Some aspects of the use of metal-halide discharge lamps for film lighting

SOME ASPECTS OF THE USE OF METAL-HALIDE DISCHARGE LAMPS FOR FILM LIGHTING

Summary

The cyclic variations of film exposure, which can occur when discharge lamps run from an alternating supply are used for film lighting, are examined in general terms. Supply-frequency limits (of the order of 50 Hz) within which the fluctuations of displayed picture luminance resulting from the film exposure variations are imperceptible, are derived for a particular metal-halide discharge lamp. The effect of camera shutter angle on these limits is described. Approximations arising from the method used to derive the limits are also discussed. Some relationships, based on the assumption that the variation in lamp intensity is sinusoidal in character, are derived in the Appendix.
# SOME ASPECTS OF THE USE OF METAL-HALIDE DISCHARGE LAMPS FOR FILM LIGHTING

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List of Symbols

$A_n$ Area under $n$ cycles of ripple waveform
$A_q$ Area under fraction $q$ of a cycle of ripple waveform
$A_{q_{\text{max}}} A_{q_{\text{min}}}$ Maximum and minimum values of $A_q$
$C$ Number of film frames per cycle of exposure variation
$E_{\text{max}} E_{\text{min}} E_{\text{mean}}$ Maximum, minimum and mean film exposure values.
$E_{t_1}^{t_2}$ Film exposure between times $t_1$ and $t_2$
$f_c$ Camera frame frequency
$f_{L_{\text{fl}}}$ Picture luminance fluctuation frequency
$f_{L_{\text{fr}}}$ Luminance fluctuation frequency due to $f_s$ ripple component
$f_p$ Replay frame frequency
$f_r$ Lamp ripple frequency
$f_s$ Lamp supply frequency
$f_{s_{\text{min}}}$ Minimum ‘safe’ lamp supply frequency
$f_{s_{\text{d}}} f_{s_{\text{ed}}}$ Derived data relating to $f_s$ and $f_c$ values
$g_E$ Fractional part of $M_E$
$I_v$ ‘a.c. component’ of lamp ripple waveform
$I_{\text{max}} I_{\text{min}}$ Maximum and minimum light intensity
$I_t$ Light intensity at time $t$
$K$ Shutter angle in units of 180°
$l$ Constant
$M_E$ Ratio of maximum and minimum film exposure values
$M_{E_{\text{max}}}$ Maximum value of $M_E$
$m$ Whole number of ripple cycles in camera period
$m'$ Whole number of cycles of $f_s$ component of ripple in camera period
$n$ Whole number of ripple cycles in exposure interval
$n^*$ Value of $n$ for which $M_{E_{\text{max}}} \leq$ defined value
$n_{\text{min}}$ Lowest possible value of $n^*$
$p$ Lamp ripple ratio
$q$ Fractional part of ripple cycle in exposure interval
$q'$ Fractional part of ripple cycle in camera period
$R_E$ Exposure fluctuation ratio
$S$ Constant
$t$ General time variable
$t_{1} t_{2}$ Particular instants of time
$t_c$ Camera frame period
$t_e$ Exposure interval
$t_L$ Picture luminance fluctuation period
$t_p$ Replay frame period
$t_r$ Lamp ripple waveform period
$t_s$ Lamp supply period
$t_{so} t_{r0} t_{so}$ Originally-available data relating to $t_o, t_r$ and $t_s$ values
$t_{ed} t_{ed}$ Derived data relating to $t_c$ and $t_o$ values
$\alpha$ General angular variable
$\theta$ Camera shutter angle, expressed in degrees
1. Introduction

There is a growing interest in the use of metal-halide discharge lamps as light sources for film work. The light from such a lamp varies very considerably in intensity at a frequency of twice that of its supply (e.g., at 100 Hz when using conventional 50 Hz 'mains'). Under some conditions this 'ripple' component can result in a cyclic frame-to-frame variation in film exposure. When the film is projected or scanned in a television system the exposure variations will give rise to corresponding cyclic variations of overall picture luminance. The possible frequency of these luminance variations ranges from zero (i.e., no exposure variation), through low values when many frames are involved in one exposure variation cycle, up to a maximum value equal to half the replay frame frequency, when successive film frames have received alternately higher and lower exposures.

It is important to ensure that any variations in picture luminance caused by the use of metal-halide lamps remain imperceptible, since considerable impairment of the displayed picture can otherwise occur. Previous work has been directed towards defining a limit to the magnitude of the variation in film exposure, such that the resulting fluctuations in picture luminance remain imperceptible, without at the same time imposing an unnecessarily severe restriction on the amount of exposure variation. It has in fact been found (see Section 3.2) that this limiting value depends markedly on the frequency of the luminance fluctuations. The magnitude of the variation in film exposure depends on the relationship between the time-function or 'waveform' of the variation of light intensity, the frequency of the power supply to the lamp and the camera frame frequency and shutter angle. The frequency of the picture luminance fluctuations can be derived from a knowledge of the lamp supply, camera frame and replay frame frequencies. The derivation of these relationships is discussed in general terms in the following section of this report. Knowing these relationships, the previously-derived limits of exposure variation can be used to determine the conditions under which metal-halide lamps may be used for film work without the appearance of luminance fluctuation effects in the displayed picture. Supply-frequency tolerances for a particular metal-halide lamp, which would be readily obtainable from a nominal 50 Hz generator, are derived in Section 3 of this report for the case in which the camera frame frequency and the replay frame frequency each adopt their conventional practical values of 25 Hz.

2. Theory

2.1. Light intensity and film exposure parameters

An example of the variation of the light intensity from a metal-halide lamp with time is shown in Fig. 1. Although the direction of current through the lamp reverses

$$I_\nu = I_{\text{max}} - I_{\text{min}}$$

for each cycle of this ripple waveform, successive cycles of the waveform are nevertheless very similar. The effect of any dissimilarity between successive ripple cycles is discussed in Section 2.3. The light intensity varies between maximum and minimum values ($I_{\text{max}}$ and $I_{\text{min}}$ respectively), and thus may be considered (ignoring this dissimilarity) as consisting of a constant component of magnitude $I_{\text{min}}$ to which is added a 'ripple' component of peak magnitude $I_\nu$, where

$$p = I_{\text{min}}/I_{\text{max}}$$

The ripple component has a period of $t_r$ and, a corresponding frequency of $f_r$, while the alternating lamp supply has a period of $t_e$ and frequency $f_e$ ($f_e = 1/t_e$ etc.). It is apparent that

$$n = \frac{1}{d q_f}$$

![Fig. 1 - Illustration of the variation in light intensity from a metal-halide discharge lamp](image)

![Fig. 2 - Relation between exposure interval and variation in light intensity](image)
\[ t_e = 2 \cdot t_r \]  

(3)

or

\[ f_s = 2 \cdot f_r \]  

(4)

The cine camera has a frame period of \( t_c \) and exposure interval of \( t_e \) (Fig. 2). The relationship between \( t_e \) and \( t_c \) is determined by the shutter angle \( \theta \), such that

\[ t_e = \frac{\theta}{360} \cdot t_c \]  

(5)

where \( \theta \) is expressed in degrees.

Furthermore, the exposure interval may be expressed in terms of the sum of a whole number of ripple waveform cycles (denoted by the integer \( n \)) and a fractional part of such a cycle (denoted by the quantity \( q \), where \( q < 1 \)): hence

\[ t_e = t_r \cdot (n + q) \]  

(6)

This relationship is also shown in Fig. 2.

