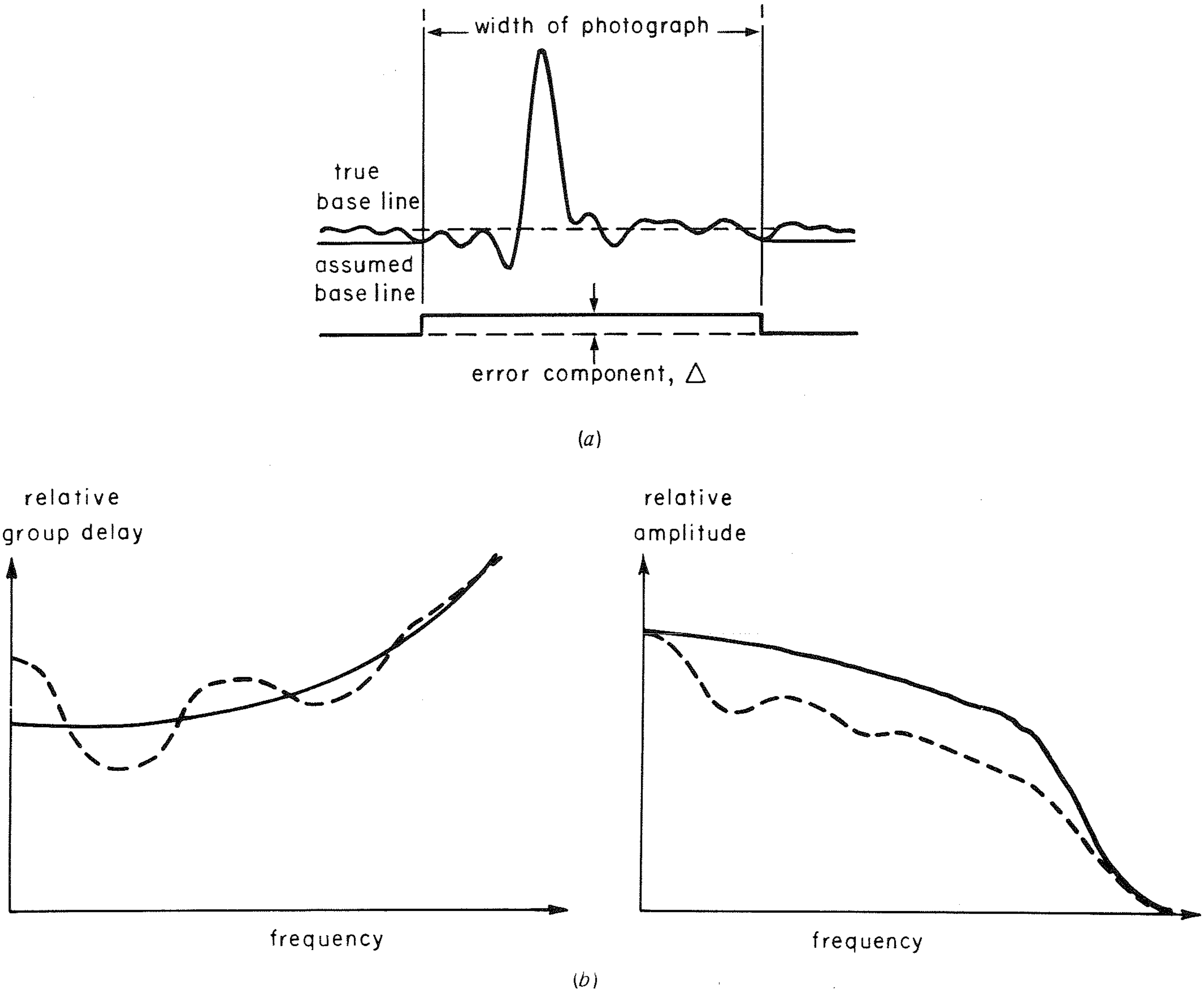

Fig. 2

(*a*) Base line error caused by taking first point as reference (*b*) Effect of offset on computed amplitude and group delay

———— true – – – – computed with offset

form ring extending to this edge of the photograph the Y value of the point may be above or below the true base line of the pulse. If the first point is below the true base line by an amount Δ then the pulse will be distorted by the addition of an error component of height Δ and width equal to the total width of the photograph as shown in Fig. 2(*a*).

The Fourier analysis will then yield a spectrum which is equivalent to the sum of two spectra; one for the pulse and the other for the error component. For even quite small offsets the effect is readily apparent on the overall spectrum (see Fig. 2(*b*)) when the required result is known. This is because the error component contains a large d.c. content which can be comparable with the true d.c. content of the pulse.

Fortunately it is possible to correct base line errors in the computation, before Fourier analysis is performed, using the pulse-to-bar ratio of the photographed waveform to normalise its d.c. content (see Appendix).

