Tropospheric radio wave propagation over irregular terrain:
the computation of field strength for uhf broadcasting

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TROPOSPHERIC RADIO WAVE PROPAGATION OVER IRREGULAR TERRAIN:
THE COMPUTATION OF FIELD STRENGTH FOR UHF BROADCASTING

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(RA-91)
# TROPOSPHERIC RADIO WAVE PROPAGATION OVER IRREGULAR TERRAIN: THE COMPUTATION OF FIELD STRENGTH FOR UHF BROADCASTING

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TROPOSPHERIC RADIO WAVE PROPAGATION OVER IRREGULAR TERRAIN:
THE COMPUTATION OF FIELD STRENGTH FOR UHF BROADCASTING

Summary

The report describes techniques for application in a digital computer programme to calculate the field strength of u.h.f. transmitters, with a particular emphasis on the determination of field at long distances which may cause a co-channel interference limitation to a broadcast service.

Theoretical calculation methods for certain idealised conditions are specified and proposals made for their application to practical conditions of propagation over irregular terrain. The influence of tropospheric conditions is considered and means of deriving factors which must be determined from empirical evidence are indicated.

Recognition is given to the fact that any system is liable to suffer from a lack of radio and geographical data, and the methods devised are those using a practical amount of such data.

List of Principal Symbols

The number in parenthesis indicates the equation defining the specified parameter.

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1. Introduction

The purpose of this report is to propose a system for a computer programme to calculate field strengths received from broadcast transmitters in the frequency range 400 to 1000 MHz, both within and beyond the service limit, although in the methods proposed particular emphasis has been placed on the latter case which is important in dealing with the co-channel interference problem. There tends to be an interdependence between calculation methods and data, and in this report the terrain data assumed are that of a matrix with elements spaced at 0.5 km consisting of ground heights together with some indication of buildings and trees at each matrix element. Nonetheless it is hoped that the methods proposed here will be sufficiently flexible for extension, so that if finer data are available the system will not require serious modification. Thus compatibility of methods should exist with calculations of small service areas, where greater detail and more data are essential. Similarly the modifications should not be too serious for an extension of the system to a wider frequency range.

The complexity of conditions pertaining to the propagation of radio waves near the earth’s surface often results

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in the use of empirical techniques to determine the field in given circumstances, little regard being given to the mechanisms involved. However, only slight variances on these circumstances can produce highly erroneous deductions. On the other hand theoretical work based on the solution of Maxwell’s equations is complex and analytic solutions are only complete for boundary conditions that have a simple geometry, e.g. knife edges, spheres, cylinders etc., and in some cases it is necessary to assume the boundary is perfectly conducting as in the case of much work on wedge diffraction. However, techniques based on the work of Hufford\(^1\) and developed by Ott and Berry\(^2\) and Ott,\(^3\) allow solutions of Maxwell’s equations to be obtained with arbitrary boundary conditions and work on such methods is still in progress. Nonetheless it must be stressed that theoretical techniques are not only demanding in the amount of mathematical understanding and computation required but are also demanding in the amount of data required. Thus the data about ground heights, trees and buildings contained in a matrix with elements spaced at 0.5 km enables a certain use of theory in diffraction calculations but even here we must resort to some form of statistical approach as the detail is far from complete, e.g. we do not know precisely how close the nearest trees are to a receiver, a fact which may seriously modify the field. A further important modifying influence on the received field is that of tropospheric effects, e.g. the inhomogeneities of the refractive index. However, in this case we will assume no particularised data bank in which case calculations on the phenomena of refraction, ducting, scattering, etc., must contain much of a statistical and empirical nature although guidance will be taken from the existing theories.

2. Basic propagation formulae for simple boundary conditions

In this section we will recommend formulae necessary for the calculation of fields in a homogeneous atmosphere in the presence of simple geometric obstacles. These problems will be dealt with in isolation leaving to Section 3 the work of how the formulae shall be adapted to the highly irregular situation of the true world.

Each of the sub-sections below will provide formulae for the loss in decibels relative to free space field. The symbol \(A\) is used to designate this relative loss, with a subscript to identify the particular way it is produced.

Free space field, \(F\), in decibels relative to \(1\mu V/m\) is given by:

\[
F = 106.9 - 20 \log_{10} d + P^* \tag{1}
\]

where \(d\) is the distance in km, and \(P\) is the e.r.p. in dB relative to 1 kW radiated from a half-wave dipole.

2.1. Knife-edge diffraction

The classical theory for the field behind an absorbing knife-edge results in the complementary Fresnel-Kirchhoff integral which gives the value of field relative to that obtained in free space. This ratio or loss factor is given by:

\[
\frac{E}{E_0} = \frac{1 - \frac{1}{2} \int_0^\infty \int_0^\infty e^{-\pi r^2/2} dt}{2} \tag{2}
\]

The parameter \(r\) is given by

\[
v = \theta \left[ \frac{2ab}{(a+b)\lambda} \right]^{1/2} \tag{3}
\]

where \(a\) = distance source to edge

\(b\) = distance edge to receiver

\(\theta\) = diffraction angle (positive in shadow)

\(\lambda\) = wavelength

\(\theta\) may be obtained from the approximate relationship

\[
\theta \approx h \left( \frac{a+b}{ab} \right)
\]

\(h\) is height of the edge above the source/receiver line and thus \(v\) may be computed as follows:

\[
v = h \left[ \frac{2(a+b)}{ab \lambda} \right]^{1/2} \tag{4}
\]