2.2. The relationship between exposure interval and film exposure variation

Film exposure is proportional only to the total energy received by the film, assuming constancy of the spectral content of the light reaching the film and that the film is used under conditions in which reciprocity failure is not significant. Thus in Fig. 2 or Fig. 3(a) the exposure is proportional to the area under the ripple waveform curve bounded by the starting and finishing times of the exposure interval. It is convenient to regard this exposure as occurring in two stages: first for a whole number of ripple cycles (the 'integer area' shown by dark shading in Fig. 3(a)) and then for a fraction of a ripple cycle (the 'fractional area' shown by light shading in Fig. 3(a)). Since the value of the integer area is independent of the phase during the cycle at which the boundaries of the area occur, these boundaries can be considered as occurring at the same phase of the ripple cycle during successive exposure intervals. Thus in Fig. 3(b) the boundaries of this area are arbitrarily shown as coinciding with minima of the ripple waveform. However, the phase of the ripple cycle at which the fractional area is considered as starting (Fig. 3(b)) must be the same as the phase at which this portion of the exposure interval actually starts, as shown by the vertical alignment of the two fractional areas in Figs. 3(a) and 3(b). This is equivalent to saying that the start of the fractional area in Fig. 3(b) must occur at the same phase of the ripple waveform as the start of the complete exposure interval in Fig. 3(a).

Because the relationship between the ripple period \( (t_r) \) and the camera frame period \( (t_c) \) is in general non-integral, the start of successive exposure intervals will occur at different phases of the ripple cycle (see Fig. 2): thus the corresponding starts of the fractional areas in Fig. 3(b) will also occur at these different phases. The value of the fractional area will therefore change during successive exposure intervals as different parts of the ripple waveform are sampled. The value of film exposure as a function of time therefore consists of a relatively large constant component, determined by the integer area \( A_n \) in Fig. 4) upon which is superimposed a fluctuating component determined by the successive values of the fractional area. The maximum and minimum exposure values \( (E_{\text{max}} \text{ and } E_{\text{min}}) \) occur when the fractional area coincides respectively with the maximum and minimum excursions of the ripple waveform \( (A_{q_{\text{max}}} \text{ and } A_{q_{\text{min}}}) \text{ in Fig. 4.} \) The ratio of these values \( \left( M_E \right) \) is therefore given by

\[ M_E = \frac{E_{\text{max}}}{E_{\text{min}}} = \frac{A_n + A_{q_{\text{max}}}}{A_n + A_{q_{\text{min}}}} \]  

(7)

Equation (7) shows qualitatively how the value of \( M_E \) depends on the values of \( n \) and \( q \). Since successive

Fig. 3 · Integer and fractional exposure areas  
(a) Practical situation (see Fig. 2)  
(b) Equivalent representation

Fig. 4 · Equivalent representations of exposure interval for  
(a) maximum and (b) minimum exposure

(PH-153)
cycles of the ripple waveform are identical, the value of the integer area \( (A_n) \) is directly proportional to \( n \). Thus an increase in \( n \) will, for a given value of \( q \), give rise to a larger constant component of exposure value in both the numerator and denominator of Equation (6), and thus reduce the value of \( M_E \). If the value of \( q \) is zero, then the value of \( M_E \) will be unity and no fluctuation of exposure will occur. As \( q \) increases from zero, two effects occur. For small values of \( q \) (i.e. much less than 0.5) the ratio between the values of the fractional areas \( A_{q_{\text{max}}} \) and \( A_{q_{\text{min}}} \) is at a maximum, but as the fractional areas are relatively narrow they represent only a small part of the total film exposure. Thus the value of \( M_E \) (Equation (7)) does not depart greatly from unity. As \( q \) increases further, the ratio between the values of \( A_{q_{\text{max}}} \) and \( A_{q_{\text{min}}} \) decreases, but the fractional areas become broader and represent an increasing part of the total film exposure. At first the broadening of the fractional areas is the predominant effect, and the value of \( M_E \) increases with increase of \( q \). For values of \( q \) somewhat less than 0.5 the value of \( M_E \) reaches a maximum, and for still greater values of \( q \) the lessening value of the ratio between \( A_{q_{\text{max}}} \) and \( A_{q_{\text{min}}} \) predominates and the value of \( M_E \) decreases again with further increase in \( q \), becoming unity when \( q = 1 \) (which is the same as saying that the integer area occupies \( n + 1 \) ripple cycles and that the value of \( q \) is zero). The actual values of \( q \) for which \( M_E \) is at a maximum depend on the value of \( n \) and also on the form of the lamp ripple component. Detailed analysis (see Appendix) shows that for large values of \( n \) the value of \( q \) for maximum \( M_E \) asymptotes to 0.5.

If there is no integer-area component of film exposure (i.e. \( t_n/t_r < 1 \) in Equation (6)) the value of \( M_E \) will equal the ratio between the values of the fractional areas \( A_{q_{\text{max}}} \) and \( A_{q_{\text{min}}} \), and approaches the value \( 1/p \) (Equation (2)) for low values of \( q \). This behaviour is not important when conventional camera frame frequencies are involved, but must be taken into account when considering slow-motion filming when the camera frame frequency is higher than normal (see Section 2.3).

The relationship between the value of \( M_E \) and the value of \( n \) and \( q \) is illustrated by the full line in Fig. 5. Integral values of the ratio \( t_q/t_r \) occur when \( q \) is zero (Equation (6)) and at these points the value of \( M_E \) is unity. The maxima in the relationship decrease in value (as shown by the dotted line in Fig. 5) as the value of \( n \) increases. In general terms it can be seen that for low values of \( n \) the value of \( M_E \) depends strongly on the value of \( q \); in fact, as will be discussed later in this report, the value of \( M_E \) is acceptably low only if \( q \) is either small compared with unity or differs from unity by a small amount. As the value of \( n \) increases the value of \( M_E \) depends less on the value of \( q \), and for a certain value of \( n \) a point is reached where the value of \( M_E \) is acceptably low for all values of \( q \). Under this condition of operation no account need be taken of the frequency of the lamp supply in order to avoid luminance fluctuations on the displayed picture. This aspect is discussed in some detail in the Appendix (see Fig. 14).

2.3. The frequency of the picture luminance fluctuations

The frequency of the fluctuations in luminance of the displayed picture depends on the lamp supply frequency, the camera frame frequency and the replay frame frequency.
The method of derivation of this relationship is similar to that used in the previous section of this Report for deriving the magnitude of the exposure fluctuations. The camera period \( t_c \) can be considered as consisting of an ‘integer period’ occupying a whole number \( (m) \) ripple cycles and a ‘fractional period’ occupying a fraction \( (q') \) of a ripple cycle. Considering first the case where the camera period is somewhat longer than a whole number of ripple cycles, so that

\[
t_c = t_r (m + q'), \quad q' < 0.5
\]  

(8a)

it is evident that each successive camera period starts a fraction \( q' \) of a ripple cycle later than its immediate predecessor. Thus the number of camera cycles \( (C) \) which elapse between successive coincidences of a particular point in the camera period with the same phase of the ripple waveform is given by

\[
C = \frac{1}{q'}
\]  

(9)

The value of \( C \) gives the number of film frames over which a cycle of exposure variation occurs. A conceptual difficulty arises over the meaning of a non-integral value of \( C \), such as in general will be the case. This difficulty can be resolved by regarding \( C \) as a measure of the length of film, in units of frame height,* along which one cycle of exposure variation occurs. The individual film frames then represent samples of the cycle of exposure variation, as described in Section 2.2, and a fractional value of \( C \) represents interpolation between these samples.