4. Analysis procedure

4.1. Input data

Referring to Fig. 3 the first two digitised points (A and B) for which data is read into the computer denote the rotation of the time axis of the photograph with respect to the X axis of the digitising table. These points are digitised widely spaced by using a ruled line to extend them well beyond each edge of the photograph. This minimises the effect on the measured angle of quantising errors, or operator error in the digitising of the points. The data from these points is then used to apply a rotational correction to all the points which follow. The correction procedure can cope with rotational errors up to the point where the digitised X values do not always increase for increasing time. In practice the angular error is unlikely to be more than ±5°.

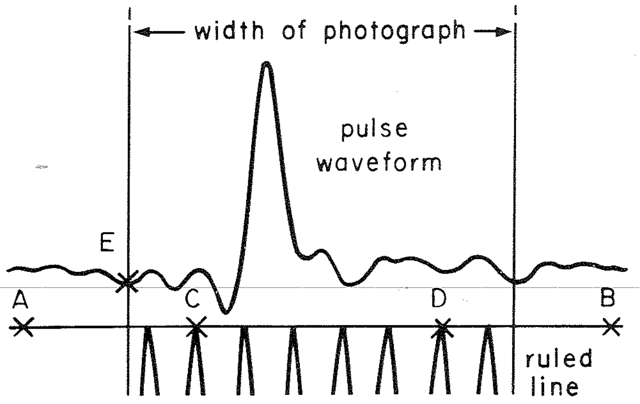

Fig. 3 - Digitising reference points from a photographed waveform

The next two points (C,D) denote the time scale of the photograph. These are points on the time calibration waveform which are spaced by some whole number of cycles. The maximum possible number of cycles is measured consistent with this number being available from all photographs. In practice 8 cycles of the 5 MHz waveform are measured using the pulse tips as reference points: the interval measured then corresponds to 1·6 μs. Measuring an even number of cycles also removes the effect of a small amount of sub-harmonic which is sometimes present on the timing waveform.

Typically 100 points are digitised to describe the pulse waveform. These points are taken sufficiently close together, paying particular attention to the bends in the waveform curve, such that a linear interpolation procedure will be able to determine accurate values for ordinates spaced at 10 ns intervals. The Y value of the first point is used as the initial base line of the pulse before base line offset correction is applied.

4.2. Initial analysis

4.2.1. Rotation correction

From the first two points, (A and B in Fig. 3) which define the angle of the time axis, the correction procedure for all other co-ordinates is derived.

4.2.2. Origin adjustment

The co-ordinates of the first waveform point (E in Fig. 3) are used as the origin values for the time and amplitude scales. The first waveform point is assumed to be at zero amplitude zero time, and the X and Y values of this first point are deducted from the X and Y values of all succeeding points. The co-ordinates of these points are then corrected for rotation.

4.2.3. Time scaling

The X value of each ordinate is then converted from the arbitrary scale produced by the combination of digitiser and photograph parameters to a true time scale where the X value is numerically equal to the time in microseconds from the origin (left hand edge of photograph).

4.2.4. Amplitude scale

The amplitude scale is also arbitrary but it is not necessary to re-scale this.

4.2.5. Peak pulse magnitude

The peak magnitude of the pulse (Y_{max}) is determined for later use in the offset correction routine.

4.2.6. Time interpolation

Interpolation is then carried out to produce waveform ordinate values at fixed 10 ns time intervals; linear interpolation is used. The number of ordinates produced, depends upon the time range contained by the photograph, and is typically about 200.

4.2.7. Evaluation of base line offset

The Y value of all these ordinates is summed and used, together with the number of ordinates, the apparent maximum height of the digitised pulse, and the pulse to bar ratio, to calculate the base line offset. This calculation is described in the Appendix. The pulse to bar ratio is derived from a different photograph and entered in numerical form before the digitised data. The other data required for this calculation concerning the half amplitude width of the generated 1T pulse is a constant which is entered as part of the computer programme.

4.2.8. Base line correction

The calculated base line offset is then subtracted from the Y values of all the waveform ordinates.

4.2.9. Fourier analysis

The time scaled, base line corrected, pulse waveform is then assumed to exist for all time having a magnitude of zero at points where ordinates are not available.

A Fourier analysis is performed on this isolated pulse. With ordinates at 10 ns time intervals the analysis could be performed up to 50 MHz but since the system bandwidth is known to be less than 10 MHz, analysis beyond this frequency is not performed.

4.2.10. Waveform smoothing

Omission of higher frequency components effectively band limits the pulse to 10 MHz and has the effect of smoothing random errors created by the digitising process. An inverse Fourier analysis is then performed to create a smoothed pulse waveform.

4.2.11. Use of resulting data

Data for the amplitude and phase response and the smoothed time response obtained by inverse Fourier analysis are stored for subsequent use with data from other input waveforms. The group delay response is derived from the phase response and the amplitude group delay and time responses can be drawn by a computer plotter if required.