The extension of the theory for more than one edge results in multiple integrals which are difficult to handle although Millington et al.\(^4\) have considered the two edge situation. To overcome the problem of multiple edges many simple geometric means have been proposed, such as those of Bullington,\(^5\) Epstein and Peterson,\(^6\) the Japanese\(^7\) and Deygout.\(^8\) The Bullington method consists of constructing a single virtual edge at the crossing point of the horizon lines from each terminal, but whilst being simple it cannot be considered accurate. The Epstein and Peterson method consists of the product of losses obtained from each diffraction edge, the loss for an edge being the value obtained by assuming the path is from the previous edge (or transmitter for first edge) to the subsequent edge (or receiver for last edge). This method was analysed by Millington et al and shown to have significant errors for the case where losses are large and for the case of two edges on line of sight. Analysis of these errors shows them to be highly variable, of either sign and in certain instances very large. The Japanese method is similar to that of Epstein and Peterson but the difference is that whilst retaining the same diffraction angle the distance on one side of the path is taken always from the terminal rather than the adjacent edge. This method is accurate for any number of edges if, and only if, each edge produces a large loss, but if any edge has a small loss the errors may be large and of either sign. Also the method becomes non-reciprocal under this condition.
The Deygout method consists of obtaining a loss for each diffraction edge in turn as if the remaining edges were absent. The larger of these losses is used initially, and the edge which produced this loss is used to divide the path in two and the process repeated in the two halves as if the edge were a terminal. This process is repeated indefinitely until each of the diffracting edges have been used. The resulting loss factor is the product of the factors corresponding to the larger losses in each of the steps. This construction is shown in Figure 1 for the case of three edges. This method does not always produce a very accurate result but it has an error which is regular. The analysis of this error for two edges is as follows:

\[ \frac{E}{E_0} = f_1 f'_2 \left[ 1 + \cos \alpha \right] \]  
\[ \frac{E}{E_0} = \frac{1}{2} \left( 1 - \frac{\alpha}{\pi} \right) = f_1 f'_2 \left[ 2 - \frac{2\alpha}{\pi} \right] \]

The above analysis is not valid for \( \nu_2 = \nu_1 = 0 \) but Millington gives a result for this condition of:

\[ \frac{E}{E_0} = \frac{1}{2} \left( 1 - \frac{\alpha}{\pi} \right) = f_1 f'_2 \left[ 2 - \frac{2\alpha}{\pi} \right] \]

Thus the terms in square brackets in Equations (10), (11) and (12) represent the error factors in the Deygout method and the errors in decibels are plotted in Fig. 2(a). This shows the regularity of these errors and the proposed correction to be applied is the compromise shown in Fig. 2(b), namely \( 6 - A_1 + A_2 \) cos \( \alpha \), where \( A_1 \) and \( A_2 \) are the losses in decibels corresponding to the values of \( \nu_1 \) and \( \nu_2 \).

![Fig. 1 - Geometrical construction for the Deygout method](image)

Let the \( \nu \) parameters for two edges each in the absence of the other be \( \nu_1 \) and \( \nu_2 \) and let the second edge have the parameter \( \nu_2' \) under the Deygout construction. Now if the values of these parameters are larger than unity we are reasonably justified in using the asymptotic approximation of the Fresnel integrals, giving loss factors as follows:

\[ f_1 = \frac{1}{\sqrt{2 \pi \nu_1}} \]  
\[ f_2' = \frac{1}{\sqrt{2 \pi \nu_2'}} \]

Now

\[ \nu_2' = \nu_2 \csc \alpha - \nu_1 \cot \alpha \]

where \( \alpha \) is a spacing parameter defined by Millington as:

\[ \alpha = \tan^{-1} \sqrt{\frac{b(a + b + c)}{ac}} \]

where \( a, b \) and \( c \) are distances source to first edge, first edge to second edge, and second edge to receiver, respectively. Again quoting Millington, a rigorous loss factor, under these conditions is given by:

\[ \frac{E}{E_0} = \frac{\sin \alpha}{2\pi^2 (\nu_1 \csc \alpha - \nu_2 \cot \alpha)(\nu_2 \csc \alpha - \nu_1 \cot \alpha)} \]

Therefore (5) through (9) gives:

\[ \frac{E}{E_0} = f_1 f'_2 \left[ 1 + \frac{\nu_2 \csc \alpha - \nu_1 \cot \alpha}{\nu_1 \csc \alpha - \nu_2 \cot \alpha} \right] \]

and if \( \nu_2 = \nu_1 \), we get:

\[ \frac{E}{E_0} = f_1 f'_2 \left[ 1 + \cos \alpha \right] \]

It is therefore proposed that in calculating for multiple knife-edges the loss for each edge is calculated in isolation and the largest loss is taken as \( A \) dB. After using this edge to separate the path in two (as Fig. 1) check is made to see if these new paths have obstacles above the line of sight; if so the largest loss is taken in each case to form \( A_1 \) and \( A_2 \) and the losses these same edges give if each alone existed on the full paths are also determined as \( A_1' \) and \( A_2' \). The total knife-edge loss assumed is then given by:

\[ \text{Total loss} = A + A_1 + A_2 + A_1' + A_2' \]
The two expressions of the form \((6 - A_1 + A_2)\cos \alpha - (6 - A_1 + A_2)\cos \alpha_3\) are only included when positive. The values of \(\alpha_2\) and \(\alpha_3\) are determined from:

\[
\cos \alpha_2 = \sqrt{\frac{a(c + e)}{(a + b)(b + c + e)}} \tag{14}
\]

and

\[
\cos \alpha_3 = \sqrt{\frac{(a + b)e}{(a + b + c)(c + e)}} \tag{15}
\]

where \(a, b, c\) and \(e\) are the distances shown in Fig. 1.

