UDC 621.391.812
621.391.827.4

BBC

1969
THE QUEEN'S AWARD TO INDUSTRY

RESEARCH DEPARTMENT

REPORT

L.F. AND M.F. PROPAGATION: a study of ionospheric cross-modulation measurements

No. 1972/23

Research Department, Engineering Division
THE BRITISH BROADCASTING CORPORATION

RESEARCH DEPARTMENT

L.F. AND M.F. PROPAGATION: A STUDY OF IONOSPHERIC CROSS-MODULATION MEASUREMENTS

Research Department Report No. **1972/23**
UDC 621.391.812
621.391.827.4

It uses SI units in accordance with B.S. document PD 5686.

P. Knight, M.A., M.I.E.E.

P. Rainger
Head of Research Department

(RA-102)

Research Department Report No. **1972/23**

L.F. AND M.F. PROPAGATION: A STUDY OF IONOSPHERIC CROSS-MODULATION MEASUREMENTS

July 1972

Research Department Report No. **1972/23**
UDC 621.391.812
621.391.827.4

L.F. AND M.F. PROPAGATION: A STUDY OF IONOSPHERIC CROSS-MODULATION MEASUREMENTS

Summary

Measurements of ionospheric cross-modulation found in the literature have been standardised to a common disturbing-transmitter power and subsequently compared. It is shown that disturbing transmitters radiating in the medium-frequency broadcasting band can cause as much cross-modulation as those operating in the low-frequency band, because of the influence of the Earth's magnetic field. A semi-empirical formula for the maximum cross-modulation which is likely to be observed is proposed.

1. Introduction

When the region of the ionosphere traversed by a sky-wave broadcast-signal is strongly illuminated by a high-power disturbing transmitter, the audio-frequency modulation of the latter may be superimposed on the carrier of the former and cause interference. This cross-modulation, also known as the Luxembourg effect, is caused by a non-linear process in the ionosphere. The mechanism has been described in an earlier report[1] and elsewhere.[2] Briefly, the disturbing transmitter varies the collision frequency* of the ionosphere in step with its modulation and this, in turn, varies the attenuation suffered by the traversing wave, leading to cross-modulation.

Ionospheric cross-modulation is mainly confined to the low-frequency (l.f.) and medium-frequency (m.f.) broadcasting bands, and the high-power transmitters which are in common use today may cause serious interference to sky-wave broadcasting services. Because of the large numbers of transmitters in these bands, ionospheric cross-modulation is difficult to distinguish from co-channel interference and even more difficult to measure. Numerous measurements have, however, been made in the past when these bands were less congested and a preliminary analysis of these measurements has already been presented.[3] A more detailed comparison of these measurements is contained in this report and a semi-empirical formula for the calculation of maximum cross-modulation levels is proposed.

2. Standardisation of ionospheric cross-modulation measurements

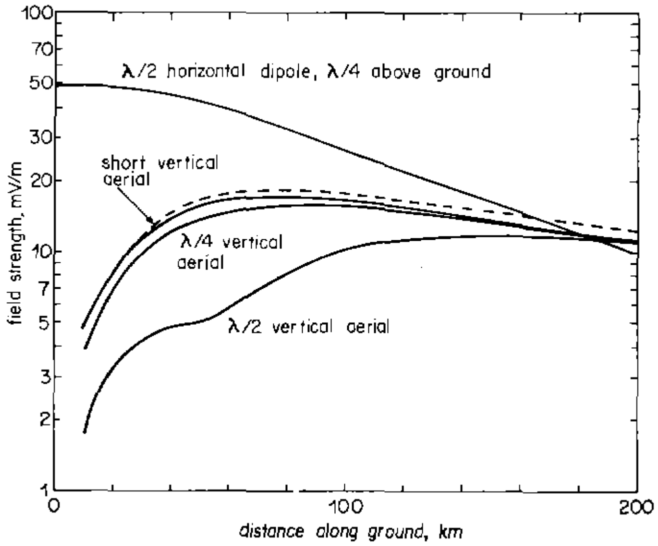

Fig. 1 - Field strength at base of ionosphere when 100 kW is radiated from various types of aerial

– – – – – 0·2 MHz (L.F.) ———— 1·0 MHz (M.F.)

Ground conductivity in vicinity of aerials, 10^{-2} S/m

The measurements of ionospheric cross-modulation listed in the next section were made with a variety of disturbing-transmitter powers and modulation characteristics, and different types of transmitting aerial were used. Before they can be compared they must be standardised to a reference transmitter and aerial, using methods described in this section.

The level of cross-modulation depends mainly on the strength of the disturbing wave at the base of the ionosphere, and this is determined both by the transmitter power and by the type of aerial employed. Fig. 1 shows the field strength at the base of the ionosphere when 100 kW is radiated from different aerials; the height of the base of the ionosphere is assumed to be 80 km and the aerials to be situated on ground of good conductivity (10^{-2} S/m).

With aerials up to a quarter wavelength (λ/4) high the greatest field strength at the base of the ionosphere is produced by radiation which leaves the aerial at 45° to the vertical, and therefore occurs at a horizontal distance of

---

* Average rate at which free electrons collide with molecules.

about 80 km from the transmitter; the maximum is, however, very broad, as may be seen from Fig. 1. Aerials short compared with λ/4 are always used at l.f., but at m.f. aerials 0·5λ high are frequently used to suppress high-angle radiation, and these produce a lower field strength at the ionosphere, as Fig. 1 shows. On the other hand horizontal aerials, which are sometimes used at m.f., may give rise to higher field strengths at the base of the ionosphere. For example, Fig. 1 shows that the field strength directly above a horizontal dipole λ/4 above ground is nearly three times as great as the maximum field strength due to a short vertical aerial radiating the same power. The increase is partly due to the shorter distance to the ionosphere and partly to the higher power gain of the horizontal aerial. A small contribution (0·5 dB) also comes from the more efficient reflection of horizontally-polarised waves in the ground below the aerial.