4.3. Combined responses

When data from two or more pulse responses are available their amplitude spectra may be multiplied or divided and the phase responses added or subtracted to obtain the response of combined networks (multiplying), or to isolate the response of a component network (dividing). The resulting amplitude and phase responses are inverse Fourier analysed to produce a time response. The group delay response can be derived from the phase response. The amplitude, group delay and time responses may be drawn out on a plotter.

4.3.1. Response to a 'CEEFAX pulse'

A procedure of multiplication and division of spectra as outlined above can be used to derive the 1T pulse response of a network from its response to an isolated CEEFAX data pulse. Here, dividing the 1T pulse response by the pulse response of the generated 1T pulse gives the impulse response of the network (convolution division of the time responses). Then this impulse response is multiplied by the time response of a single CEEFAX data pulse as it would have been generated for insertion into the network (convolution of the time responses).

4.3.2. Division by zero

When performing a convolution division, care must be taken to ensure that the spectrum forming the divisor has no frequency components of zero amplitude. This may be ensured by adding a small level of uniform amplitude spectrum (which is equivalent to random noise) on the rare occasions when problems are expected.

4.3.3. Filtering

The spectrum resulting from convoltuion division may have spurious strong components due to inaccuracies in the level of the low amplitude components of the divisor spectrum, these strong resulting components being at the frequency of the low amplitude divisor components. Normally such components are above 5·5 MHz, which is the bandwidth of interest, and they may be removed by 'filtering' the resulting spectrum.

The filter function is always used on the resulting spectrum when division has been performed. The 'filter' response used for System B rolls off from 5 MHz and is extinct at 7 MHz. Failure to use the filter will cause excessive ringing on the output time response.

4.4. CEEFAX eye pattern prediction

4.4.1. A CEEFAX eye pattern

A CEEFAX eye-pattern is useful in assessing how readily a CEEFAX waveform can be decoded to retrieve the binary information as a series of '1's or '0's. It is formed by subdividing the waveform into sections equal in length to one data bit duration and centred in the centre of each bit cell. The time axes of these sections of the waveform are then superimposed to give a pattern whose shape resembles that of an eye.

The space in the middle of an eye pattern gives a measure of the immunity of the signal to additional noise at different sampling instants, without the signals being misinterpreted in the decoding process and thus introducing errors. This space is bounded by the waveforms giving the lowest voltage '1's and the highest voltage '0's. In practice it is more important to define the eye boundary than the distribution of the waveforms as the latter is critically dependent on the information being transmitted.

The vertical extremities of the eye pattern are also useful in determining the immunity of the signal to other impairments such as multipath propagation.

In the computer procedure which is to be described, only the boundary of the space in the centre of the eye and the vertical extremities of the eye pattern are evaluated.

4.4.2. Initial procedure

A CEEFAX eye pattern of a linear system can be predicted from its response to a single CEEFAX pulse. Such a response can either be measured directly or derived from a 1T pulse response as detailed in Section 4.3.1. above.

In order to provide a well resolved eye pattern it is necessary to interpolate between the waveform samples formed during previously described analysis procedures. The time response data was stored at 10 ns intervals (Section 4.1). By Fourier analysing the pulse represented by this data, filtering and then applying the inverse Fourier analysis, a new set of data is formed which represents the waveform at 2 ns intervals.

The amplitude values are then rescaled (normalised) so that if CEEFAX pulses, as represented by the data, were superimposed at successive time displacements of one CEEFAX clock period (144 ns) the resulting steady state magnitude would be unity.

4.4.3. Evaluation of limits of eye

It is possible to represent the magnitude of a pulse at its own sampling instant and its magnitude at preceeding and succeeding sampling instants (i.e. its intersymbol components) by means of a diagram (Fig. 4).

The main pulse magnitude and the magnitudes of the intersymbol components will be different for different phasings of the sampling instant relative to the centre of the pulse.

The main pulse and intersymbol components can be derived for various sampling phases and used to evaluate four parameters:

1. the lowest '0' value	maximum negative overshoot
2. the highest '0' value	bottom of eye
3. the lowest '1' value	top of eye
4. the highest '1' value	maximum positive overshoot

These are obtained respectively from algebraic summations of the ordinates at intervals of one bit duration as follows:

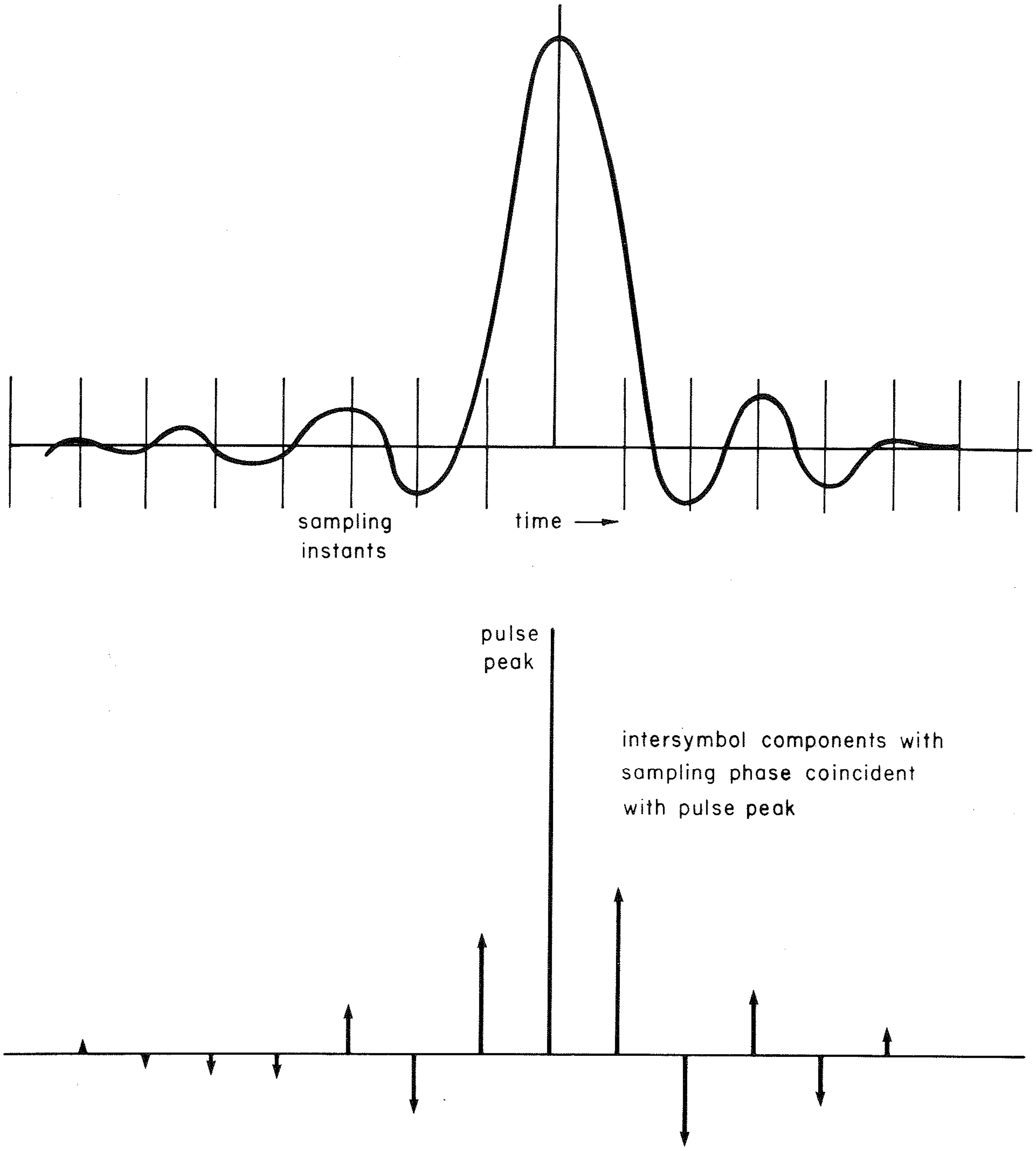

Fig. 4 - Intersymbol components of a pulse

1. all negative intersymbol components
2. all positive intersymbol components
3. the main pulse component and all negative intersymbol components
4. the main pulse component and all positive intersymbol components

The computer program evaluates intersymbol components by taking values of the CEEFAX data pulse spaced at 144 ns intervals i.e. 72 intervals apart for a pulse represented at 2 ns intervals. It then determines which of these components is a maximum positive value and uses this as the main pulse component. The four parameters above are then evaluated. This process is repeated for all possible sampling phases within a CEEFAX clock period (72 spaced at 2 ns intervals) and, for each phase, the four parameters are plotted to form an eye pattern as shown in Fig. 5. The eye pattern is arranged to start and finish with a sample phase virtually midway between the centre of the CEEFAX pulses by dictating that sampling phase relationship, represented by the centre of the diagram, is the one which gives the largest magnitude of main pulse component.

4.4.4. Code sequence for worst-case eye-height

It is also possible to determine the code sequence which gives the worst-case eye-height for each sampling phase by inspection of the sign of the intersymbol components.

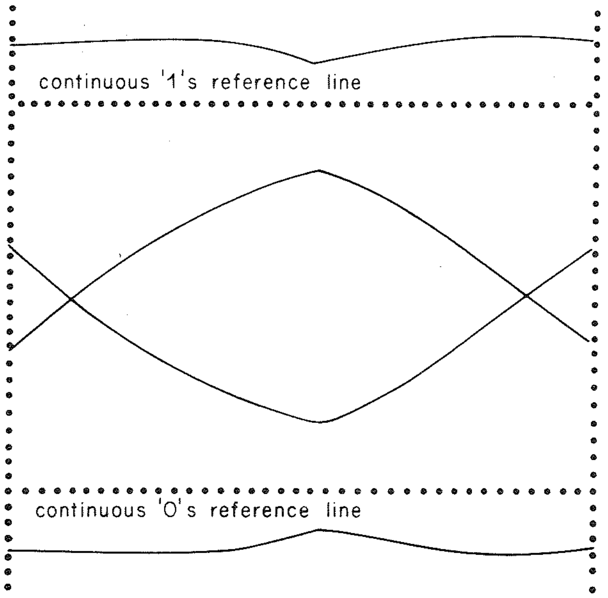

Fig. 5 - A typical computed eye pattern

Positive intersymbol components can degrade the eye height by raising the level of the bottom of the eye, and negative intersymbol components can degrade the eye height by reducing the level of the top of the eye. At the same time positive intersymbol components could enhance the top of an eye and negative intersymbol components could enhance the bottom of an eye.