**2.2. Curved surface diffraction**

Contributions to the solution of Maxwell’s equations in the presence of a sphere began with Rayleigh in 1871 and near the turn of the century many distinguished scientists worked on the problem. However, at this stage the problem was only solved in the form of a harmonic series not usable for radii large in terms of wavelengths. The first usable form of solution for this case came with the Watson transformation followed by the work of Van Der Pol and Bremner in which the result is in the form of a residue series. Further work was done on the problem in a slightly different manner by Domb and Pryce. In addition we should mention the work of Leontovich and Fock. These authors give results which are numerically identical but have a slightly different notation. The Appendix gives a residue series solution but for our present purposes we will adopt a simplified approach following the line of Vogler. This method is based on the use of the first term of the residue series, giving an acceptable approximation with several restrictions which are discussed later. The loss over a spherical surface is thus given by:

\[
A_r = G(x_o) - F(x_1) - F(x_2) - 20 \tag{16}
\]

where \(x_o = Bd\)

\(x_1 = Bd_{L_1}\)

\(x_2 = Bd_{L_2}\)

\(B = 448\lambda^{-1/3} r^{-2/3}\)

\(r = \) radius of sphere

\(d = \) total path distance

\(d_{L_1} = \) horizon distance for transmitter

\(d_{L_2} = \) horizon distance for receiver.

The functions \(G(x_o)\) and \(F(x)\) may be approximated in the manner of Longley and Rice. First, for \(G(x_o)\) we use

\[
G(x_o) = 0.05751 x_o - 10 \log_{10} x_o \tag{17}
\]

The term \(F(x)\) is an involved combination of advanced mathematical functions; e.g. Airy Integrals (see Appendix). Thus, to keep the expressions to be used in terms of elementary mathematical functions, the term \(F(x)\) is divided up as shown:

\[
1 < x \leq 200 \\
F_1(x) = 40 \log_{10} x - 117 \tag{18}
\]

\[
x > 2000 \\
F_3(x) = 0.05751 x - 10 \log_{10} x \tag{19}
\]

\[
200 < x \leq 2000 \\
F_2(x) = SF_1(x) + (1 - S) F_3(x) \tag{20}
\]

where

\[
S = 0.0134 x \exp \left(-0.005x\right) \tag{21}
\]

For distances inside the optical cut-off point more terms of the residue series are required but as this introduces greater complication it is not considered worthwhile, at least for the present. For the regions well within line of sight a result based on a two ray theory could be obtained but this requires a high degree of data resolution and at u.h.f. it is not considered that the resultant interference pattern is sufficiently troublesome to warrant inclusion. However, this does leave a region where the field is between the free space and optical cut-off values, for which a method needs to be devised. At u.h.f. the former point is given approximately by the distance at which the direct and reflected rays have a path difference of \(\lambda/6\), i.e. the point where the \(v\) parameter is approximately equal to \(-0.78\). It is also a reasonable approximation to state that the loss, in the region of concern, is linear with \(v\). In the particular application here it is not immediately obvious how to define the \(v\) parameter, but we may do this by using the geometric optics reflection point at the surface and thus we get:

\[
v = \left(\frac{d_1}{2r} \frac{h_t}{d_1}\right) \sqrt{\frac{2d_1d_2}{\lambda(d_1 + d_2)}} \tag{22}
\]

where \(h_t = \) height of transmitter above the surface of the sphere.

\(d_1 = \) distance transmitter to reflection point

\(d_2 = \) distance receiver to reflection point.

Unfortunately \(d_1\) and \(d_2\) cannot be determined explicitly, but must be obtained by a process of iteration using the following:

\[
d = d_1 + d_2 \\
d_1 = d \left(1 + \frac{h'_t}{h_t'}\right)^{-1} \tag{23}
\]

\[
d_2 = d \left(1 + \frac{h'_t}{h_t'}\right)^{-1} \\
h'_t = h_t - \frac{d_1^2}{2r} \\
h'_t = h_t - \frac{d_2^2}{2r}
\]
We may now obtain an estimate of the loss in the troublesome region using:

\[
A_r = \left[ G(x_1 + x_2) - F(x_1) - F(x_2) - 20 \right] \left[ 1 + \frac{v}{.78} \right] (24)
\]

for \(-.78 < v < 0\) and

\[
A_r = 0
\]

(25)

for \(v \leq -.78\).

In using methods such as those given here for the determination of fields over a spherical surface care must be taken because significant errors may result from the approximations made. These approximations fall into many categories such as: the basis of the physical theory used, the mathematical approximations used in restating the theory, the termination of a series, the use of approximate parameters to describe the geometry. To avoid trouble from these errors restrictions must be placed on the use of a method, such as already mentioned and formulated by Vogler, these are illustrated by the dotted lines on Fig. 3. In addition it is not valid to use the parameters \(d_{11}\) and \(d_{12}\) if the terminals are spaced from the surface by more than 1/100 of the radius of the surface, giving the 'Vogler Geometry Applicable' line in Fig. 3. This means that with the heights in use the method is satisfactory for radii of the order of that of the earth, but it is not satisfactory for the case of a rounded hill isolated from the terminals.

Alternative methods exist for this problem of isolated curved obstacles and are compared in BBC Research Department Report RA-21 and by Hacking. The obstacles in these cases are considered as cylinders of either parabolic or circular cross-section. The methods involve the assumption of an incident plane wave and result in separate terms, one in the form of a knife-edge loss and the other involving the radius of curvature. In addition to the methods considered in Reference 17 a technique has been developed by Dougherty and Maloney which gives a wide range of application. This method is favoured by the present authors although it should be pointed out that there is an error in Reference 19 Equation (10) resulting in an error in Fig. 4 of that paper. However, the corrected version is given in a subsequent paper by Dougherty and Wilkerson.