The period \( t_L \) of the fluctuation in luminance on the displayed picture is the time taken to replay the portion of film of length \( C \) film frame units. Thus

\[
t_L = C t_p
\]  

(10)

where \( t_p \) is the replay frame period.

Substituting Equation (9) and then Equation (8a) into Equation (10) gives

\[
t_L = t_p \frac{t_r}{t_c - mt_r}
\]  

(11)

Substituting corresponding frequencies \( (f_p, f_c \) and \( f_D \) respectively) for ripple, camera frame and replay frame periods into Equation (11) gives

\[
t_L = f_c \frac{f_c}{f_p} \frac{1}{f_r - mf_c}
\]  

(12)

while expressing Equation (12) in terms of the luminance fluctuation frequency \( f_L \) and replacing the ripple frequency by the lamp supply frequency \( f_s \) (Equation (4)) gives

\[
f_L = \frac{f_p}{f_c} (2f_s - mf_c)
\]  

(13a)

If the camera period is somewhat shorter than a whole number of ripple cycles, so that

\[
t_c = t_r (m - q') \quad q' < 0.5
\]  

(8b)

it can be seen, following the above argument, that

\[
f_L = \frac{f_p}{f_c} (mf_c - 2f_s)
\]  

(13b)

Since the sign of the frequency value is not significant in this context, Equations (13a) and (13b) can be combined into the form

\[
f_L = \frac{f_p}{f_c} \left( \frac{2f_s - mf_c}{2f_s - mf_c} \right)
\]  

(14)

where the value of the integer \( m \) is chosen to give the lowest possible value of \( f_L \). In choosing \( m \) in this manner, the significance of its value as indicating, to the nearest whole number, the number of ripple cycles in the camera period (see Equations (8a) and (8b)) should be remembered.

The lowest number of film frames over which a cycle of exposure variation can occur is two, when alternate frames are differently exposed. This corresponds to the conditions when \( q' = 0.5 \) in Equations (8a) or (8b). In this case the value of luminance fluctuation frequency is half the replay frame frequency; i.e.

\[
f_L = \frac{f_p}{2}
\]  

(15)

Under these conditions there are effectively two samples of exposure variation per complete cycle. Thus the magnitude of the exposure variations as recorded on the film depends on the effective phase of this sampling, and can range from zero to a maximum value. Furthermore, if the value of \( q' \) differs slightly from 0.5, the sampling phase will change slowly with time and the exposure variation magnitude will similarly change. In these circumstances the relation between picture luminance and time is not described precisely by the value of luminance fluctuation frequency. Under the conditions discussed in Section 3, however, at least four film frames are involved in one cycle of picture luminance fluctuation, and effects due to the sampling effect of discrete film frames are ignored.

If the camera frame and replay frame frequencies are the same, Equation (14) reduces to

\[
f_L = \left| \frac{2f_s - mf_c}{2f_s - mf_c} \right|
\]  

(14a)

Furthermore, since the System I television transmission system employs a field frequency of 50 Hz, it is operational practice to adopt a value of 25 Hz for both these fre-
f \_L = \left| \frac{f _s - 25m}{f _c} \right| (14b)

where \( f \_L \) and \( f _s \) are expressed in Hz.

Equation (14) must however be used when considering the use of slow-motion effects (\( f _c > f _p \)) or accelerated-motion effects (\( f _c < f _p \)).

It is worth noting the effect of lack of identity of successive cycles of the light ripple waveform, due to dependence of the light output on the direction of current through the lamp. In these circumstances a component at the supply frequency (\( f _s \)) is introduced into the ripple waveform. This ripple component will give rise to an additional component of picture luminance fluctuation, of frequency \( f \_L \). Following through the derivation of Equation (14) but considering the ripple waveform component of frequency \( f _s \), leads to the relationship

\[
f \_L = \left| \frac{f _p}{f _c} \right| f _s - m \frac{f _c}{2} \tag{16a}
\]

The integer \( m \) is the number of complete cycles of the \( f _s \) component of the ripple waveform in the camera period. The relationship between this additional component of picture luminance fluctuation and the principal component (\( f \_L \)) may be examined by considering the case when \( f \_L = 0 \). In this case Equation (14) gives

\[
f _s = \frac{m}{2} f _c
\]

and substituting this value of \( f _s \) into Equation (16) gives

\[
f \_L = f _p \left| \frac{m}{2} - m \right| \tag{16a}
\]

In interpreting Equation (16a), it must be remembered that since the quantity \( m \) is the number of complete ripple cycles in the camera period, it is also the number of complete half-cycles of the supply-frequency (\( f _s \)) component of the ripple waveform in this period. If, therefore, \( m \) is even, then \( m = m / 2 \) and Equation (16a) shows that \( f \_L = 0 \). In fact, it can be seen from Equations (14) and (16) that if \( f \_L \) is not zero, then \( f \_L = f _p / 2 \). Since the visibility of picture luminance fluctuations increases with frequency (see Fig. 8, Section 3.2), the presence of this extra component is not likely to affect the visibility of the luminance fluctuations as a whole, as determined by the frequency \( f \_L \) derived from Equation (14) or (14a), particularly as the magnitude of this component will be relatively small. * However, if the value of \( m \) is odd, there will be an odd number of half-cycles of the \( f _s \) component during the camera period, and the number of complete cycles during this period is therefore given by \( m = (m - 1) / 2 \). In this case Equation (16a) shows that \( f \_L = f _p / 2 \). Under these conditions alternate film frames will receive different exposures from the \( f _s \) component of the ripple waveform (see Equation (15)).

Taking a more general view of the situation where the value of \( m \) is odd, it can be seen that if the frequency of the picture luminance fluctuations due to the main ripple component (\( f \_L \)), as determined by Equation (14) or (14a), has a low value and the fluctuations consequently have low visibility, then the corresponding luminance fluctuations due to the \( f _s \) component in the ripple waveform will approach the highest possible frequency of \( f _p / 2 \) and may be of greater visibility despite their lower magnitude.

It may be noted that for a lamp supply frequency of around 50 Hz and camera frame and replay frame frequencies of 25 Hz, \( m = 4 \) in Equation (14a) or (14b) and being even indicates, as discussed above, that under these circumstances practical dissimilarities between successive cycles of the ripple waveform may be ignored (see Section 3).

2.4. Discussion

Equation (6) (Section 2.1) may, by substitution from Equations (3) and (5) and remembering the reciprocal relationship between frequency and period, be written as

\[
n + q = \frac{0}{180} \frac{f _s}{f _c} \tag{17}
\]

Turning to Equation (7) (Section 2.2), the relative values of the integer and fractional areas (i.e., the values of the terms in 'A' on the right-hand side of this equation), and thus the magnitude of the variation in film exposure (\( M _E \)), are determined by the values of \( n \) and \( q \) and also by the characteristics of the variation in light intensity from the lamp under consideration (i.e., the waveform of the ripple component and the magnitude of the lamp ripple ratio (\( \rho \), Equation (2i)) Under certain conditions the lamp ripple waveform and ratio may be regarded as invariant:** furthermore, the camera frame frequency may also be regarded in this manner in the context of normal operational practice, since its value is then assumed to be fixed at 25 Hz. In these circumstances the value of \( M _E \) is seen to depend only on the lamp supply frequency (\( f \_L \)) and the camera shutter angle (\( \theta \)).