When evaluating the code which gives the worst case bottom of eye the code required will have a '0' in the main pulse position and '1's in only those positions which produce positive intersymbol components coincident with the main pulse position. In other words the code is arranged so that only intersymbol components which reduce the eye height will occur.

Similarly the code which gives the worst case top of the eye has a '1' in the main pulse position and '1's in only those positions where they give rise to negative intersymbol components coincident with the main pulse position.

The worst case codes for the top and bottom of the eye are complementary.

5. Observed accuracy of analysis method

5.1. General

Tests were made to discover how accurate and consistent the results of the analysis as described above can be expected to be. The effects of non-linearities were also analysed.

5.2. Effects of errors in digitising the pulse waveform

By digitising the same waveforms three times, twice by one engineer and once by another engineer, the effects of errors in digitising have been assessed.

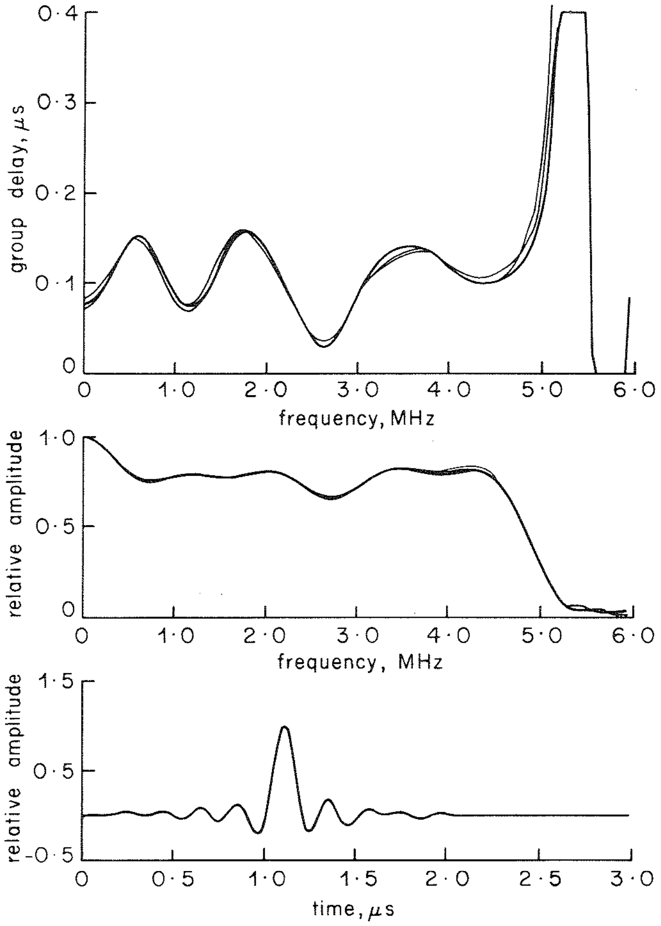

Fig. 6 - Results of three digitisations of one pulse waveform

Fig. 6 shows the results of the three computer analyses superimposed. From this it can be seen that inconsistencies in amplitude are about 5% of the low frequency components, and inconsistencies in group delay are about 15 ns up to 5 MHz. The measured eye heights and peak-to-peak data amplitude in the three cases are 48·5%, 48%, 48·5% and 120%, 121%, 121%. Direct measurements made from the photograph of eye height taken from the eye display are eye height 53%, peak-to-peak amplitude 120%.

5.3. Effects of errors in measuring the pulse-to-bar ratio (1T)

From some ten measurements of pulse-to-bar ratio taken from the photographs errors of about 3 to 4% in the measurements have been discovered. To test the effects of such errors the same data was applied to the computer with pulse-to-bar ratios of 67·5% and 70%.

The results are shown in Fig. 7 and give a difference in the computed amplitude response of about 4% of the amplitude at all except the very low frequencies. This is to be expected as, in the computer program, the pulse-to-bar ratio is used to normalise the relationship between the low frequency component and the higher frequency components. Some small differences in computed group delay response, less than 5 ns, occurred at low frequencies.