It should be pointed out that it is doubtful if any of the techniques for dealing with the radius of hill tops are usable when the data consists only of a coarse matrix of heights. Thus whilst recognising the importance of the roundedness of hills in the modifying effect they have upon fields we cannot be deterministic about this modification with the coarse data. However, we will continue to outline the method so that it may be used should data permit.

The diffraction loss, as defined by the Dougherty method, is separable into three terms given in the following equation:

\[
A(v, \rho) = A(v, o) + A(o, \rho) + U(\rho \rho)
\]

(26)
where \( v \) is the Fresnel limit parameter for an equivalent knife-edge with the same diffraction angle as the curved surface, and \( \rho \) is the index of curvature given by:

\[
\rho = \frac{1}{\lambda} e^{-1/3} \left( \frac{d}{d_1 d_2} \right)^{1/3}
\]

(27)

where \( \lambda \) is the wavelength, \( r \) the radius of curvature of the surface and \( d_1, d_2 \) are distances from the terminals.

The functions \( A(\alpha, \rho) \) and \( U(\nu \rho) \) may be approximated using the following:

\[
A(\alpha, \rho) = 6 + 7.19\rho - 2.02\rho^2 + 3.63\rho^3 - 0.75\rho^4
\]

with \( \rho < 1.4 \)

\[
U(\nu \rho) = (43.6 + 23.5 \nu \rho) \log(1 + \nu \rho) - 6 - 6.7 \nu \rho
\]

with \( \nu \rho < 2 \)

\[
U(\nu \rho) = 22 \nu \rho - 20 \log \nu \rho - 14.13
\]

with \( \nu \rho \geq 2 \)

The overall restriction on this method is that \( \rho \) should not be greater than 1.4 and in order to analyse the meaning of this restriction let us take a case where violation is most likely to occur. This is when the diffracting hill is equally close to both terminals. We then obtain, using Equation (27), the following restriction:

\[
1.4 \geq \frac{1}{\lambda} e^{-1/3} \left( 2rh \right)^{-3/2} 2^{1/3}
\]

(29)

where \( h \) is the height of either terminal.

This may be reduced to

\[
4h \geq r \left( \frac{2}{3} \lambda \right)^{2/3}
\]

(30)

The restriction that this gives results in the 'Dougherty and Maloney Applicable' line in Fig. 3. All of the lines of this graph are calculated for frequency of 600 MHz (u.h.f. band).

2.3. Diffraction by wedge-shaped obstacles

Certain terrain profiles may be approximated by a wedge shape and it is thus desirable to have a method of calculating fields over such obstacles. This problem has been tackled by several authors using a full wave approach, for example Tyras. However, in the context of the present application the problem may be treated as a modified Fresnel diffraction problem using a four ray technique, the additional rays arising through ground reflections, being associated with images of the source and receiver. This type of technique was described in a paper by Knight, Davies and Manton for a slightly different purpose.

For our particular purpose we wish to consider the geometry shown in Fig. 4 and we will make the assumption that the reflection coefficient at the faces of the wedge is minus one. The received field relative to free space, under these circumstances may then be given by:

\[
\left| \frac{E_w}{E_o} \right| = \frac{1 - j}{2} \left[ \int_{v_{SR}}^{0} e^{\frac{\pi}{12} (r^2 - v_{SR}^2)} dt - \int_{v_{SR}'}^{v_0} e^{\frac{\pi}{12} (r^2 - v_{SR}^2)} dt \right] - \int_{v_{SR}'}^{0} e^{\frac{\pi}{12} (r^2 - v_{SR}^2)} dt - \int_{v_{SR}'}^{v_0} e^{\frac{\pi}{12} (r^2 - v_{SR}^2)} dt - \int_{v_{SR}''}^{v_0} e^{\frac{\pi}{12} (r^2 - v_{SR}^2)} dt
\]

(31)

where

\[
v_{SR} = (\psi - \phi_1 - \phi_2) \left( \frac{2ab}{(a + b)\lambda} \right)^{1/2}
\]

\[
v_{SR}' = (\psi + \phi_1 - \phi_2) \left( \frac{2ab}{(a + b)\lambda} \right)^{1/2}
\]

\[
v_{SR}' = (\psi - \phi_1 + \phi_2) \left( \frac{2ab}{(a + b)\lambda} \right)^{1/2}
\]

\[
v_{SR}' = (\psi + \phi_1 + \phi_2) \left( \frac{2ab}{(a + b)\lambda} \right)^{1/2}
\]

The required diffraction loss in decibels is then given by:

\[
A_w = 20 \log_{10} \left| \frac{E_0}{E_w} \right|
\]

(33)

3. Application of formulae to practical boundary conditions

So far formulae have been given for idealised conditions and it is now necessary to describe how these formulae can be applied to the far from 'ideal' state of the real world. In addition, we must cater for the fact that the data to be used are far from complete.
It is not as yet practical to calculate u.h.f. fields over an irregular shaped profile without stylising this profile, although this seems to have been achieved at lower frequencies. It is hoped that the future will bring similar techniques for higher frequencies but this would require much further work.

In the absence of such developments the best approach seems to be that of approximating the profile into shapes with a simple geometry, this geometry being amenable to existing techniques of calculating fields. Thus the first and obvious approximation is to assume the profile consists of a few knife-edges, these being made to simulate the most significant hills. Unfortunately, this alone is not sufficient as errors are likely to be large and biased in the direction of large fields. The reason for this is that the remaining surface is ignored and in many practical examples this will produce large reductions in signal. To obtain a measure of this we will assume that the profile can be represented by a simple geometric surface which, although generally much smoother, still approximates to the true profile. This surface will be defined by either a rounded or a wedge shape. However, because of the way it is constructed and because of its smoothness, this will tend to result in calculated values of field which are too low. It is therefore proposed that calculations of field are made for both knife-edge and surface approximations to the profile. An interpolation dependent on roughness of the actual profile is made between the two results.