Examination of Equation (14b) (Section 2.3) shows that the picture luminance fluctuation frequency (\( f \_L \)) also depends, in the context of normal operational practice, only on the value of \( f _p \), if the variation in the frequency of the lamp supply is regarded as being restricted so that the value of \( m \) in Equation (14b) does not change. There is therefore a 'one-to-one' relation between \( M _E \) and \( f \_L \), depending only on the shutter angle \( \theta \).

* This is a valid assumption if the variation in the frequency of the lamp supply \( f _p \) is restricted, and that the power input to the lamp, the waveform of the lamp power supply and the lamp ballasting arrangements are not changed.

---

* Since in practice successive cycles of ripple waveform are similar, even though not necessarily identical.
Consider first the situation where $\theta = 180^\circ$. In this case Equation (17) becomes

$$n + q = \frac{f_s}{f_c} \tag{18}$$

If $q = 0$ (i.e., the exposure interval occupies a whole number of ripple waveform cycles) there will be no exposure variation and $M_E$ will be unity. Equation (18) then gives

$$f_s = nf_c$$

Substituting for $f_c$ in Equation (14a) gives

$$f_L = f_S \left(2 - \frac{m}{n}\right) \tag{19}$$

It can be seen from Equation (19) that $f_L = 0$ when $m = 2n$: thus in this case zero exposure variation and zero luminance fluctuation frequency occur at the same lamp supply frequency, when there are $n$ cycles of ripple frequency in an exposure interval, and $2n$ cycles of ripple frequency in a camera frame period. A small departure from this condition (i.e., $q$ non-zero but small in Equation (18)) will give rise to low values of both exposure variation and luminance fluctuation frequency. Since the visibility of picture luminance fluctuations is relatively low for low values of fluctuation frequency (see Section 3.2) this situation is favourable to the luminance fluctuations remaining imperceptible. On the other hand, if the camera shutter angle is not equal to $180^\circ$, Equation (17) may be written

$$n + q = K \frac{f_s}{f_c} \tag{18a}$$

where the factor $K (\neq 180)$ is non-integral. Substituting for $f_c$ in Equation (14a) when $q = 0$ (and therefore $M_E = 1$) gives

$$f_L = f_s \left(2 - K \frac{m}{n}\right) \tag{19a}$$

If the camera shutter angle does not differ too greatly from $180^\circ$, the relationship $m/n = 2$ will still hold. The presence of the non-integral factor $K$ in Equation (19a) therefore indicates that the lamp supply frequency for which $M_E = 1$ does not correspond with zero luminance fluctuation frequency. There will of course be no resulting fluctuations in picture luminance when this condition is exactly obeyed, but very small departures of the value of $q$ from zero may result in visible fluctuations of picture luminance, since the relatively high value of fluctuation frequency will enhance the visibility of the luminance fluctuations. The situation in which the camera shutter angle differs from $180^\circ$ is therefore unfavourable to the luminance fluctuations remaining imperceptible.

If the camera shutter angle is not equal to $180^\circ$, as described above, but the lamp supply frequency is such that the luminance fluctuation frequency (Equation (14a)) is precisely zero, then the value of $M_E$ will not be zero. Nevertheless, no picture luminance fluctuations should be visible since the fractional area (Fig. 3(b)) will occur at the same phase of the ripple waveform during successive exposure intervals. This situation implies, however, a stable long-term phase relationship between the lamp supply and camera frame frequencies such as could be obtained, for example, by crystal control of the camera frame frequency and operation of the lamp from the public electricity supply. In practice, situations can arise in which this stable relationship is absent, even though the lamp supply frequency is nominally a precise multiple of the camera frame frequency: for example, when the lamp is supplied from a mobile generator whose governor exhibits some 'hunting' around the correct nominal frequency. Under these conditions fluctuations in picture luminance will occur, the time variations of these fluctuations depending on the factors (e.g., the governor characteristics in the above example) which determine the variation in the lamp supply frequency\(^*$ about its mean value. In such circumstances the value of $M_E$ will indicate the greatest possible variations in film exposure that can occur.

If the appropriate value of $m$ in Equation (14a) or (14b) is an odd integer, the number of ripple cycles in the exposure duration will be approximately half-integral i.e., $q \approx 0.5$ in Equation (6). This situation gives rise to a relatively high value of $M_E$ (see Section 2.2 and Appendix). If the luminance fluctuation frequency $f_L$ is precisely zero the situation will be as described in the previous paragraph and fluctuations in picture luminance will occur if the phase relationship between the lamp supply and camera frame frequencies is unstable. If the value of $f_L$ is not zero, fluctuations of picture luminance will occur as previously discussed. In both these cases the luminance fluctuations may be perceptible even for low values of $f_L$ because of the large value of $M_E$ which is obtained under these conditions. Because the maxima in the value of $M_E$ are relatively broad compared with the cusps of minimum value, the precise value of the shutter angle will, for low values of $m$, have little effect on the value of $M_E$: thus this situation is unfavourable to the luminance fluctuations remaining imperceptible irrespective of the precise value of shutter angle.** For large odd values of $m$, however, the departure of the shutter angle from $180^\circ$ may be great enough to include a precise whole number of ripple cycles in the exposure interval i.e., $q = 0$ in Equation (6) and $M_E = 1$ from Equation (7). These conditions are met with when running the lamp from supplies having frequencies much greater than 50 Hz. A similar situation occurs, even for low values of $m$, if the shutter angle departs greatly from $180^\circ$. For instance, an argument can be developed for a shutter angle of $120^\circ$ showing that if $m = 3$ in Equation (14a) or (14b), then $n = 1$ and $q = 0$ in Equa-\(^*$ or the camera frame frequency, if this is unstable with time.

** This situation exists when the lamp supply frequency equals 60 Hz and the camera frame frequency equals 24 Hz. In this case $m = 5$ in Equation (14) (or Equation (14a), if the replay frame frequency also equals 24 Hz).
tion (6), so that $M_E = 1$ from Equation (7). As the shutter angles found in practical motion-picture cameras are all of the order of $180^\circ$, however this consideration is of only academic interest and will not be pursued further.

3. Derivation of lamp supply frequency limits

3.1. General

The calculations which follow serve as an example of the application of the theory outlined in Section 2. In these calculations any dissimilarity between successive ripple cycles has been ignored (see Section 2.3). Limits were required to the frequency $^*$ of the supply to a particular luminaire containing a 575-W metal-halide discharge lamp, within which no perceptible luminance fluctuations would occur on pictures originating from film exposed in a particular type of motion-picture camera having a shutter angle $^{**}$ of $172^\circ$. In deriving these limits, both the camera frame frequency and the replay frame frequency were to be assigned values of 25 Hz. The available data on the exposure variations which resulted from the use of this luminaire was in the form or a relation between the value of the ratio of the maximum and minimum exposures ($M_E$, Equation (7)), and the exposure interval $t_e$ (Fig. 2), assuming a constant supply frequency of 50 Hz. Transformation of the independent variable from the exposure interval (as used in this ‘original’ data) to the lamp supply frequency (as required for the ‘derived’ data) is therefore required: in other words, the relationship is required between the exposure interval in the original data and the lamp supply frequency in the derived data for which the film exposure variation is equivalent. In the present context the term ‘equivalent’ means that in the two cases under consideration the diagrams (Fig. 4, Section 2.2) from which the maximum and minimum exposure values are obtained are identical, apart from differences in the time or intensity scales on which the diagrams are constructed. Such identity is only obtained if all of the following parameters are the same in the two cases:

(a) The form of the lamp ripple component.
(b) The lamp ripple ratio ($p$ in Equation (2)).
(c) The whole number of ripple waveform cycles occupied by the exposure interval ($n$ in Equation (6)).
(d) The fractional part of the ripple waveform cycle occupied by the exposure interval ($\theta$ in Equation (6)).