It is reasonable to assume that cross-modulation caused by radiation from any particular type of aerial depends on the maximum power density at the base of the ionosphere. In standardising to a reference aerial radiating a stated power, therefore, maximum power densities at the ionosphere rather than actual transmitter powers must be compared. For example, 12·5 kW radiated from the horizontal aerial described above may be assumed to have the same maximum effect as 100 kW radiated from a short vertical aerial.

2.1. Variation of cross-modulation with power density

The maximum power density at the base of the ionosphere when 100 kW is radiated from a short vertical aerial is a convenient reference. Cross-modulation is proportional to the power density of the disturbing transmission, and to its modulation depth, provided the cross-modulation is less than 10%. If either the measured or the standardised cross-modulation exceeds this value, a more precise relationship between power and cross-modulation, derived in this section, must be used instead.

It has been shown[1] that the increase in the attenuation of the wanted wave, on a typical path, is proportional to the instantaneous power of the disturbing transmitter. A more detailed theoretical study[4] has shown that this relationship is to be expected for all disturbing waves at l.f. and for ordinary[†] disturbing waves at m.f. It may not apply, however, to extraordinary[†] disturbing waves near the gyromagnetic frequency[*] unless they are relatively weak.

When the linear relationship between the instantaneous power and the attenuation increase applies the attenuation of the wanted wave may be written in the form

$$A = A_0 + kP \qquad \text{(dB)} \tag{1}$$

where A_0 is the attenuation of the wanted wave in the absence of the disturbing wave, P is the instantaneous power of the disturbing wave and k is a constant. When the disturbing transmitter is sinusoidally modulated, P varies between $P_u(1 - M)^2$ and $P_u(1 + M)^2$, where M is the modulation coefficient and P_u is the power radiated by the disturbing transmitter when unmodulated. Substitution of these values of P in Equation (1) shows that A then varies by $4kMP_u$ dB over the modulation cycle. The ratio of the maximum and minimum field strengths of the wanted wave (E_{max} and E_{min} respectively) is therefore given by

$$20 \log_{10} \frac{E_{max}}{E_{min}} = 4kMP_u \tag{2}$$

The modulation impressed on the wanted wave is approximately sinusoidal and the transferred modulation coefficient T is given by

$$T = \frac{E_{max} - E_{min}}{E_{max} + E_{min}} \tag{3}$$

From Equations (2) and (3) it may be shown that

$$\begin{aligned} T &= \tanh\,(0{\cdot}2303\,kMP_u) \\ &= \tanh\,(qMP_u) \end{aligned} \tag{4}$$

where q is a further constant.

Fig. 2, which is a hyperbolic tangent curve, shows the relationship between T and the product MP_u in arbitrary units. Fig. 2 may be used to estimate values of T for specified values of M and P_u from measurements made with other values. It shows that T is proportional to M and P_u provided it is less than 0·1 (10% cross-modulation). The linear relationship does not apply to larger values of T, because T cannot exceed 1·0 (100% cross-modulation).

2.2. Dependence of transferred modulation on modulation frequency

The change in the attenuation of the wanted wave does not occur instantly when the power of the disturbing

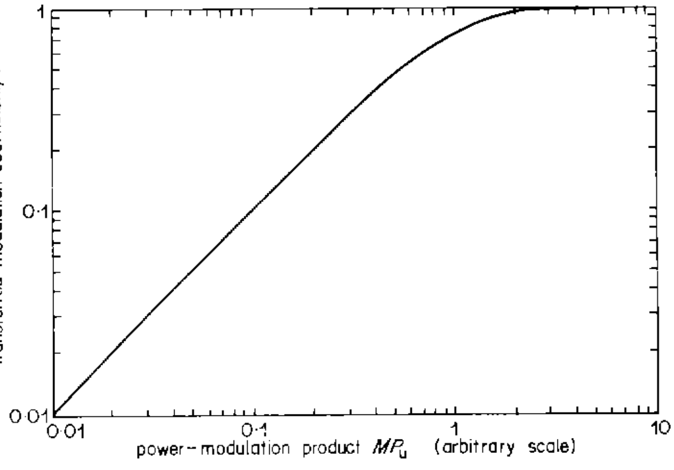

Fig. 2 - Variation of cross-modulation with power and modulation depth of disturbing transmitter

---

[†] The terms refer to the two modes of wave propagation in the ionosphere viz the ordinary wave and the extraordinary wave.

[*] In Europe the gyromagnetic frequency is about 1·25 MHz.

transmitter varies, but is subject to a time constant which is of the order of 1 ms. Consequently the transferred modulation level decreases as the modulation frequency of the disturbing transmitter increases. It has been shown[2] that the transferred modulation coefficient T is related to its limiting value T_0 at very low modulation frequencies by the expression

$$T = T_0 \left[1 + \left(\frac{\omega_m}{G\bar{\nu}}\right)^2\right]^{-\frac{1}{2}} \tag{5}$$

where ω_m is the angular modulation frequency, G is a constant and $\bar{\nu}$ is the average electron-collision frequency at the height where cross-modulation occurs.

Measurements of the time constant have shown that $G\bar{\nu}$ varies between 750 and 3000 sec^{-1}, the value depending on the height at which cross-modulation occurs. Fig. 3 shows the variation of T/T_0 for three values of $G\bar{\nu}$ within this range; the value of 1500 may be regarded as typical. Fig. 3 may be used to estimate values of T for specified modulation frequencies from measurements made with other frequencies.

3. Comparison of ionospheric cross-modulation measurements

Published measurements of ionospheric cross-modulation, made with a variety of disturbing transmissions, are listed in Table 1. Whenever possible Table 1 gives median values for measurement periods of several minutes or longer. Higher values observed for shorter periods with fading signals have been disregarded.