3.1. Knife-edge approximation

The knife-edge calculations should be made by the methods of Section 2.1 but to this must be added assumptions about what constitutes a knife-edge. The computer operates on a finite series of heights at intervals representing the profile along the path and it would be unrealistic to assume each one of these constituted a knife-edge. It is therefore proposed that heights should be grouped to form virtual edges. This grouping can be effected by first determining the sequential horizon points i.e. which heights would be touched if a 'string' were stretched between the terminals. These heights are then considered as belonging to a group if they are spaced less than 0.5 km or if their spacing relative to the full path is such that the Millington $\alpha$ parameter is less than 17.5°. The virtual edge is then taken as the crossing point of the straight lines which exist in the 'string' before it touches the outer two heights of a group.

If the 'string' does not touch any of the heights an optical path exists. In this case it is considered that a calculation should be made for a single knife-edge, below the line of sight and the height which gives the minimum negative value of the $\nu$ parameter is selected as the effective edge.

3.2. Smooth surface approximation

To represent a path profile by a smooth surface it is necessary to assume simple geometric conditions and for this purpose paths are put into three categories; those with more than one edge, those with one edge and those which are optical.

In the case with more than one edge it is thought advisable to follow the system as laid down by Rice et al. This involves the application of Equation (16) of the present report, but instead of using one radius, four are used as shown in Fig. 5. The relevant radii together with the values to be applied to Equation (16) are given in the following equations.

\[
\begin{align*}
    x_1 &= 448\lambda \frac{-1/2a_1 - 2/3d_{L1}}{a_1}, \\
    x_2 &= 448\lambda \frac{-1/2a_2 - 2/3d_{L2}}{a_2}, \\
    x_0 &= 448\lambda \frac{a_1}{a_1 + a_2} - 2/3d_{L1} + a_1 - 2/3d_{L2} + \frac{a_2}{a_1} - 2/3d_{L2}
\end{align*}
\]

(35)

where

\[
\begin{align*}
    d_t &= \frac{d^2}{2a_e} + \frac{\theta}{\theta} + \frac{h_t - h_i}{\theta} - d_{L1}, \\
    d_e &= \frac{d^2}{2a_e} + \frac{\theta}{\theta} + \frac{h_t - h_i}{\theta} - d_{L2}, \\
    a_t &= \frac{d^2}{2h_{te}}, \\
    a_e &= \frac{d^2}{2h_{re}}, \\
    a_r &= \frac{(d_t + d_e)dt}{\theta dt}
\end{align*}
\]

(36)

where $a_e$ is the effective earth radius (see Section 4.2).

These formulae may now be evaluated provided we are able to give a meaningful definition to the terms $h_{te}$ and $h_{re}$, which are known as the effective heights of transmitter...
The method proposed for approximating the wedge shape and make calculations in the manner of Section 2.3. For each terminal or a virtual edge produced by the projection to the terrain is that we must first determine the peak of the horizon lines. The direction of the faces of the wedge although Reference 23 still makes use of them by the spherical or cylindrical approximations are of course poor for one extreme of the calculation but if poor horizon becomes short, say less than 3 km, the method of using four radii (Equations 35) is erroneous. In fact, Rice et al. use the central 80% of the terrain between a terminal and its horizon, so we will adopt this criterion. However, in their case they obtain the mean height of this terrain above sea level to determine \( h_{se} \) and \( h_{te} \) but this may give aerials below effective ground level unless other criteria are added. To avoid this problem we will add to their value of the effective height an amount equal to half the difference between the ground height at the terminal and the height of the horizon point. In this way a sea level profile still remains as such and we also keep a reasonable form for profiles that approximate to wedge shapes or plateaus.

When the distance between transmitter and receiver horizons becomes short, say less than 3 km, the method of using four radii (Equations 35) is erroneous. In particular, these errors are serious for paths with a common horizon. In such circumstances a knife-edge is of course satisfactory for one extreme of the calculation but if poor clearance exists to this edge we still need a system for the surface part of the calculation. In such circumstances spherical or cylindrical approximations are of course poor although Reference 23 still makes use of them by the introduction of a term to account for ground reflection. The present authors feel that a far more satisfactory approximation would be to simulate the surface by a wedge shape, and make calculations in the manner of Section 2.3. The method proposed for approximating the wedge shape to the terrain is that we must first determine the peak of the wedge. This should be coincident with the edge visible to both terminals or a virtual edge produced by the projection of horizon lines. The direction of the faces of the wedge must then be determined. Here we propose that the faces should be a ‘least squares’ fit to the actual terrain, with the constraint that they pass through the above determined peak. In addition a further constraint is required which is to stop the heights of the terminals above the wedge being less than five metres.

In the case of an optical path it is considered that to produce a surface the most relevant criterion is that it shall have a minimum path difference which is the same as the minimum path difference produced by the actual ground. It is then considered that the surface should pass through the heights which are representative of ground at the transmitter and receiver ends of the path in a similar way to that discussed for the multiple edge condition, but in this case instead of the horizon dictating the section to be taken it is the edge giving the minimum clearance. We thus may construct a single circle which passes through these end points and touches (common tangents) the ellipse produced by the locus of points having the same minimum path difference as that of the actual ground. This construction is shown in Fig. 6.