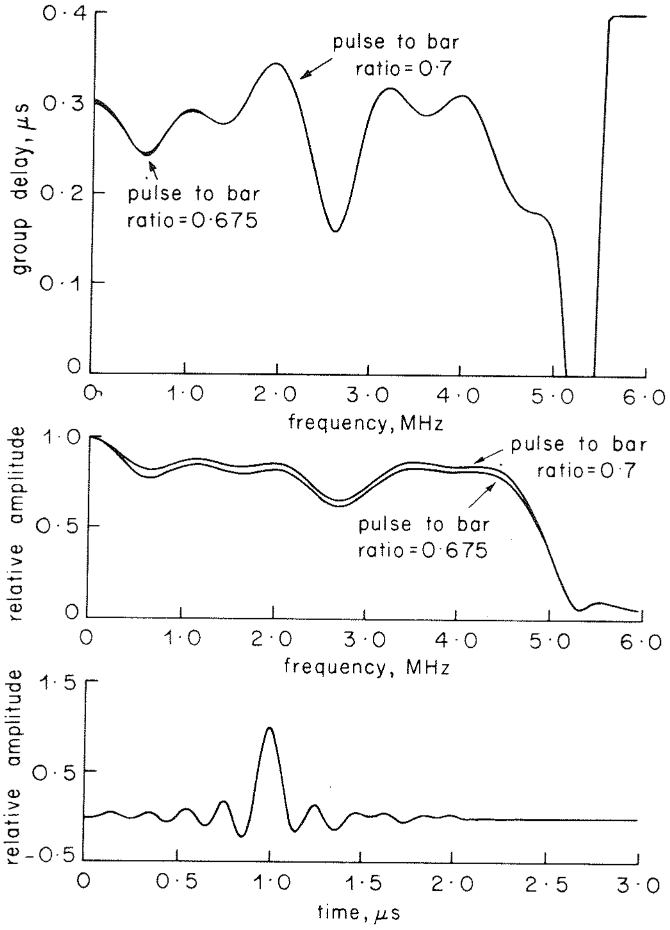

Fig. 7 - Effects of pulse-to-bar ratio on computed results

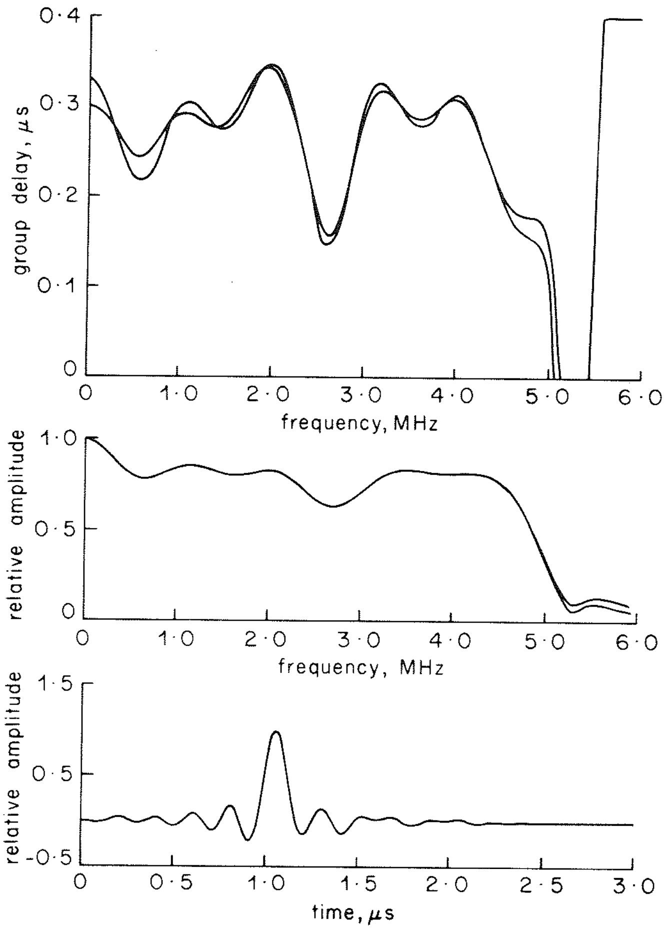

Fig. 8 - Effects of base line tilt on computed results

5.4. Effects of errors in digitising the horizontal reference line

To test the effects of inaccuracies in digitising the horizontal reference line a tilt of two plotter points across the width of the photograph was introduced. The effects of this error are shown in Fig. 8. The only difference this caused in the amplitude response was at high frequencies. Considerable inconsistency was caused in the group delay response.

5.5. Effects of errors in digitising the timing reference

Errors in digitising the timing reference will cause a distortion of the time scale, and hence of the inferred pulse area. This is equivalent to a proportional change in the pulse-to-bar ratio. Such errors will be small because eight cycles of the timing reference are measured. These cover a distance of typically 80 mm of the photograph and are digitised to an accuracy of 254 μm (i.e. 0·01 inch). Hence these errors are insignificant.

5.6. Accuracy of combined responses

Overall, the errors caused by inaccuracies have been found to be typically 7% amplitude and about 30 ns group delay when the spectrum of one digitised pulse is computed. Where processes of successive multiplication or division are used, such as when computing the responses of systems and subsystems, the accuracy is reduced at each operation. The results of one such process are reasonably accurate but after three such processes, the results are not usable.