The methods described in Section 2 have been used to estimate the cross-modulation level, T_{300}, which would have been observed had the disturbing transmitters radiated from short vertical aerials with a power of 100 kW and been modulated at 300 Hz to a depth of 80%. This relatively low power was adopted as a reference because many of the measurements were made with powers of this order; its use ensured that large extrapolations from measured values were seldom required.

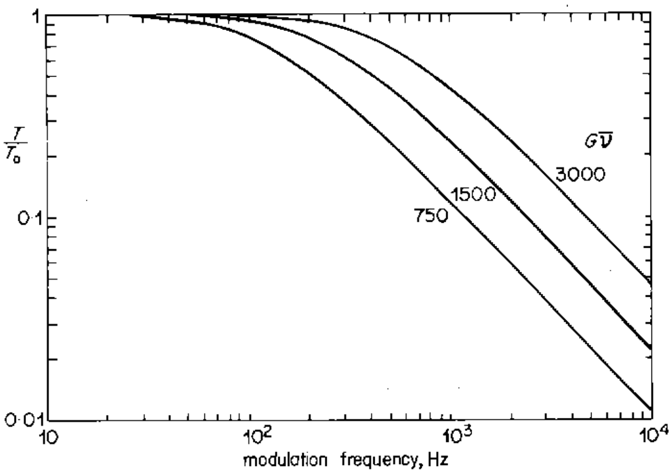

Fig. 3 - Variation of cross-modulation with modulation frequency

The choice of 300 Hz for the reference modulation frequency was made because it was used for many of the experimental transmissions. In standardising measurements made with other modulation frequencies or reported in terms of the limiting cross-modulation T_0 applicable to very low frequencies, $G\bar{\nu}$ was assumed to be 1500 sec^{-1} unless a measured value was quoted.

TABLE 1

Measured cross-modulation

Date	Location and gyromagnetic frequency, MHz	Disturbing transmitter				Wanted frequency kHz	Cross-modulation		Reference
		Frequency kHz	Power kW	Modulation Hz	Modulation %		Measured %	Standardised %	
1934	W.Europe 1·22	230	200[1]	200 400	80	556	3·7 3·3	1·75	5
1935	W.Europe 1·22	230	200[1]	400	80	150 – 200[2] 550 – 1500[2]	1·3 – 1·6 0·8 – 1·5	0·8 – 0·95 0·5 – 0·9	5
1935	Germany 1·23	191	60	300	70	1031 658	1·0 – 1·5 2 – 4	2·0 – 3·0 4 – 8	6 (Fig. 6) 6 (Fig. 7)
1937	England 1·25	1150	30	125	75	160 and 182	3	7	7

Superscripts refer to notes at the end of the Table.

TABLE 1 (Cont'd)

Date	Location and gyromagnetic frequency, MHz	Disturbing transmitter				Wanted frequency kHz	Cross-modulation		Reference
		Frequency kHz	Power kW	Modulation			Measured %	Standardised %	
				Hz	%				
1938	Germany 1·23	638	120	400	70	592	0·57 – 1·3	0·7 – 1·5	8 (Table 6)
			85				0·45 – 1·1	0·8 – 1·8	
			50				0·27 – 1·1	0·8 – 3·1	
		785^{3}	120^{1}	400	70	841	0·03 – 0·09	0·07 – 0·21	8 (Fig. 7)
		841^{3}	100^{1}	400	70	785	0·03 – 0·12	0·08 – 0·34	
1946	England 1·25	1050^{3}	120	300	60	200	2·3	5·2	9
					80		3·7	6·2	
1947	England 1·25	200	150	300	80	767	2·2	1·5	10 (Fig. 11)
		167	167	See Note 4		767	8	3·6	10 Table 4
		90·2	20			767	<0·1	<0·4	
		200	170			1050	4	1·8	
		1013	60			1050	3	3·7	
		68	80			1050	<0·1	<0·1	
		167	167			1050	1	0·5	
		1013	60			200	2	2·5	
		200	170			1013	4	1·8	
		167	167			200	4	1·8	
		200	170			167	2	0·9	
		200	170	300	80	1050	0·40 – 1·27	0·24 – 0·75	11 (Fig. 2)
		200	170	See Note 5	80	1050	0·6 – 2·5	0·2 – 0·9	11 (Table 1)
		200	170		80	1050	0·77 – 2·6	0·3 – 0·95	11 (Table 2)
		167	170		80	767	4	1·6	11 (p. 142)
1948	Italy 1·19	1128	5	230	30 – 35	485	2·3 – 3·8	75 – 92	13 (Fig. 2)
				450	35		1·5 – 2·0	75 – 85	12 (Fig. 3)
1948	England 1·25	167	170	300	80	767	2·2	1·3	15 (Fig. 4)
		200	170	See Note 5	80	1050	2·2 – 3·0	1·0 – 1·6	15 (Fig. 7)
		167	170		80	767	0·5 – 2·6	0·2 – 0·8	
		200	170		80	767	1·3 – 1·7	0·5 – 0·6	
		1013	60		80	767	0·3 – 0·9	0·2 – 0·7	
		167	170		80	767	1·7 – 2·3	0·6 – 0·8	
		200	170		80	767	1·3 – 1·7	0·5 – 0·6	15 (Fig. 9)
		1013	60		80	767	0·5 – 0·8	0·5 – 0·8	
		1122	60		80	767	<0·5	<0·5	15 (p. 505)
		1312	60		80	767	<0·5	<0·12	
		1474	60		80	767	<0·5	<0·12	
		167	170		80	767	3·3	1·2	
		1122	60		80	167	0·5	0·5	15 (p. 506)
		1312	60		80	167	0·5	0·12	
		1474	60		80	167	18, 40	4·0, 9·3	
		167	170	60	60	767	2·1 – 2·3	0·79 – 0·87	15 (p. 507)
		167	350	60	60	767	4·25 – 4·4	0·78 – 0·82	
		167	520	60	60	767	6·4 – 6·8	0·79 – 0·84	
1949	Italy 1·19	1100	3·3	230	80	475	2 – 9	45 – 98	14 (Fig. 3)
		1200				475	1·5	35	
		1300				475	2 – 9	45 – 98	
		1200				430	6	90	18 (Fig. 6)