3.3. Interpolation between knife-edge and smooth surface losses

It is now necessary to determine a field from the two sets of calculations, knife-edges and smooth surfaces. It is assumed that this result may be obtained by interpolating between the two sets of calculations using roughness of the true surface as interpolation parameter. Such effects are discussed in References (17, 18) together with the results of experiments to determine the effects of roughness. It is proposed that the required interpolation parameter be based on a standard deviation, \( \sigma \), of the actual terrain heights from the equivalent heights of the assumed surface. It is then necessary to produce a weighting factor as follows:

\[
W_s = \left[ 1 + \frac{\sigma}{\lambda} \right]^{-1} \tag{37}
\]

where \( G \) is determined by use of a computer optimisation process to provide a minimum difference between calculated and measured values of loss. In the case of the wedge we may produce two such weighting factors, one for each face of the wedge, giving \( W_{s1} \) and \( W_{s2} \). The interpolation between a knife-edge and a surface result may then be determined by multiplying the second term in Equation (31) by \( W_{s1} \), the third term by \( W_{s2} \), and the fourth by \( W_{s1} W_{s2} \). When we are interpolating between multiple knife-edges and the surface made up of the four radii the result is given by the following:
This formula should also be used for the case when we have a line of sight path with poor clearance.

In the case of an isolated rounded surface where the method of Dougherty and Maloney is applicable (see Section 2.2) it is considered that a more sensitive measure of roughness may be necessary. This being based on the environmental clutter which exists on the surface, let us call the weighting factor in this case $W_r$.

Now using this concept we must modify Equation (26) to give:

$$A'_r(v, \rho) = A(v, \alpha) + W_r [A(\alpha, \rho) + U(v\rho)]$$

(39)

Nonetheless it must again be pointed out that when the data consist of only a coarse matrix we do not intend to make use of this calculation.

### 3.4. Environmental clutter

The title environmental clutter is meant to include the effects of trees, buildings, and other objects above the ground surface. However, it is not practical to obtain from maps definitive data of the existence of such objects and thus a calculation system must allow for this lack of definition. The figures to be used in the matrix data bank of environmental clutter have been obtained by the BBC using 1 in. to 1 mile ordnance survey maps and consist of four figures at each element representing woods, orchards, houses and factories. The last two are distinguished only for the sake of population counting, not for propagation calculation. These four figures are integers from 0 to 10 and a symbol representing greater than 0 but much less than 1 and they are to be interpreted as meaning the coverage, in tenths, of each area on the map by the features mentioned. However, it must be pointed out that the maps in question greatly exaggerate the coverage by buildings, this exaggeration being different on maps of a different scale. Thus we must allow for such a distortion, and a limited investigation indicated that a reasonable correction to the value obtained from a 1 in. to 1 mile map is to multiply the value by 0.3.

At u.h.f. the above mentioned objects are largely opaque, except maybe for sparse woods. Thus when the objects are further than 1 km away from a terminal they will simply be considered as raising the ground profile height by 50 ft for woods, 20 ft for orchards and 30 ft for all buildings. In dealing with the last kilometre there is a problem of a statistical nature in that we have ill defined data and many receivers within the area of concern. Thus any one receiver has a certain probability that its path is obstructed by a certain amount. In addition to this several other perturbations exist, for example reflections from off profile obstacles give multipath effects, there is diffraction round as well as diffraction over obstacles and also the energy transmitted through trees is of relevance. Because of all this complication it does not seem worthwhile being highly analytic about the problem but simply obtain equations based on intuitive ideas and experimental results to provide a measure of the resulting losses.

Firstly all points on the path profile beyond 1 km from the receiver are raised by the heights previously stated, if a non-zero value of occupancy exists, i.e. any clutter beyond 1 km is treated as a solid obstacle of appropriate height. For the part of the path within 1 km of the receiver but outside the immediate vicinity of the receiver location area it is proposed to raise the profile by a height value dependent on the occupancy of clutter and the distance from the receiver. Because the probability of obstructing the path is of linear form whereas the occupancy factor is an area function the square root of these factors is used. The equation obtained to define the amount by which the ground height is raised is:

$$h(\text{ft}) = (30\sqrt{B} + 50\sqrt{T})$$

(40)

where $B$ and $T$ are the building and tree occupancies at the distance $d$ from the receiver.

Although this artifice of raising the ground height is considered appropriate for determining losses for edge diffraction it is not considered satisfactory to have these elevations for the surface loss calculation. This means that it is possible that the edge diffraction will produce a higher loss than the surface and in this case the interpolation should not take place, the loss being solely determined by the edge diffraction calculation.

An additional loss is required to allow for the effects of clutter existing at the receiving location area and this is applied to both edge and surface diffraction results. This loss is obtained by use of the following:

$$A_c = 0 \text{ for } \beta > 0.05 \text{ rad.}$$

$$A_c = (6\sqrt{B_o} + 15\sqrt{T_o}) (1 - 20\beta) \text{ for } 0.05 > \beta > 0.0.$$  \hspace{1cm} (41)

$$A_c = 6\sqrt{B_o} + 15\sqrt{T_o} \text{ for } \beta \leq 0.0.$$  \hspace{1cm} (41)

where $B_o$ and $T_o$ are occupancy factors for buildings and trees at the receiving location.

$\beta$ is the angle of arrival of the signal relative to the horizontal ($\beta$ positive for ray line above horizontal).

The square root of the occupancy factors is again taken to account for the area to linear transformation required and the coefficients, 6 and 15, were determined from consideration of both the attenuation due to diffraction over the obstacles and the attenuation on passing through clutter.