From Equation (7) it can be seen that under the above conditions the value of the ratio ($M_E$) of the maximum and minimum exposure values will be the same in the two cases. It may be noted that a variation in the time scale on which Fig. 4 is constructed permits a variation in the period of the lamp ripple component (which is required since the lamp supply frequency is treated as a variable in one of the cases under consideration), but at the same time implies a corresponding variation in the exposure interval such as to preserve (see Equation (6)) the identity of the values of $n$ and $\theta$ between the two cases. It must however be remembered that the absolute values of the integer and fractional parts ($A_n$ and $A_\theta$ in Fig. 4) are directly proportional to the period ($t_e$) of the ripple component if the above conditions are observed. There will thus be a change in the absolute value of exposure with change of ripple component period (or in other words, with lamp supply frequency). It is assumed throughout this discussion that such changes in absolute exposure are not significant: for example, that they have been allowed for by an appropriate change in the aperture of the camera lens.

Let quantities referring to the originally-available data be designated by the subscript ‘o’ ($t_{eo}$, $n_o$, etc) and quantities referring to the derived results be designated by the subscript ‘d’ ($t_{ed}$, $n_d$, etc). The relationship is required between the exposure interval in the original data ($t_{eo}$) and the lamp supply frequency in the derived results ($f_{sd}$) for which the film exposure variation is equivalent. In the case of the original data, Equation (6) may be written

$$t_{eo} = \frac{n_o + q_o}{f_{eo}}$$  \hspace{1cm} (20)

Remembering that the ripple period $t_{eo}$ is half the lamp supply period $t_{so}$ (Equation (3)), Equation (20) can be written as

$$n_o + q_o = \frac{2t_{eo}}{t_{so}}$$  \hspace{1cm} (21)

In the case of the derived data the exposure interval ($t_{ed}$) is given (Equation (5)) by

$$t_{ed} = \frac{\theta}{360} \cdot t_{cd}$$  \hspace{1cm} (22)

Also, from Equations (6) and (4), and remembering the reciprocal relationship between period and frequency,

$$t_{ed} = \frac{1}{2f_{sd}} (n_d + q_d)$$  \hspace{1cm} (23)

From Equations (22) and (23)

$$f_{sd} = \frac{180}{\theta \cdot t_{cd}} (n_d + q_d)$$  \hspace{1cm} (24)

Now equivalent film exposure variation is obtained (see conditions (c) and (d) above) when $n_o = n_d$ and $q_o = q_d$. Thus from Equations (21) and (24)

$$f_{sd} = \frac{360}{\theta \cdot t_{cd} \cdot t_{so}}$$

or in terms of camera and supply frequencies

* Only lamp supply frequencies around 50 Hz are considered in this section.

** Other shutter angles are considered in Section 3.3.
\[ f_{sd} = \frac{360 f_{cd} f_{so}}{\theta} t_{e0} \]  (25)

For the present calculation \( f_{cd} = 25 \text{ Hz}, f_{so} = 50 \text{ Hz} \) and \( \theta = 172^\circ \) (see above): hence Equation (25) becomes

\[ f_{sd} = 2616.3 t_{e0} \]  (26)

where \( f_{sd} \) is expressed in Hz and \( t_{e0} \) in seconds.

In the above derivation of the relationship between the exposure interval in the original data and the lamp supply frequency in the derived data, all the initial conditions for equivalence of exposure variation have been assumed to hold with precision. In fact, condition (b) above is not obeyed exactly, since it is known that the lamp ripple ratio \( \rho \) varies with the lamp supply frequency, and therefore varies to some extent under the conditions relating to the derived data, while remaining constant under the conditions relating to the original data. Over the lamp supply frequency range at present under consideration, however, condition (b) is obeyed with adequate accuracy.

Fig. 6 shows the original data relating relative exposure variation \( |M_E| \) and exposure duration \( t_{e0} \). Fig. 7 shows the corresponding derived data, in which the values of lamp supply frequency \( f_{sd} \) are related by Equation (28) to the exposure duration values in Fig. 6. It should be noted that for a lamp supply frequency of 50 Hz the frequency of the fluctuations of picture luminance is nominally zero (see Equation (14b)) and the interpretation of Fig. 7 is then as discussed in Section 2.4.

3.2. Derivation of lamp supply frequency limits for a shutter angle of 172°

The limits defining the greatest permissible magnitude of exposure variation for the resulting fluctuations of picture luminance on replay to remain imperceptible are shown in Fig. 8. The limits are expressed in terms of the 'exposure fluctuation ratio' \( R_E \), given (in decibels) by

\[ R_E = 20 \log_{10} \frac{E_{\text{mean}}}{E_{\text{max}} - E_{\text{min}}} \]  (27)

where \( E_{\text{max}}, E_{\text{min}} \) and \( E_{\text{mean}} \) are the maximum, minimum and mean film exposure respectively. It should be noted that an increase in the value of \( R_E \) implies a decrease in the relative variation of exposure.

* These limits assume an overall transfer characteristic obeying a power-law relationship having an exponent ('gamma value') of 1.5.
The relationship between the exposure fluctuation ratio \( R_E \) and the ratio \( M_E \) between the maximum and minimum exposure values (Equation (7)) may be derived by writing

\[
M_E = 1 + g_E
\]

Equation (7) may be written

\[
R_E = 1 + \frac{E_{\text{max}} - E_{\text{min}}}{E_{\text{min}}}
\]

Hence

\[
E_{\text{max}} - E_{\text{min}} = g_E \cdot E_{\text{min}}
\]

Furthermore, if the exposure variation relationship is symmetrical, so that

\[
E_{\text{mean}} = \frac{(E_{\text{max}} + E_{\text{min}})/2 + E_{\text{min}}) - E_{\text{mean}} - E_{\text{min}})/2}{2}
\]

it can be seen that

\[
E_{\text{mean}} = \left(1 + \frac{g_E}{2}ight) E_{\text{min}}
\]

Equation (27) therefore becomes

\[
R_E = 20 \log_{10} \left( \frac{2 + g_E}{2g_E} \right)
\]

(28)

For values of \( M_E \) less than 1·1 (i.e., for all values included in Figs. 6 and 7) an error of less than 0·5 dB is introduced by writing

\[
R_E = 20 \log_{10} \frac{1}{g_E} = 20 \log_{10} \frac{1}{M_E - 1}
\]

(28a)

Since all lamp supply frequencies considered in this Report are in the region of 50 Hz, \( m = 4 \) in Equation (14b). Re-arranging Equation (14b) gives the relation between the lamp supply frequency used in the derived data \( f'_{cd} \) and the luminance fluctuation ratio \( f_L \) (both in Hz) as

\[
f_{cd} = \frac{100 \pm f_L}{2}
\]

(29)

Note that, as would be expected from the argument used in deriving Equation (14), there are two possible lamp supply frequencies for a given luminance fluctuation frequency. Fig. 9 shows (full lines) the limit relationship of Fig. 8 plotted using lamp supply frequency values derived from Equation (29). The two branches of this curve correspond to the two possible lamp supply frequencies for a given luminance fluctuation frequency, as noted above. Also shown (dotted line) is the derived data of Fig. 7, replotted in terms of exposure fluctuation ratio values. It can be seen that the calculated exposure fluctuation ratio values lie above the limit relationship curve over a lamp supply frequency range of approximately 52·1 – 52·5 Hz. It must be remembered that these limiting values only apply for the conditions listed at the beginning of this Section: in particular, even a small change of camera shutter angle from the assumed value of 172° will produce a significant change in the limiting values. This aspect is discussed in the following Section. It must also be noted that different types of metal-halide lamps are characterised by different ripple waveforms and lamp ripple ratios, and that these parameters can change significantly with a change of power input to the lamp or of the ballasting arrangements. Furthermore, techniques involving the use of a group of lamps, each run from a different supply phase or run from the same supply but through different phase-shifting ballasting networks, may be used to reduce the overall lamp ripple ratio. The variation of the lamp ripple ratio with supply frequency has already been mentioned above: this has been discounted for the conditions involved in the present calculation, where the lamp supply frequency remains within 3·5 Hz of the nominal value of 50 Hz, but can become significant for larger changes in supply frequency.