Superscripts refer to notes at the end of the Table

TABLE 1 (Cont'd)

Date	Location and gyromagnetic frequency, MHz	Disturbing transmitter				Wanted frequency kHz	Cross-modulation		Reference
		Frequency kHz	Power kW	Modulation			Measured %	Standardised %	
				Hz	%				
1949	England 1·25	167	120	See Note 5	See Note 6	767	6	3	16 (p. 12)
		167	120			1050	1 – 3	0·5 – 1·5	
		167	120			1122	0·5	0·25	
		200	180			1050	1·5 – 4	0·5 – 1·4	
		200	180			804	2	0·7	
		200	180			70	<0·1	<0·03	
		167	120			200	10 – 20	5 – 10	
		668	100			1050	2	1	
		68	80	50		1050	<0·1	<0·1	
		1325 – 1355	100	60		767	3	0·5	16 (p. 15)
		200	170	300		1050	0·13	0·08	17 (Fig. 6)
		200	170	300		1050	3·5	2·0	17 (Fig. 8)
		1325 – 1355	100[1]	60		767	1 – 4	0·15 – 0·6	17 (Fig. 9)
1951	Australia 1·53	1255	36[7]	See Note 8		590	1·2	1·0	19 (Fig. 3)
		1385					3·3 – 5·0	2·6 – 4·0	
		1455					6·0 – 7·9	4·8 – 6·3	
		1530					3·9 – 9·9	3·1 – 7·9	
		1605					5·6 – 9·0	4·5 – 7·2	
		1680					6·0 – 7·5	4·8 – 6·0	
		1755					2·7	2·2	
1954	Australia 1·53	1390	36[7]	See Note 8		590	2·5 – 5	2·0 – 4·0	20 (Fig. 4)
		1420					3 – 7	2·4 – 5·6	
		1450					4 – 7	3·2 – 5·6	
		1480					5 – 9	4·0 – 7·2	
		1510					5 – 9	4·0 – 7·2	
		1540					4 – 10	3·2 – 8·0	
		1570					4 – 9·5	3·2 – 7·6	
		1600					5 – 9	4·0 – 7·2	
		1630					5 – 8	4·0 – 6·4	
		1660					3 – 6	2·4 – 4·8	
		1690					2 – 6	1·6 – 4·8	
1965	W. Europe 1·21	180	2700	400	80	233	3·3 – 4·8	0·15 – 0·21	21 (Table 1)
		233	3250			180	3·8	0·14	
		180	3050	400	70	233	5·0 – 14·0	0·23 – 0·63	21 (Tables 2 and 3)
			1500				2·5 – 6·1	0·23 – 0·56	
			600				1·0 – 6·2	0·23 – 1·42	
		233	3630	400	70	180	6·8 – 9·1	0·26 – 0·34	
			1650				5·1 – 5·3	0·42 – 0·44	
			800				2·5 – 6·8	0·43 – 1·16	
		180	2200	400	70	233	5·5 – 10	0·34 – 0·62	21 (Fig. 8)
			1100				4·5 – 6·5	0·56 – 0·81	
			400				4·5	1·4 – 1·7	
		233	2600	400	70	180	3 – 9	0·16 – 0·47	
			1200				1·5 – 4·5	0·17 – 0·51	
			600				0 – 5·5	0 – 1·3	

TABLE 1 (Cont'd)

Date	Location and gyromagnetic frequency, MHz	Disturbing transmitter				Wanted frequency kHz	Cross-modulation		Reference
		Frequency kHz	Power kW	Modulation Hz	Modulation %		Measured %	Standardised %	
1968	W. Germany 1·23	1538	700[7]	400	70	971	5·5 – 10·3	0·11 – 0·21	22 (Table 2)
						665	16·3 – 22·5	0·33 – 0·45	
			350			971	1·3 – 5·1	0·14 – 0·55	
						665	3·7 – 11·0	0·40 – 1·2	
1970	W. Europe 1·23	200	400	Programme		566	5	1·2	Unpublished BBC measurements
		1538	700[7]			539	3 – 25	0·06 – 0·5	

NOTES

1. Power not stated in Reference. Value derived from other sources.
2. Many wanted transmitters were observed. Results quoted are for paths with reflection points close to disturbing transmitter.
3. 0·5λ vertical transmitting aerial.
4. Results quoted are 'coefficients of transferred absorption', equal to $T_0(1 + M^2/2)/2M$.
5. Results quoted are values of T_0.
6. Modulation depth not stated, but assumed to be 80% as in earlier series of measurements.
7. Horizontal transmitting aerial.
8. Pulse transmission: power quoted is peak value.

Cross-modulation was assumed, on theoretical grounds, to be independent of the power of the wanted transmitter. Provided the wanted wave has no significant effect on the collision frequency in the region where cross-modulation takes place, its attenuation will be modified only by the disturbing transmitter. Even if the wanted wave is strong enough to disturb the ionosphere, the change in cross-modulation level will be small.[1]

Most of the disturbing transmissions were radiated from vertical l.f. and m.f. broadcasting aerials operating at their normal frequencies and standardisation to a short vertical aerial presented no difficulty. When transmitting aerials with horizontal directivity were used (as in Reference 21) the effective radiated power* in the direction where cross-modulation takes place, rather than the actual power, was standardised to 100 kW.

Some disturbing transmissions were radiated from unconventional aerials, or from aerials operating at frequencies for which they were not intended. Thus the special transmissions from Ottringham on frequencies between 1122 and 1474 kHz, described in References 15, 16 and 17, were radiated from a pair of 154 m (504 ft) masts spaced 61 m (200 ft), intended for use at lower frequencies. At 1474 kHz, for example, these masts were 0·75λ high and most of their radiation was concentrated at high angles, the estimated maximum power density at the base of the ionosphere being 4·5 times that due to a short vertical aerial radiating the same power. Allowance must, of course, be made for this greater power density in standardising the measurements made with these transmissions. The vertical aerial at Mainflingen[21] is also about 0·75λ high and has a similar radiation pattern.