Attenuation due to passage through woods is discussed by Saxton and Lane\textsuperscript{26} where a loss of about 0-15 dB/m is obtained for the frequencies considered here. Now with Equation (41) we are concerned with determining the loss due to either passage through or diffraction over trees and these losses are approximately equal for a distance of 100 metres when the direct path is along the horizontal, hence the coefficient 15. The loss is reduced relative to this maximum for increasing angles of approach.
Equation (41) is based on the assumption of a receiving aerial height of 10 metres and should not be used for other conditions. Nonetheless it should not be difficult to devise formulae for other specific aerial heights, say for mobile radio purposes, or possibly obtain a formula which contains a suitable height factor. However, for our particular application we are considering a domestic receiving aerial 10 metres above ground level or an aerial in open conditions for re-broadcast purposes. In the latter case the loss $A_e$ should not be included.

4. Tropospheric effects

In previous sections we have assumed that the medium above the earth's surface was homogeneous. We must now consider the effects of the lack of homogeneity obtained in the troposphere.

4.1. Scatter propagation

The term scatter propagation is often given a wide meaning to cover most atmospheric effects that increase the field above that resulting from diffraction round the earth with a homogeneous atmosphere. This includes such phenomena as partial reflection, which can be produced by discontinuities of refractive index in a layer form. Generally speaking a division may be made into theories which are based on the idea of 'blobs' of refractive index variation which are isotropic in space, and others which are based on the idea of patchy layers. The former division is considered in a paper by Booker and Gordon and the latter by Friis, Crawford and Hogg. Methods for calculating the field under scatter conditions are given by Rice et al. and also by Longley and Rice but they are quite complex and it is not considered that this complexity is justified in view of the tenuous nature of both theory and empirical evidence. This view is reinforced by a paper, comparing methods, written by Larsen.

Most of the methods of the literature have one important thing in common, this being that a most dominant parameter is the angular distance, $\theta$, which is shown in Fig. 7.

This angular distance may be defined as follows:

$$\theta = \theta_1 + \theta_2 + \frac{d}{r}$$

where $\theta_1$ and $\theta_2$ are the angles in radians, at the terminals, between the horizontal and the horizon.

The frequency dependence of the scatter field is a function of the type and size of the refractive index variations. However there is evidence, for example Crawford, Hogg and Kummer, that it is reasonable to assume that the loss from free space is directly proportional to frequency at the median field level but not so at other levels.

To keep this part of the calculation as simple as possible it is proposed that the loss from free space should be given by the following:

$$A_e = C_1 + C_2 \log_{10} \theta$$

where $C_1$ and $C_2$ are to be determined from u.h.f. experimental data, these data being restricted to that collected in N.W. Europe as the level and type of scatter propagation varies with varying regions. Strictly speaking $C_1$ is a function of frequency but if we confine ourselves to determining a single value to be used only for u.h.f. the errors should be small.

Fig. 7 - Angular distance

4.2. Time variability

A particular concern in assessing co-channel interference is the enhanced signals experienced for small percentages of time. There are two major ways of dealing with this problem. The first is to determine an increase of field from a median time value, this being the approach adopted in References (23) and (16). The second is to look at certain time percentages in their own right and base calculations on the dominant mechanisms at those times. The latter method may be slightly more complex but the increased accuracy which should result is thought to make it preferable.

The increases in signal are mainly due to refractive index ($n$) conditions in the atmosphere. The meteorological parameters, temperature $T$ degrees Kelvin, pressure $P$ and water vapour pressure $e$, both in millibars may be converted to refractive index thus:

$$(n - 1)10^6 = N \approx \frac{776}{T} \left( P + 4810 \frac{e}{T} \right)$$
With the conditions normally experienced \( n \) will gradually decrease with height and as a result ray paths curve slightly towards the earth.

This may be allowed for by use of an effective earth radius thus:

\[
a_e = ka \tag{45}
\]

where \( a \) is the actual earth radius.

It is a well used assumption that under normal conditions \( k = \frac{4}{3} \), but a general expression for \( k \) is given by:

\[
k = \frac{1}{\left( \frac{a}{n} \right)^{1/3} \left( \frac{dn}{dh} \right)^{-1/6}(m - \frac{4}{3})^{-2/3}} \tag{46}
\]

where \( h \) is the height.

It is possible for \( k \) to become infinite or negative and it is then possible for the radio wave to be propagated in ducted modes. This phenomenon is discussed by several authors such as Kerr and Budden. The physical situation when ducting occurs is analogous to that in a waveguide, where the field may be resolved into a sum of elementary waves or modes. However, in the case of a guide all modes are confined whereas in the case of a duct there is a leakage from the upper side. With the aid of several assumptions Budden obtains an expression of the form:

\[
\frac{E}{E_0} \approx 0.77d^{1/3} \left( \frac{dn}{dh} \right)^{1/3} - \frac{1}{6} \left( \lambda^2 - \frac{4}{3} \right)^{-2/3} \tag{47}
\]

where \( m = 1, 2 \ldots \), and designates the propagation mode number.

A further assumption often valid at u.h.f. is that the duct only supports the first mode, thus:

\[
\frac{E}{E_0} \approx 0.93d^{1/3} \left( \frac{dn}{dh} \right)^{1/3} - \frac{1}{6} \tag{48}
\]

Putting this in a decibel scale and incorporating the terms involving refractive index lapse rate and wave length in a term labelled \( D_1 \), we get a formula which should only be applied when it gives an increase to free space field. This being:

\[
A_d = D_1 - \log_{10}d \tag{49}
\]

Lack of detailed meteorological data means that \( D_1 \) must be determined by reference to measured radio data. A further problem exists because of the limited horizontal dimensions of the duct. Let us assume that given certain conditions a duct extends along a propagation path for a distance \( l \). It is proposed that Equation (49) should be used up to the distance \( l \) after which the proposal is that the following formula be used:

\[
A_l = D_1 - (10 + D_2) \log_{10}l + D_2 \log_{10}d \tag{50}
\]

The values \( D_2 \) and \( l \) must again be determined by an empirical fit. We now have the following factors as time variables: \( k, D_1, D_2 \) and \( l \) as well as \( C \) from the previous section on scatter propagation. These factors will, of course, be a function of geographical location but if we restrict ourselves to N.W. Europe it should be sufficient to sub-divide the factors into two categories only, these being land and sea.