5.7. The effects of non-linearities

A severe problem in the analysis of the German test results has been the effects of quadrature distortions. All the domestic receivers available for the tests in Germany introduced some degree of quadrature distortion. These non linearities result in distortions of the 1T pulse waveforms which are different for a positive and negative pulse.

Figs. 9(*a*) and 9(*b*) show the positive and negative responses for a domestic receiver with an envelope detector. The two responses are so different that it would be impossible to compute accurately data shapes at the output of such a receiver using this computing technique.

5.8. Consistency between directly measured eye-heights and computer predictions

Eye-height (defined as the voltage difference, in the centre of the eye pattern, between the lowest '1' and the highest '0') has been used as a single measurement for comparing eye patterns.

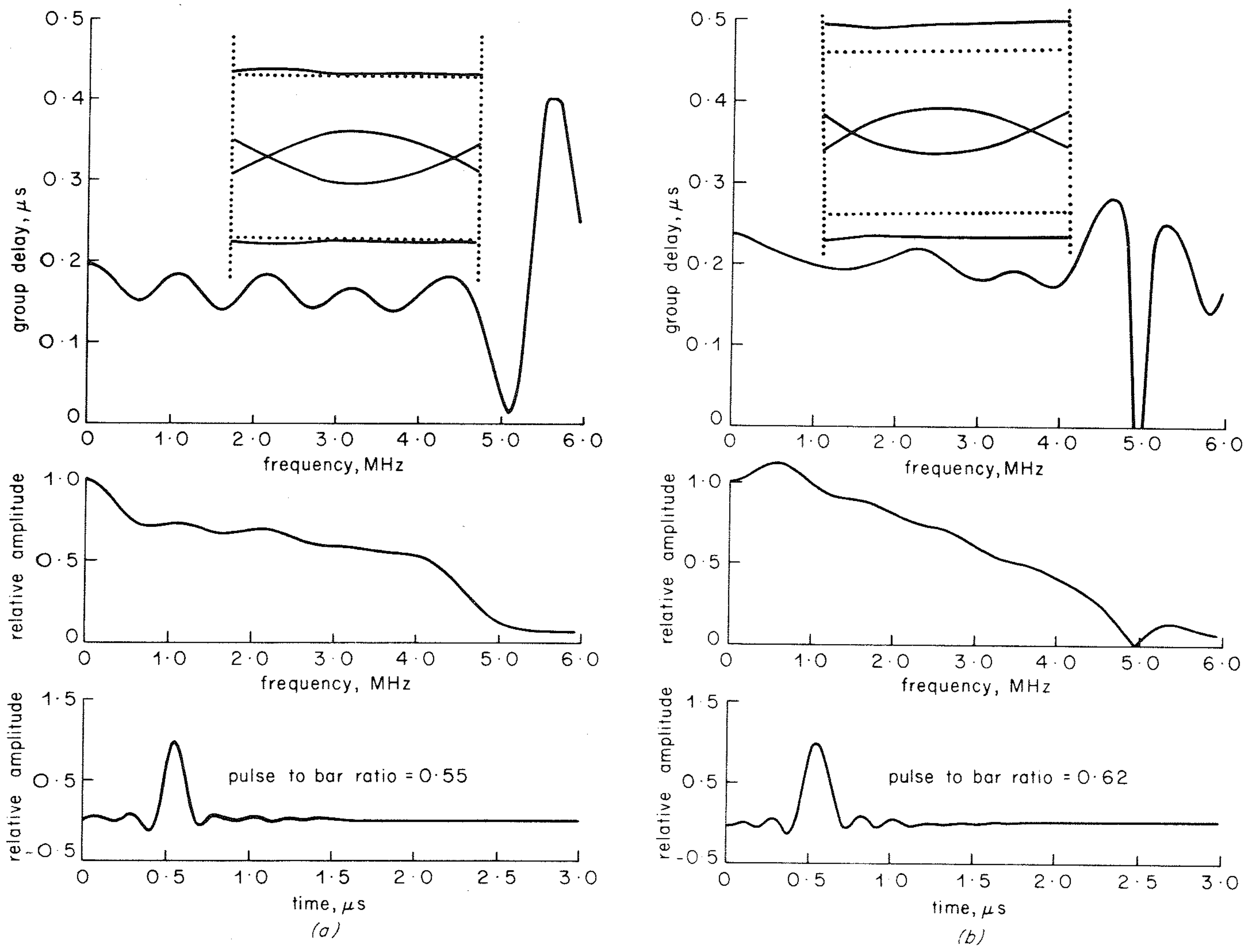

Fig. 9 - Results for a system with quadrature distortion

(*a*) positive pulse (*b*) negative pulse

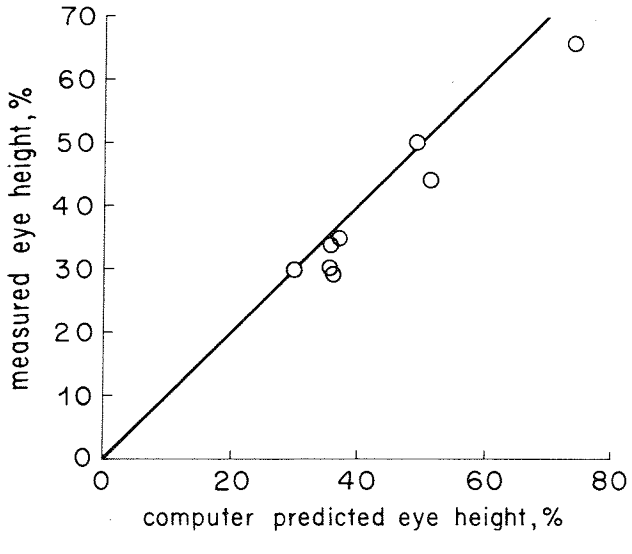

Fig. 10 - Correlation between practical measurements and computer predicted eye-heights

Fig. 10 gives eight results where the eye-height has been computed and measured from a photograph of a special eye-display on an oscilloscope.[2] This figure shows good agreement between the predicted and measured values of eye-height, the mean difference between the two being about 5% eye-height.

6. Conclusion

Provided that there are not significant non-linearities in the system being analysed the results of an individual analysis will be accurate to within 0·5 dB (5%) and 15 ns for amplitude and group delay respectively. Computed eye-height will be within about 5% of the measured eye-height.

The inaccuracies result from difficulty in interpreting the information on the photograph. If the sources of error had been realised before the photographs were taken it might have been possible to reduce the errors from some causes such as base line tilt. To make this type of analysis fully effective a direct digital recording of the waveforms would be needed in place of the photographs.

Where two or more multiplications or divisions are used to give a result for a condition which was not measured, the combined effect of the errors from analysis makes the result of little practical value.