Some of the transmissions from Mainflingen were radiated from a directional horizontal aerial, the estimated maximum power density at the base of the ionosphere exceeding that due to a short vertical aerial radiating the same power by a factor of 9·5.

The Australian transmissions, which were intended for the study of cross-modulation at frequencies near the gyromagnetic frequency, made use of a horizontal dipole $\lambda/4$ above ground; Fig. 1 shows that the maximum power density due to this type of aerial is 8 times that due to the reference aerial. Pulse modulation was employed, the peak power of 36 kW being the same as that radiated by a conventional 11 kW transmitter, modulated to a depth of 80%, at the peak of the modulation cycle. The Australian transmission is therefore equivalent to 88 kW radiated from a short vertical aerial.

In these pulse transmissions, the duration of the pulse (1 ms) was insufficient for the ionosphere to reach its steady-state condition. The measured cross-modulation was therefore less than it would have been had the pulse been longer. Appendix 1 shows how T_0 and T_{300} may be calculated from the measured values of T.

---

* The actual radiated power multiplied by the aerial gain in the direction of interest.

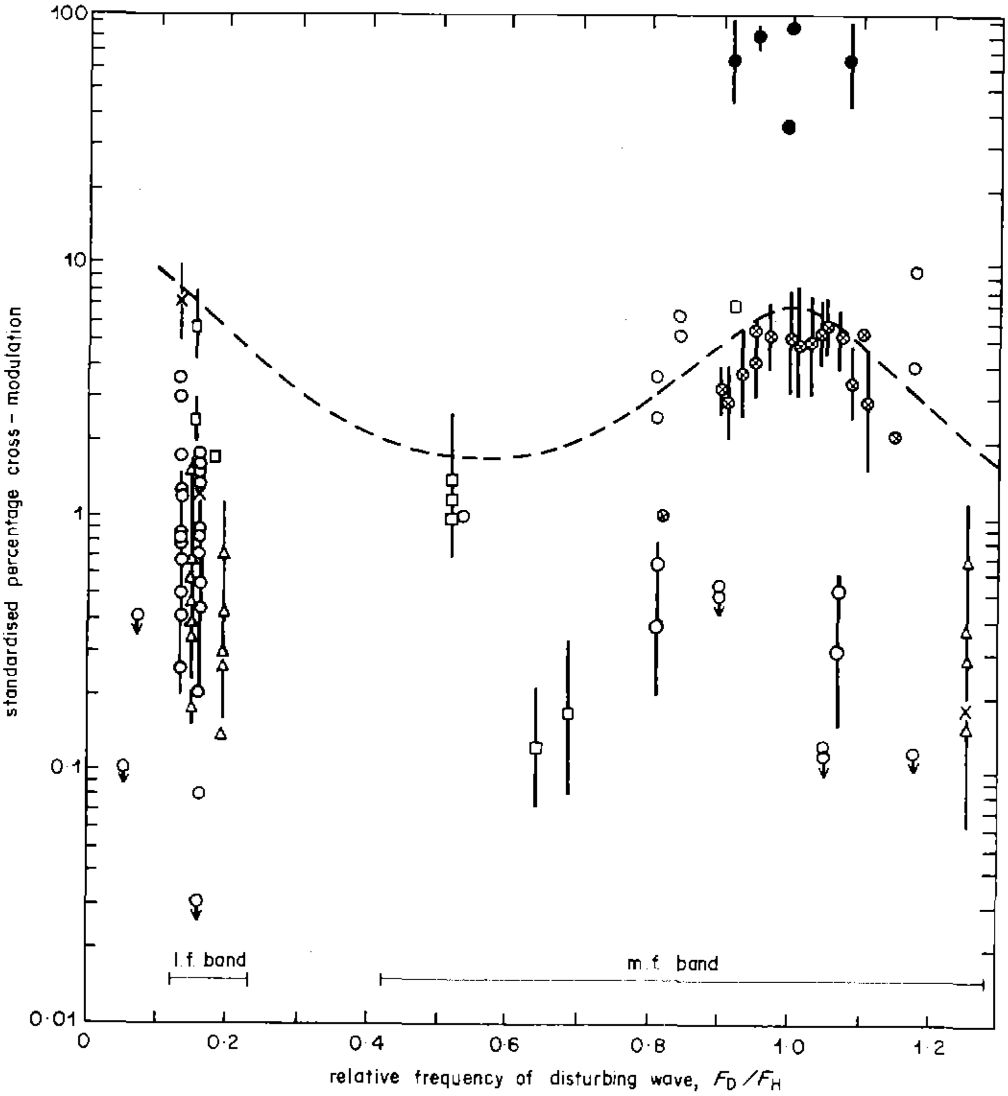

Fig. 4 - Comparison of cross-modulation measurements

- □ Pre-war measurements (References 5, 6, 7 and 8)
- ○ Measurements made at Cambridge and Birmingham (References 9, 10, 11, 15, 16 and 17)
- ● Italian measurements (References 12, 13, 14 and 18)
- ⊗ Australian measurements (References 19 and 20)
- △ Post-war measurements made in W. Europe (References 21 and 22)
- x Other measurements
- — Semi-empirical upper limit

Vertical lines represent a range of median values measured during the course of a night, or on different nights. Arrows pointing downwards indicate measured values which are less than the value shown.

4. Discussion

The standardised measurements are listed in Table 1 and compared in Fig. 4. They show considerable variation, some of which may be due to a dependence on the frequency of the wanted wave. Since the rates of attenuation of waves in the lower ionosphere tend to decrease with the square of the frequency, waves of higher frequencies would be expected to suffer less attenuation in the disturbed region of the ionosphere, and therefore less cross-modulation.