### 4.3. Mixed land/sea path

Much of the published work on this subject is for medium wave frequencies where the problem is associated with the difference in electrical constants between the two areas. However, in the case of higher frequencies electrical constants are relatively insignificant and instead the two areas are distinguished by other differences, in particular the kind of refractive index conditions that exist in the troposphere above these areas and also the obvious difference of the heights of the surface. This height of surface is automatically accounted for by the use of a matrix of terrain heights, but no such data exist for refractive index conditions.

For short distances this lack of tropospheric data presents little problem. Also the fields quoted for 50% of time should not suffer any great error from this lack of data. This is because the assumption of a four-thirds earth radius is roughly applicable in both cases and scatter propagation may be tackled by using the constants that are appropriate to the position of the centre of the common volume. However, when tropospheric conditions are such that sea path fields are considerably enhanced relative to land path fields a problem does arise.

Excluding ducting conditions a satisfactory result may be obtained by use of a combination of effective earth radii. Thus assume we have determined two sets of earth’s radii multiplying factors, \( k \), one set for land and one for sea, the members of each set consisting of the values to be associated with the time percentages for which calculations are required. We may now construct a profile over the mixed path where the curvature applied is appropriate to the particular section, provided we stipulate that each section joins with a common tangent. Having obtained such a profile we can make a calculation in the same manner as described for ordinary paths, but again using the scatter propagation constants associated with the area of the common volume. For the sake of simplicity and because refractive indices are less likely to have stable lapse rate conditions near coastlines it is proposed that if a stretch of sea is less than 50 km in a mixed path we do not change the radius in this section from that of the land sections.

When the sea path field is considerably enhanced by ducting a mixed path result is most difficult to produce because of the many unknowns that exist. For example, what are the chances of this duct extending over flat lands or the effect of the land reducing the duct out at sea? How well does a land based transmitter couple into a sea based
duct? What is the height of the particular duct in question? In short, the number of variables for such conditions is immense and we cannot hope to be definitive about this problem at present. We therefore propose that a first attempt should be that under sea ducting conditions the profile to have the parameters and the calculations of a sea path.

A question we may now ask is in view of the fact that all relevant transmitters and receivers are land based under what conditions do we consider that we have a sea path. The decision here is that taking the appropriate land curvature for the time percentage in question, if both terminals ‘see’ the sea and no mid-path land exists we will consider about this problem at present. We therefore propose that a sea path.

5. Conclusions

This report has recommended and outlined methods which can be used in the production of a computer programme to calculate field strengths at u.h.f.

However, there is a need for further work on the study of certain aspects of the problem and much compilation and analysis of available measured field data is required to determine the values of constants which are to be empirically based.

It should also be pointed out that the scope of this report is that of the propagation factors in the proposed computer programme. This leaves many peripheral problems associated with achievement of the required aim which is the calculation of the effective coverage of a u.h.f. broadcast network.

6. References


**APPENDIX**

The Residue Series Solution for Diffraction by a Spherical Surface

There exist two residue series, one for each polarisation, but the one for the vertical case is the more complex because it involves terms that are functions of the electric constants of the ground. However, for frequencies greater than 100 MHz adequate results are obtained using the simpler horizontal polarisation series for both conditions, we will thus quote this result here. The field relative to that in free space is given by:

\[
\frac{E}{E_0} = \sqrt{\frac{4\pi D}{d}} \sum_{n=1}^{\infty} f_n(H_1) f_n(H_2) e^{-\frac{\sqrt{3} + j}{d} d_D} \tag{A.1}
\]

The symbols having the following meanings:

\[D = \frac{d}{d_0}\]

where \(d\) is the distance between terminals.

\[d_0 = \left(\frac{r^3}{\pi}\right)^{1/3}\]

\[a_n\]

\[H_{1,2} = \frac{h_{1,2}}{h_0}\]

\[h_{1,2} = \text{heights of aerials above surface.}\]

\[h_0 = \frac{\sqrt{2}}{2\pi} \left(\frac{r^2}{\pi}\right)^{1/3}\]

\[= \text{a standard height.}\]

\(a_n\) represents numbers which are the zeros of the Airy integral, the first five of which are given below:

\[n\]

\[a_n\]

\[= 2.3381 \quad 4.0879 \quad 5.5206 \quad 6.7867 \quad 7.9441\]

The functions \(f_n(H_{1,2})\) are given by:

\[f_n(H_{1,2}) = \frac{\text{Ai}(-a_n + \frac{\pi}{3} H_{1,2})}{\text{Ai}(-a_n)} \tag{A.2}\]

where \(\text{Ai}\) signifies an Airy integral.

It will be noticed that in the body of the report, Equation (16), the heights as such are not used as parameters. Instead the horizon distance that such heights would give, are the values used. Thus, using the fact that the horizon distance, \(d_L = \sqrt{2dh}\), we get:

\[
\frac{h}{h_0} = \frac{d_L^2}{2\pi} \left(\frac{\pi}{r^2}\right)^{1/3} = \frac{d_L^2}{2\pi} \left(\frac{\pi}{r^2}\right)^{2/3} = \left(\frac{d_L}{d_0}\right)^2 \tag{A.3}\]

Hence the change of variable is easily achieved.

SMW/AMM