### 3.3. Relation between lamp supply frequency limits and shutter angle

Equation (24) can be written

\[
f_{cd} = \frac{180f_{cd}}{q} + \frac{180f_{cd}}{\theta} q
\]

(30)

The first term in the right-hand side of Equation (30) represents the lamp supply frequency for which \( q = 0 \), and therefore for which the frame-to-frame exposure variation is zero (see Section 2·2). The second term determines the shape of the two branches of the curve (Fig. 7 and dotted line in Fig. 9) relating film exposure variation to lamp supply frequency. Both terms are dependent on the shutter-angle value (\( \theta \)). The error involved in ignoring this dependence on shutter angle is about 4·4% if the results...
calculated for a shutter angle of 172° (i.e., as shown in Figs. 7 and 9) are taken as referring to a shutter angle of 180°. In the case of the first of these two terms in Equation (30) this error is by no means negligible as it would produce an error of about 2.3 Hz in the lamp supply frequency for zero film exposure variation. This is greater than five times the supply frequency range for which, using a shutter angle of 172°, picture-luminance fluctuation effects are imperceptible (see Section 3.2). However, in the case of the second term, the effect of ignoring the dependence on shutter angle is only to introduce a corresponding error in the ‘differential’ lamp supply frequency value (i.e., the frequency difference between the lamp supply frequency for zero film exposure variation and the frequency for any given magnitude of exposure variation). For a shutter angle of 172° the value of the differential lamp supply frequency for luminance fluctuation effects to remain imperceptible is about 0.25 Hz (see Section 3.2) and the error involved in ignoring the dependence on shutter angle is therefore only about 0.01 Hz. This error is negligible: thus lamp supply frequency limits for shutter angles other than 172° may be obtained from Fig. 9 by bodily translating the dotted exposure-variation curve along the supply frequency axis, centring it on the supply-frequency value, appropriate to the shutter angle under consideration, corresponding to zero film exposure variation (as deduced from the first term on the right-hand side of Equation (30)). This procedure leads to the supply-frequency limit chart shown in Fig. 10, in which the unshaded region indicates the range of acceptable lamp supply frequencies for any given shutter angle between 170° and 180°. The narrow unshaded area in this figure corresponding to a lamp supply frequency of 50 Hz, for which the resulting luminance fluctuations are nominally of zero frequency, indicates that operation at all shutter-angles may be possible without the occurrence of picture luminance fluctuations. This aspect is discussed in Section 2.4.

3.4. Discussion

In a practical camera, the shutter is not usually in the image plane. The exposure interval is therefore not characterised by an abrupt start and finish, but usually by a transition from zero to full illumination of the film occupying a significant fraction of the camera operating cycle at the start of the exposure interval, and another similar transition back to zero film illumination at the end of this interval. Because of this indeterminacy of the precise exposure duration, which could differ from one part of the camera field-of-view to another, and because of the critical dependence of the supply frequency limits on this duration (see Fig. 10, remembering that shutter angle is a measure of exposure duration), it is possible that the lamp supply-frequency tolerances appropriate to different parts of the film frame might not overlap and therefore that some luminance fluctuation effects might always be expected to occur for these conditions of operation.

It is interesting to examine the effect on the limits of supply frequency of a decrease in the magnitude of the exposure variations. Fig. 11 shows a supply-frequency limit chart derived in the same way as in the case of Fig. 10,

---

* Note that in these calculations \( n = 2 \).

** Some small errors can result from the graphical nature of this procedure.

---

Fig. 10 - Lamp supply frequency limits for a 575-watt metal-halide lamp, as a function of camera shutter angle

Approximate relationship: see text
Camera and replay frame frequencies — 25 Hz

Fig. 11 - Lamp supply frequency limits for a hypothetical metal-halide lamp (see text), as a function of camera shutter angle

Approximate relationship: see text
Camera and replay frame frequencies — 25 Hz
except that the exposure-variation curve (the dotted lines in Fig. 9) has been bodily translated upwards by 6 dB. This represents the case of a hypothetical lamp giving rise to exposure variations of half the magnitude of those produced by the measured metal-halide lamp.* A considerable easing of the supply-frequency tolerances for shutter angles in the region of 174° — 176° is evident as compared with the original results (Fig. 10) obtained from the metal-halide lamp: in addition, operation in the region of 50 Hz appears possible with a tolerance of at least ±0.2 Hz for a shutter angle of 172° (irrespective of the ‘zero frequency’ picture luminance fluctuation effects discussed in Section 2.4), this tolerance further increasing with increase in shutter angle.

4. Conclusions

Because the light from metal-halide discharge lamps varies very considerably in intensity at twice the lamp supply frequency, exposure variations can occur if such lamps are used to provide the scene lighting in motion-picture film work. These exposure variations may be great enough to give rise to fluctuations in the luminance of the replayed picture. Relationships between the characteristics of the light variation from the lamp (the ‘ripple’ characteristics), the lamp supply frequency, the camera frame frequency and shutter angle, and the replay frame frequency have been derived. These relationships may be used, in conjunction with data relating the visibility of such picture luminance fluctuations with their magnitude and frequency, to predict operating conditions which will not give rise to visible picture luminance fluctuations.

As an example, approximate limits to the lamp supply frequency, in the region of 50 Hz, have been derived for camera frame and replay frame frequencies of 25 Hz (i.e. normal operating conditions), for a particular 575-watt metal-halide lamp. These supply-frequency limits are strongly dependent on camera shutter angle, leading to the possibility, if the shutter angle is not well defined, of an inability to eliminate luminance fluctuation effects completely for lamp supply frequencies around 50 Hz. It might be possible to avoid picture luminance fluctuations by using a lamp supply frequency of precisely twice the camera frame frequency (e.g. 50 Hz and 25 Hz respectively), depending on the short-term stability of these frequencies.

The lamp ripple characteristics are dependent on the operating conditions of the lamp, including the power input, ballasting arrangements and supply frequency (although independence of this latter parameter is assumed over the limited frequency range considered in the limit relationships considered in this Report). The ripple characteristics also differ between different types of metal-halide lamp. For these reasons the limits discussed above must be regarded as specific to the particular type of metal-halide lamp under consideration, and to the particular conditions under which it was operated.

If the lamp ripple characteristics are represented by a sinusoidal function, it becomes possible to derive an explicit mathematical formula for the amount of film exposure variation. This procedure considerably simplifies the calculation of exposure variation magnitude, which otherwise must be obtained by a process of numerical integration: it gives results which agree with this latter method of calculation to within about one decibel. However, the derived formula is thought not to apply with any reasonable accuracy in the situation where the lamp is supplied from a non-sinusoidal (e.g. square wave) power source. The derived formula can be used in predicting the minimum ‘safe’ lamp supply frequency for a particular lamp, above which no luminance fluctuation effects will be apparent irrespective of the relationships between the lamp and camera parameters.