The system is not suitable for non-linearities of the type caused by envelope or quasi-synchronous detection.

7. References

1. CEEFAX: field trials at VHF System B – Bavaria April, 1975 – BBC Research Department report in course of preparation.
2. CEEFAX: measurement techniques. BBC Research Department Report in course of preparation.

Appendix

Base Line Offset Correction

The assumption, in defining the correct position of the base line of the pulse, is that the d.c. component of the pulse is incorrect in a linear system when related to the d.c. component of another part of the test-line, e.g. the bar. It is also assumed that the d.c. component can be measured directly in the area between the pulse waveform and its true base line (any overshoots of the pulse below the base line will enclose an area which is subtracted from that above the base line).

Referring to Fig. 11, the pulse is defined in a series of n + 1 ordinates ($y_o \rightarrow y_r \rightarrow y_n$) at intervals δt over a range in time of n.δt, obtained by interpolation between the digitised points. The ordinate describing the peak of the pulse (in general obtained directly, without interpolation), is y_{max} and the difference between the true base line and the assumed base line is Δ.

Then the true pulse area:

$$P = \sum_{r=o}^{n} y_r \delta t - \Delta.n.\delta t \qquad (1)$$

And the true pulse height:

$$h_p = y_{max} - \Delta \qquad (2)$$

Now the measured pulse-to-bar ratio of the waveform is defined as

$$R = \frac{h_p}{h_b} \quad \text{where } h_b \text{ is the bar height.}$$

Substituting for h_p in (2) gives

$$h_b = \frac{y_{max} - \Delta}{R} \qquad (3)$$

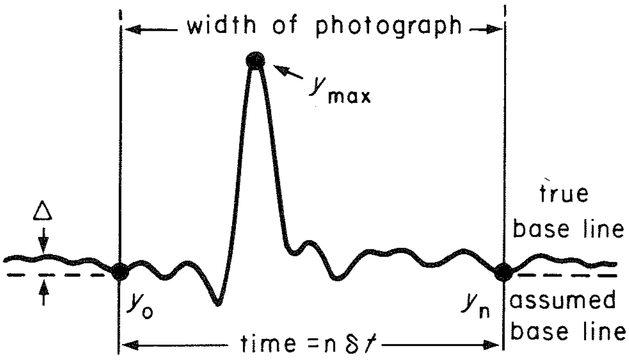

Fig. 11 - Calculating the position of the true base line of a pulse waveform

The Area P′ under the 1T pulse at source for a perfect sin² pulse of half amplitude deviation T and height equal to the bar height

$$P' = h_b T$$

is equal to P, the area under the pulse being analysed.

$$\therefore \qquad P' = h_b T = P = \sum_{r=o}^{n} y_r.\delta t - \Delta.n.\delta t$$

Substituting h_b from (3) and solving for Δ gives:

$$\Delta = \frac{R \sum_{r=o}^{n} y_r . \delta t - y_{max} T}{R.n.\delta t - T}$$

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(PH-157)