In an attempt to reduce the scatter, the standardised measurements were weighted first according to the square of the wanted frequency, and then according to a semi-empirical formula which took account of the influence of electron collisions on the frequency dependence at lower frequencies.* The scatter was not reduced in either case. Although some of the variation must be due to the frequency of the wanted wave, it may be smaller than theory suggests, because the total attenuation of wanted waves is largely independent of frequency within the l.f. and m.f. bands.† Most of the variation probably arises because of the variety of geographical arrangements of transmitters and receivers which were used for the measurements. Some of the arrangements, such as those described in References 8, 12, 13, 14, 18, 19, 20 and 22, were deliberately chosen to secure the maximum effect. Others were dictated by the positions of existing transmitting and receiving sites and were not necessarily the optimum for the observation of the greatest possible cross-modulation.

Some of the variation may be due to changes in the absorption of the wanted wave from night to night, or during the course of the night. An apparent dependence on the power of the wanted transmitter which has been reported[21] is thought to be due to these changes.

---

* The rate of attenuation of a wanted wave varies approximately as $\nu/(\nu^2 + \omega^2)$ where ν is the effective collision frequency and ω is the angular wave frequency. At the height where cross-modulation occurs, ν^2 and ω^2 are of comparable magnitude and the $1/\omega^2$ relationship does not apply.

† This situation arises because waves of the lower frequencies penetrate less deeply into the ionosphere. Thus the total attenuation they experience is comparable to that suffered by waves of higher frequencies, even though the rate of attenuation is greater.

If the Earth's megnetic field were absent, cross-modulation would decrease approximately as $1/F_D^2$, where F_D is the frequency of the disturbing transmitter. The Earth's field, however, causes a cross-modulation maximum at the gyromagnetic frequency, the width of the maximum depending on the collision frequency at the height where cross-modulation takes place. The enhanced cross-modulation near the gyromagnetic frequency was first reported in Italy[12,13,14] and has been carefully measured in Australia.[19,20]

The exceptionally large standardised values derived from measurements made near the gyromagnetic frequency in Italy are difficult to explain; cross-modulation levels as large as 9% were measured when powers of only 3·3 and 5 kW were radiated. The 5 kW transmitter (Vatican Radio) radiated from a wire 25 m long;[23] this would be expected to behave as a short vertical aerial. The 3·3 kW transmitter was the RAI station at Florence and presumably radiated from a vertical aerial. Theory suggests that the linear relationship between power and attenuation may not apply near the gyromagnetic frequency; if so the use of Fig. 4 for the standardisation of these low powers would give rise to errors and may explain why the standardised values are not consistent with those derived from other measurements.

Mention must also be made of the large values of T_o (18% and 40%) measured on two occasions at Birmingham when 1474 kHz was radiated from Ottringham.[15] The disturbing transmission was radiated from an aerial 0·75λ high and consequently the reflection point for the wanted wave, also radiated from Ottringham, was strongly illuminated. The standardised values of T (4·0% and 9·3%) are consistent with the largest values measured elsewhere. Thus the explanation, given in Reference 15, that the abnormally large values may be associated with a coincidence of reflection levels appears to be correct, but the proximity of the wanted frequency to the gyromagnetic frequency (1·25 MHz) is also partly responsible for the enhanced effect.

Because many of the experimental arrangements were chosen to obtain the greatest effect, Fig. 4 gives an indication of the maximum cross-modulation which is likely to occur in practice under the standardised conditions. It will be seen that, if the Italian measurements are disregarded, the maximum standardised cross-modulation observed at the gyromagnetic frequency is about 7% and is comparable with the largest values measured at l.f. This figure has been used in the derivation of the following semi-empirical formula for the maximum percentage cross-modulation likely to be observed under the standardised conditions:

$$T_{300} = 0{\cdot}28\left[\frac{2}{0{\cdot}04 + F_D^2} + \frac{1}{0{\cdot}04 + (\Delta F_D)^2}\right] \qquad (6)$$

Here F_D is the frequency of the disturbing wave in MHz and ΔF_D is the difference between F_D and the gyromagnetic frequency, also in MHz. The derivation of Equation (6) is described in Appendix 2. Since it is based on the measurements described here, its use should be restricted to regions such as Europe and Australia where the magnetic dip latitude is approximately 60°. Fig. 4 shows values of T_{300} calculated from Equation (6) assuming a gyromagnetic frequency of 1·25 MHz.

The use of Equation (6) should also be restricted to aerials which radiate linear polarisation. In temperate latitudes, a plane-polarised incident wave excites the ordinary and extraordinary waves in roughly equal proportions at near-vertical incidence. If an m.f. aerial were designed to radiate only the ordinary wave, i.e. by radiating circular polarisation with the appropriate sense of rotation, cross-modulation would be considerably reduced. On the other hand, if the extraordinary wave only were radiated, cross-modulation would be doubled because the extraordinary wave is mainly responsible for disturbing the ionosphere at m.f.

5. Conclusions

The measurements which are compared show that disturbing transmitters can cause as much cross-modulation at m.f. as they do at l.f., especially if they operate at frequencies close to the gyromagnetic frequency. No clear dependence on the frequency of the wanted wave can be detected, however, and there is some justification for assuming that cross-modulation is almost independent of wanted-wave frequencies within the l.f. and m.f. broadcasting bands.

When 100 kW is radiated from a short vertical aerial, cross-modulation depths of up to 7% may be observed. However measured cross-modulation levels are frequently much lower than this figure because wanted waves seldom traverse the most disturbed region of the ionosphere. Cross-modulation may also vary considerably from night to night, or during the course of a single night, because of changes in the ionospheric absorption of the wanted wave; the figure quoted above is a median value. Abnormally high cross-modulation may occur during deep fades of the wanted signal but it is then accompanied by other forms of distortion which may be equally disturbing.