5. References


* This situation would also arise if the limits defining the greatest permissible magnitude of exposure variation (Fig. 8) could be relaxed by 6 dB.
Appendix

Representation of the Lamp Ripple Component by a Sinusoidal Function

In general, the form of the lamp ripple component (Fig. 1) cannot readily be expressed by a simple mathematical function, and the calculation of the integral and fractional areas (Fig. 3), necessary for the derivation of the variation of exposure as a function of time, has therefore to be carried out numerically. This, in turn, requires the lamp ripple waveform to be described by a relatively large number of ordinate values. In many cases, however, a reasonable approximation to the time variation of exposure is obtained if the ripple waveform is considered to be sinusoidal. Since

$$\sin^2 \alpha = \frac{1}{2}(1 - \cos 2\alpha)$$

it is possible to represent the variation with time of the lamp intensity, incorporating a sinusoidal ripple component having a frequency twice that of the lamp supply (Fig. 12, cf. Fig. 1), by the relationship

$$I_t = I_{\min} + U_{\max} - I_{\min} \sin^2 \left( \frac{2\pi}{t_r} t \right)$$

where \(I_t\) is the lamp intensity at time \(t\) and all other symbols are as defined in Section 2.1.

$$E \left\{ t_2 \bigg| t_1 \right\} = S \int_{t_1}^{t_2} I_t \, dt$$

(32)

where \(E \left\{ t_2 \bigg| t_1 \right\}\) is the film exposure between the times \(t_1\) and \(t_2\) and \(S\) is a constant which includes film speed, camera aperture etc.

Now

$$\frac{\alpha_2}{\alpha_1} = \int_{\frac{\alpha_1}{\alpha_2}}^{\frac{\alpha_2}{\alpha_1}} \frac{\sin^2 \lambda d\lambda}{\lambda} = \frac{1}{2} \left[ \frac{\alpha_2 - \alpha_1}{\alpha_1} - \frac{1}{2} \left( \sin 2\alpha_2 - \sin 2\alpha_1 \right) \right]$$

(33)

where \(\lambda\) is a constant.

Hence from Equations (31), (32) and (33)

$$E \bigg|_{t_1}^{t_2} = \frac{S}{2} \left[ U_{\max} + I_{\min} \right] \left( t_2 - t_1 \right) - \frac{S}{2 \pi} \left[ U_{\max} - I_{\min} \right] \left( \sin \frac{2\pi}{t_r} t_2 - \sin \frac{2\pi}{t_r} t_1 \right)$$

(34)

Three particular exposure intervals are of importance. In the first place, consider the case when the exposure interval occupies a whole number \(n\) ripple cycles. In this case let \(t_1 = 0\); then \(t_2 = nt_r\). Hence Equation (34) becomes

$$E \bigg|_{0}^{nt_r} = \frac{St_r}{2} \cdot n \left( U_{\max} + I_{\min} \right)$$

(35)

If the exposure interval occupies a fraction \(q\) of a ripple cycle, maximum exposure will occur when the mid-point of the exposure interval occurs at the instant of maximum lamp intensity. In this case \(t_1 = t_r(1 - q)/2\) and \(t_2 = t_r(1 + q)/2\); hence Equation (34) becomes

$$E \left\{ t_r \left( \frac{1+q}{2} \right) \bigg| t_r \left( \frac{1-q}{2} \right) \right\} = \frac{St_r}{2} \left[ qU_{\max} + I_{\min} - \frac{1}{\pi} U_{\max} - I_{\min} \sin q \right]$$

(36)

Similarly, the minimum exposure will occur when the mid-point of the exposure interval occurs at the instant of minimum lamp intensity. Here \(t_1 = t_r(1 - q/2)\) and \(t_2 = t_r(1 + q/2)\); hence Equation (34) becomes

$$E \left\{ t_r \left( \frac{1+q}{2} \right) \bigg| t_r \left( \frac{1-q}{2} \right) \right\} = \frac{St_r}{2} \left[ qU_{\max} + I_{\min} - \frac{1}{\pi} U_{\max} - I_{\min} \sin q \right]$$

(37)

Fig. 12 - Sinusoidal variation in light intensity

Since \(t_s = 2t_r\) (Equation (31)), this expression may be written

$$I_t = I_{\min} + \left( U_{\max} - I_{\min} \right) \sin^2 \left( \frac{\pi}{t_r} t \right)$$

(31)

Film exposure between the instants of time \(t_1\) and \(t_2\) is proportional to the area under the ripple waveform curve (see Section 2.2) and thus is proportional to the integral of Equation (31) between appropriate limits. Thus
In the present context Equation (7) (Section 2.2) may be written

\[ M_E = \frac{E_{\text{max}}}{E_{\text{min}}} = \frac{\frac{n t_r}{2} + E}{\frac{n t_r}{2} + E} \left( \frac{t_r(1+q)/2}{t_r(1-q)/2} \right) \]

Thus from Equation (38) the ratio of the maximum and minimum film exposure values (\( M_E \)) may be determined from the ratio of the maximum and minimum lamp intensity values (the lamp ripple ratio \( p \), Equation (21)) and the relationship between the exposure interval and the period of the lamp ripple component (expressed in terms of an integer \( n \) and a fraction \( q \) as in Equation (6)).

The variation in exposure can also be expressed in terms of the exposure fluctuation ratio (\( R_E \), Equation (27)) and values of \( R_E \) can be derived from corresponding values of \( M_E \) using Equation (28) (and noting the assumption implicit in the derivation of this equation). The error involved in using Equation (38), rather than taking account of the actual form of the lamp ripple component, naturally depends on the degree to which this component departs from the sinusoidal form defined by Equation (31). Fig. 13 shows a practical lamp ripple waveform compared with a sinusoidal waveform having the same value of lamp ripple ratio. In this case the value of the exposure fluctuation ratio, as calculated from Equations (38) and (27), does not differ by more than one decibel from the value calculated from the actual lamp ripple waveform. The direction of error is such as to give a low value of exposure fluctuation ratio when using Equations (38) and (27): an error in this direction implies that the use of these equations will underestimate the amount of exposure variation.

The positions of the maxima in Fig. 5 (Section 2.2) may in principle be found by differentiating Equation (38) and equating to zero. In practice the positions of the maxima do not depend on the value of the lamp ripple ratio \( p \), since as far as film exposure is concerned the variation of intensity of lamp illumination can be regarded as consisting of two components: an alternating component of peak magnitude \( U_{\text{max}} - U_{\text{min}} \) and having \( p = 0 \) (see Fig. 12) and a constant component of magnitude \( U_{\text{min}} \). Only the alternating component is responsible for determining the positions of the maxima in Fig. 5, irrespective of the ‘diluting’ effect of the constant component. Equation (38) may therefore be simplified before differentiation by setting \( p \) equal to zero and thus taking account only of the alternating component described above. In this case

\[ M_E = \frac{n + q + \frac{1}{2} \sin \pi q}{n + q - \frac{1}{2} \sin \pi q} \]

Differentiating Equation (38a) gives

\[ \frac{dM_E}{dq} = \frac{1 + \cos \pi q}{n + q - \frac{1}{2} \sin \pi q} \left( \frac{n + q + \frac{1}{2} \sin \pi q}{n + q - \frac{1}{2} \sin \pi q} \right)^2 \]