The value of 7% referred to above is also given by the semi-empirical formula, Equation (6), for the calculation of the maximum median cross-modulation which is likely to be observed. Since this formula was derived from measurements made in Europe and Australia, its use should be confined to temperate latitudes and to aerials which radiate linear polarisation. Cross-modulation may be reduced at m.f. if circular polarisation is radiated, provided the sense of rotation is such that the ordinary wave predominates; with the opposite sense of rotation cross-modulation may be doubled.

Up to a level of 10%, cross-modulation is directly proportional to the power of the disturbing transmitter and to its modulation depth. If cross-modulation exceeds this figure its variation with power and modulation depth is described by Fig. 2. There is some evidence which suggests that Fig. 2 may not apply at the gyromagnetic frequency if the extraordinary wave is strongly excited.

Cross-modulation also depends on the type of aerial from which the disturbing wave is radiated. The semi-empirical formula applies to radiation from vertical aerials up to 0·25λ high. Twice as much power may be radiated from 0·5λ vertical aerials for the same effect, because they radiate less strongly at high angles. On the other hand, horizontal aerials radiate more strongly; for example 12·5 kW radiated from a horizontal dipole 0·25λ above ground has the same maximum effect on the ionosphere as 100 kW radiated from a short vertical aerial.

6. References

1. Ionospheric non-linearity. BBC Research Department Report No. RA-17, Serial No. 1968/19.
2. HUXLEY, L.G.H. *and* RATCLIFFE, J.A. 1949. A survey of ionospheric cross-modulation. *Proc. Instn. Elect. Engrs.*, 1949, **96**, III, pp. 433 – 440.
3. Ionospheric cross-modulation. CCIR Report 460, New Delhi, 1970.
4. KNIGHT, P. 1972. Radio-wave propagation in the lower ionosphere during the night, at medium and low frequencies. Ph.D. Thesis, London University, January 1972.
5. VAN DER POL, B. *and* VAN DER MARK, J. 1935. Interaction of radio waves. *Tijdschrift Nederlandsch Radio Genootschap*, **7**, Part I, pp. 12 – 17, Part II, pp. 93 – 97, 1935.
6. BÄUMLER, M. *and* PFITZER, W. 1935. Untersuchung der gegenseitigen Modulationsbeeinflussung elektrischer Wellen mit deutschen Rundfunksendern. *Hochfrequenztechnik und Elektroakustik*, 1935, **46**, pp. 181 – 186.
7. BAILEY, V.A. 1937. Resonance in the interaction of radio waves. *Nature*, 1937, May 15, pp. 838 – 839.
8. GROSSKOPF, J. 1937. Die Gegenseitung Modelungsbeeinflussung elektromagnetischer Wellen in der Ionosphäre. *Hochfrequenztechnik und Elektroakustik*, 1938, **51**, pp. 18 – 20.
9. HUXLEY, L.G.H., FOSTER, H.G. *and* NEWTON, C.C. 1947. Gyro interaction of radio waves. *Nature*, 1947, Mar. 1, pp. 300 – 301.
10. RATCLIFFE, J.A. *and* SHAW, I.J. 1947. A study of the interaction of radio waves. *Proc. Roy. Soc.*, 1948, **193**, Series A, pp. 311 – 343.
11. HUXLEY, L.G.H., FOSTER, H.G. *and* NEWTON, C.C. 1947. Measurements of the interaction of radio waves in the ionosphere. *Proc. Phys. Soc.*, 1948, **61**, pp. 134 – 146.
12. CUTOLO, M. *and* FERRERO, R. 1948. Misura del grado di modulazione parassita nel fenomeno di girointerazione. *Alta Frequenza*, 1948, **XVII**, pp. 212 – 216.
13. CUTOLO, M. *and* FERRERO, R. 1949. Photographic record and diagrams of Luxembourg effect with resonance (gyro-interaction). *Nature*, 1949, Jan. 8, pp. 58 – 59.
14. CUTOLO, M., FERERRO, R. *and* MOTOZO, M. 1950. Determinazione sperimentale della curva di risonanza nel fenomeno di girointerazione. *Alta Frequenza*, 1950, **XIX**, pp. 3 – 8.
15. HUXLEY, L.G.H. 1949. Ionospheric cross-modulation at oblique incidence. *Proc. Roy. Soc.*, 1950, **200**, Series A, pp. 486 – 511.
16. SHAW, I.J. 1950. Some further investigations of ionospheric cross-modulation. *Proc. Phys. Soc.*, 1951, **64B**, pp. 1 – 20.
17. BELL, D.A. 1950. Ionospheric interaction in disturbed conditions. *Proc. Phys. Soc.*, 1954, **64B**, pp. 1053 – 1062.
18. CUTOLO, M. 1952. Determinazione sperimentale della curve di risonanza nel moto degli elettroni lenti dell'alta atmosfera. *Nouvo Cimento*, 1952, **IX**, pp. 391 – 406.
19. BAILEY, V.A., SMITH, R.A., LANDECKER, K., HIGGS, A.J. *and* HIBBERD, F.H. 1952. Resonance in gyro-interaction of radio waves. *Nature*, 1952, May 31, pp. 911 – 913.
20. HIBBERD, F.H. 1963. An experimental study of gyro interaction in the ionosphere, at oblique incidence. National Bureau of Standards Technical Note 211, Vol. 4, pp. 45 – 64, 1964.
21. HABERKANT, E. *and* VOGT, K. 1966. Measurement of ionospheric cross-modulation affecting two high-power LF broadcast transmitters. *EBU Rev.*, 1966, Part A, No. 98, pp. 154 – 161.
22. HABERKANT, E., HEINZELMANN, G. *and* VOGT, K. 1971. The measurement of ionospheric cross-modulation. *EBU Rev.*, 1971, Part A, No. 126, pp. 50 – 55.
23. CUTOLO, M. 1963. On some resonance phenomena near the gyrofrequency obtained during the propagation of a radiowave in plasma (ionosphere). National Bureau of Standards Technical Note 211, 1964, Vol. 2, pp. 1 – 69.
24. BAILEY, V.A. 1937. The motions of electrons in a gas in the presence of variable electric fields and a constant magnetic field. *Phil. Mag.*, 1937, S7, **23**, pp. 774 – 791.