Equating this expression to zero gives

\[ (n + q - \frac{1}{2} \sin \pi q)(1 + \cos \pi q) - (n + q + \frac{1}{2} \sin \pi q)(1 - \cos \pi q) = 0 \]

which simplifies to

\[ \tan \pi q = \pi(n + q) \]

Equation (39) may be solved graphically to give the values of \( q \), as \( n = 1, 2, 3 \ldots \), at which the maxima in Fig. 5 occur. Table 1 shows the values of \( q \) obtained for some values of \( n \) between 1 and 100. It can be seen that as \( n \)

![Fig. 13: Comparison of practical and sinusoidal ripple waveforms](image)

**TABLE 1**

<table>
<thead>
<tr>
<th>( n )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>50</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q ) for max ( M_E )</td>
<td>0.430</td>
<td>0.459</td>
<td>0.471</td>
<td>0.477</td>
<td>0.482</td>
<td>0.490</td>
<td>0.495</td>
<td>0.497</td>
<td>0.498</td>
<td>0.499</td>
</tr>
</tbody>
</table>

(PH-153)
increases the value of $q$ tends to 0.5. In fact, the maxima are rather broad and in practice an error of less than 0.5 dB in the value of the maximum exposure fluctuation ratio ($M_{E_{\text{max}}}$) is obtained if the value of $q$ is taken as 0.5 rather than as given in Table 1. This error clearly decreases with increase in the value of $n$. It is therefore possible to derive a general expression for $M_{E_{\text{max}}}$ in terms of $n$ and $p$ by putting $q = 0.5$ in Equation (38). This procedure leads to the expression

$$M_{E_{\text{max}}} = \frac{(1 + p)(n + 1/2) + \frac{1}{\pi}(1 - p)}{(1 + p)(n + 1/2) - \frac{1}{\pi}(1 - p)} \quad (38b)$$

This equation may be written as

$$n = \frac{\frac{1}{\pi}(1 - p)(M_{E_{\text{max}}} + 1) - \frac{1}{2}(1 + p)(M_{E_{\text{max}}} - 1)}{(1 + p)(M_{E_{\text{max}}} - 1)}$$

$$= \frac{(1 - p)(M_{E_{\text{max}}} + 1)}{\pi(1 + p)(M_{E_{\text{max}}} - 1)} - \frac{1}{2} \quad (40)$$

Since $n$ can only adopt integer values, this relationship is not valid for all values of $M_{E_{\text{max}}}$. However, since (see Fig. 5) $M_{E_{\text{max}}}$ decreases with increase of $n$, Equation (40) may be expressed as the inequality

$$n^* > \frac{(1 - p)(M_{E_{\text{max}}} + 1)}{\pi(1 + p)(M_{E_{\text{max}}} - 1)} - \frac{1}{2} \quad (41)$$

In Equation (41) $n^*$ is an integer, being a value of $n$ for which the corresponding value of $M_{E_{\text{max}}}$ is less than (or equal to) the value referred to in the right-hand side of the equation. Thus if the right-hand side of Equation (41) is evaluated for particular values of $p$ and $M_{E_{\text{max}}}$, the next highest integer to this calculated value gives the lowest value of $n$ (i.e., the lowest number of complete ripple cycles in the exposure interval) which is required for the value specified for $M_{E_{\text{max}}}$ not to be exceeded for any value of $q$. Referring to Fig. 8 (Section 3.2), it can be seen that if the exposure fluctuation ratio always exceeds 48 dB (or, from Equation (28), if the ratio of the maximum and minimum exposure values is never greater than 1004), then no fluctuations of displayed picture luminance will be visible, irrespective of the frequency of the luminance fluctuations. By letting $M_{E_{\text{max}}} = 1004$ in Equation (41), the lowest number $(n_{\text{min}})$ of complete ripple cycles in the exposure interval, for which picture luminance fluctuations will always be imperceptible, may be determined knowing the lamp ripple ratio $p$. Under these conditions $(M_{E_{\text{max}}} + 1) = 2$ to a close approximation, Equation (41) becomes

$$n^* > 159.2 \cdot \frac{1 - p}{1 + p} - \frac{1}{2} \quad (41a)$$

![Graph](image.png)

**Fig. 14 - Lamp ripple ratio relationships giving imperceptible picture luminance fluctuations (see text)**
The value of \( n_{\text{min}} \), as discussed above, is therefore obtained by taking the next highest integer to the value of the right-hand side of Equation (41a). Fig. 14 shows the value of the right-hand side of Equation (41a) as a function of \( p \), from which the value of the integer \( n_{\text{min}} \) may be deduced.

The relation between exposure interval and ripple period for imperceptibility of picture luminance fluctuations under all operating conditions may be expressed, using symbols defined in Section 2.1, as

\[
t_e = n_{\text{min}} \cdot t_r
\]

Substituting from Equations (3) and (5) and re-arranging the expression in terms of frequencies rather than periods leads to the relationship

\[
f_{\text{smin}} = \frac{180}{\theta} \cdot f_c \cdot n_{\text{min}}
\]  
(42)

Equation (42) indicates the minimum 'safe' lamp supply frequency \( f_{\text{smin}} \) that must be used, in conjunction with a camera of shutter angle \( \theta \) degrees and frame frequency \( f_c \), in order that picture luminance fluctuations remain imperceptible under all operating conditions. Since the value of \( n_{\text{min}} \) depends on the lamp ripple ratio \( p \), the minimum 'safe' lamp supply frequency is also dependent on this parameter. Because this latter relationship is of immediate practical interest, the right-hand ordinate scale in Fig. 14 has been added to enable a determination of \( f_{\text{smin}} \) to be made directly from a knowledge of the lamp ripple ratio \( p \), without necessarily first deducing the value of \( n_{\text{min}} \). Inspection of Equation (42) shows that the calibration of this ordinate scale depends on both camera shutter angle and camera frame frequency. The scale in Fig. 14 assumes a camera frame frequency of 25 Hz, but provision has been made for including camera shutter angles in the range 170° to 190°. Before this scale can be used for a determination of \( f_{\text{smin}} \), it must be calibrated to take into account the particular shutter angle of the camera under consideration. This is achieved by drawing a vertical line through the shutter-angle value shown at the top of the scale. The points of intersection of the inclined frequency-value lines (these are labelled at each end with the frequency they indicate) with this vertical line provide the ordinate-value scale of \( f_{\text{smin}} \) for this particular shutter-angle, against which the value of the minimum 'safe' lamp supply frequency for any value of lamp ripple ratio may be read off, using the curve in the main portion of the figure. For example, given a camera shutter angle of 175°, a lamp ripple ratio of 0.385 (giving a value for \( n_{\text{min}} \) of 70) is seen to imply a minimum 'safe' lamp supply frequency of 1.8 kHz.

In using the relationship between \( p \) and \( f_{\text{smin}} \), account must be taken of the fact that the value of \( p \) is itself a function of lamp supply frequency (an increase in lamp supply frequency causes a decrease in \( p \)). The assumptions involved in the derivation of Fig. 14 (particularly that the lamp ripple waveform is sinusoidal in character) must also be remembered. It is likely that this assumption will be valid for all practical cases in which a sinusoidal lamp supply source is used together with an inductive ballast circuit, but it is by no means certain to hold for non-sinusoidal (e.g. square-wave) lamp excitation, or when resistive or capacitative ballasts are used. It may however be noted that the lamp ripple waveform tends to become more nearly sinusoidal in character as the lamp supply frequency is increased.