7. APPENDIX 1:
Standardisation of Pulse Measurements of Cross-Modulation

Pulse transmissions were used as disturbing waves for the Australian measurements, described in References 19 and 20. As the pulse length (1 ms) was comparable with the ionospheric time constant $1/G\bar{\nu}$, there was insufficient time for the ionosphere to reach its fully-disturbed state before the end of the pulse. Consequently the transferred modulation depth T was less than the limiting value T_0 which would have been measured if the pulse had been long enough. This Appendix shows how T_0 may be calculated from measured values of T.

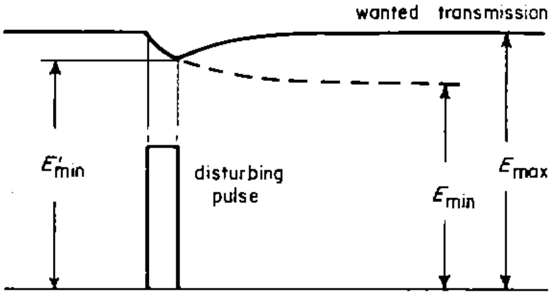

Fig. 5 - Modulation waveforms with pulse transmission

The interval between pulses (24 ms) was sufficient for the ionosphere to return to its undisturbed state before the next pulse. Consequently the modulation waveform of the wanted transmission takes the form shown in Fig. 5, E_{max} being the amplitude in the absence of the disturbing transmission, E_{min} the amplitude to which it would have been reduced had the pulses been sufficiently long and E'_{min} the minimum amplitude which actually occurs.* These three quantities are related by the equation

$$E'_{min} = E_{min} + (E_{max} - E_{min})\, e^{-G\bar{\nu}t} \tag{7}$$

where t is duration of the pulse.

The limiting value T_0 is given by

$$T_0 = \frac{E_{max} - E_{min}}{E_{max} + E_{min}} \tag{8}$$

The measured transferred modulation T is

$$T = \frac{E_{max} - E'_{min}}{E_{max} + E'_{min}} \tag{9}$$

If Equation (7) is substituted in Equation (9) and the approximation $(E_{max} + E'_{min}) \simeq (E_{max} + E_{min})$ made, it may be shown that

$$T \simeq T_0(1 - e^{-G\bar{\nu}t}) \tag{10}$$

Equation (10) enables T_0 to be calculated from T, and Equation (5) may then be used to calculate T_{300} from T_0. In the Australian measurements, values of $G\bar{\nu}$ between 750 and 900 were determined from the shape of the exponential rise of the wanted signal which follows the pulse. It may be shown that T_{300} is approximately equal to $0{\cdot}7T$ for all values of $G\bar{\nu}$ within this range.

* Fig. 3 of Reference 20 shows an oscillograph of the received waveforms.

8. APPENDIX 2:
A Semi-empirical Formula for Maximum Cross-Modulation

Equation (8) of Reference 1, a formula due to Bailey,[24] shows that the disturbance suffered by the ionosphere when a plane-polarised wave is incident upon it is proportional to

$$W = P\left[\frac{2\cos^2\psi}{\nu^2 + \omega^2} + \frac{\sin^2\psi}{\nu^2 + (\omega+\omega_H)^2} + \frac{\sin^2\psi}{\nu^2 + (\omega-\omega_H)^2}\right] \tag{11}$$

where P is the power density of the incident wave

ψ is the angle between the electric vector of the incident wave and the direction of the Earth's magnetic field

ν is the electron-collision frequency in the disturbed region of the ionosphere

ω is the angular wave frequency

ω_H is the angular gyromagnetic frequency.

Cross-modulation is approximately proportional to W.

At l.f., $\omega_H \gg \omega$ and W is greatest when $\psi = 0$; it is then equal to $2P/(\nu^2 + \omega^2)$. In Europe this situation arises to the north of disturbing transmitters radiating from vertical aerials, because the electric vector of the incident wave is then approximately parallel to the direction of the Earth's magnetic field.

Near the gyromagnetic frequency, $\omega - \omega_H$ is small and the third term in Equation (11) predominates unless $\psi = 0$. Consequently W has a maximum value, when $\psi = 90$, approximately equal to $P/[\nu^2 + (\omega - \omega_H)^2]$; this situation arises in Europe towards the south of disturbing transmitters radiating from vertical aerials.

It is assumed here that the maximum value of W for all frequencies in the l.f. and m.f. bands is given approximately by the sum of these two expressions. Thus

$$W_{\max} = P\left[\frac{2}{\nu^2+\omega^2}+\frac{1}{\nu^2+(\omega-\omega_H)^2}\right] \qquad (12)$$

Detailed calculations[4] have shown that cross-modulation is most likely to occur at a height of about 80 km, where the effective collision frequency is approximately 1·25 x 10^6 $\sec^{-1}$. If this value is inserted in Equation (12) and the latter expressed in terms of the disturbing frequency F_D in MHz, the following result is obtained:

$$W_{\max} \propto P\left[\frac{2}{0{\cdot}04+F_D^2}+\frac{1}{0{\cdot}04+(\Delta F_D)^2}\right] \qquad (13)$$

where ΔF_D is the difference between the disturbing frequency and the gyromagnetic frequency. Equation (13) shows that the values of $W_{\max}$ are approximately equal at 0·2 MHz and at the gyromagnetic frequency (1·25 MHz in Western Europe), and this conclusion is consistent with detailed theoretical calculations.[4]

Since cross-modulation is approximately proportional to W, Equation (13) was used as a basis for Equation (6), the semi-empirical formula stated earlier for the maximum percentage cross-modulation likely to be observed under the standardised conditions. The constant 0·28 in Equation (6) was chosen to give T_{300} a value of 7% at the gyromagnetic frequency, so as to be consistent with the measured values shown in Fig. 4.