Calculation of the field strength required for a television service, in the presence of co-channel interfering signals

Part 2: Effect of multiple interfering sources
RESEARCH DEPARTMENT

CALCULATION OF THE FIELD STRENGTH REQUIRED FOR A TELEVISION SERVICE, IN THE PRESENCE OF CO-CHANNEL INTERFERING SIGNALS

Part 2: Effect of Multiple Interfering Sources

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CALCULATION OF THE FIELD STRENGTH REQUIRED FOR A TELEVISION SERVICE, IN THE PRESENCE OF CO-CHANNEL INTERFERING SIGNALS

Part 2: Effect of Multiple Interfering Sources

PREFACE

The report on 'Calculation of the Field Strength Required for a Television Service in the Presence of Co-channel Interfering Signals' is in three parts, as follows:

Part 1: Effect of a single interfering source¹

Part 2: Effect of multiple interfering sources

Part 3: The computer programme².

Parts 1 and 2 deal with the theoretical background of the subject, and have been written as entities which can be read separately; for a full understanding of the work, however, the reader is advised to study Parts 1 and 2 together. Part 3 describes the form and flow diagram of the original computer programme to carry out the calculation processes described in Parts 1 and 2. Recent improvements in detail and adaptations to a larger and faster computer have been made, but are not considered here.
CALCULATION OF THE FIELD STRENGTH REQUIRED FOR A TELEVISION SERVICE,
IN THE PRESENCE OF CO-CHANNEL INTERFERING SIGNALS

Part 2: Effect of Multiple Interfering Sources

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CALCULATION OF THE FIELD STRENGTH REQUIRED FOR A TELEVISION SERVICE, IN THE PRESENCE OF CO-CHANNEL INTERFERING SIGNALS

Part 2: Effect of Multiple Interfering Sources

SUMMARY

With the great expansion in television services in Europe and elsewhere, several transmitting stations will be operating in the same frequency channel. To any one service all transmitters sharing the channel are sources of interference, and it is therefore necessary that some method of estimating the effect of several interferences should be available. The BBC has evolved a system for estimating the protection required against multiple interference, which is in current use for the planning of the UHF television services in the United Kingdom. The procedures incorporated in the system are here described.

1. INTRODUCTION

The difficulties associated with planning a television service in which a number of transmitters share a single channel, and the labour of calculating the co-channel interference levels which define the quality of service, have been discussed in Part 1 of this report. For these aspects of the problem, as well as for a description of the propagation curves evolved by the BBC for the co-channel interference calculations, the reader is referred to Part 1. The main object of the present report is to describe the procedures used in the BBC to estimate the degree of protection achieved when several interfering sources exist, from data that define the interference potential of each source taken separately.

The general principle on which these procedures rely is that the probability of protection against several interferences is the product of the probabilities of protection against each. In implementing this principle, account is taken of two conditions which prevail in practice; in the first place, radio-frequency signals vary both in time and location, and the probability of protection is therefore calculated in "time-location", i.e. the probability that a viewer of a television programme is protected is expressed so as to cover all times and all locations in the area under consideration; in the second place, wanted and interfering signals are partially correlated, and an attempt is made in the calculating procedure to reflect this fact.

The BBC method will be more readily understood by those familiar with the CCIR** method of calculating protection against a single interference, as contained in the CCIR Recommendation. The CCIR method is therefore used as the starting point of the discussion, and is illustrated in Section 3 by an example. In the same section "protected field strength" is defined. This is an important concept, and fundamental to co-channel interference calculation, whether for single or multiple interference.

The relationship between probabilities in time-location, and the separate probabilities in time and location is demonstrated in Section 4.1, while in Section 4.2 the partial correlation between wanted and interfering signals is discussed. These prepare the way for an understanding of the example in Section 4.3 illustrating the BBC method. A single interfering source is assumed at this stage to demonstrate the complementary processes of operating in time-location, using time and location data, and of resolving probability in time-location into separate time and location probabilities. The discussion continues in Section 5.1 with an illustration of probability multiplication, where the resulting probability product is seen to have a non-Normal distribution. In Section 5.2 the approximation used to make non-Normal distributions amenable to Normal distribution calculations is described. Section 5.3 contains the procedure for calculating protection against virtually continuous interference, while the method used for reducing the non-Normal distribution of the time-location probability product to separate time and location probabilities is explained in Section 5.4. Section 6 points to the

* The ITU publication 'Technical Data Used by the European VHF/UHF Broadcasting Conference, Stockholm 1961' contains a method for the calculation of multiple co-channel interference accepted at the Meeting of Experts of the CCIR Cannes 1961, but this method does not appear in any published documents of the CCIR.

** Comité Consultatif International des Radiocommunications.
more important approximations contained in the method. The complex problem of correlation between signals is discussed in the Appendix.

2. CHARACTERISTICS OF THE PROPAGATED FIELD

The characteristics of the propagated field and the manner in which they are expressed in CCIR Recommendation 370 are described in Part 1 of this report. Only a brief account is given here, to preserve continuity in the present discussion.

2.1. Variation of the Received Signal with Location

It is commonplace that signals transmitted in the v.h.f. and u.h.f. bands vary in strength from place to place over distances of a few metres. This variation with location is due mainly to topographical features in the neighbourhood of the receiving aerial and the degree of variation about the median value for the local area is virtually independent of the distance of the transmitter. This variation approximates to a Normal distribution when the signal strength is expressed in decibels* (relative to an arbitrary datum). The assumption of a Normal distribution of the location variations is generally accepted, and is incorporated in CCIR Recommendation 370. There is, however, a distinction to be made here between the CCIR approach and the location variation distribution assumed in the BBC propagation data. The BBC curves include both the variations due to terrain along the whole length of the path and to local features. The BBC method eliminates the effect of terrain from the field-strength distributions by the use of a terrain clearance correction, leaving as the random distribution only the variations due to local features. (See Part 1 of this report, Section 3.) Thus the standard deviation of the location variations in the BBC method (5 dB) is significantly smaller than any in CCIR Recommendation 370.

2.2. Variation of the Received Signal with Time

The CCIR do not assume any general rule regarding the distribution of signals with time. Instead, Recommendation 370 provides a separate series of curves of signal strength versus distance for selected time-probabilities, namely 1%, 5% (sometimes), 10% and 50%, all drawn for 50% of locations. In the procedures to be described in subsequent sections for calculating the probable interference from several sources, it has been found convenient to assume that signal variations with time are also Normally distributed. This assumption is valid for some sets of the CCIR curves, for instance, the v.h.f. and u.h.f. curves for overland paths. The steps taken to express non-Normal distributions in a form amenable to Normal-distribution calculation are not relevant here, and discussion of these steps is deferred to Section 5.2.

The BBC propagation formula, (Equation (1), Part 1) assumes that field-strength variations with time are Normally distributed, the standard deviation of the variations increasing with distance. The increase is implied by the divergent paths of 1% and 10% curves given in Fig. 6 of that report. For improving the accuracy of interference calculations, which are mainly concerned with high levels of field strength occurring for short periods, the effective median value is derived from the 1% and 10% values, making the assumption of a Normal distribution.

3. PROTECTION AGAINST A SINGLE INTERFERING SIGNAL, CALCULATED BY THE CCIR METHOD

The degree to which a wanted signal is protected against a single interference depends on the offset frequency, that is, the frequency difference between the wanted and interfering carriers, as well as on the ratio of the signal amplitudes at the receiving aerial.

To protect a wanted signal from excessive impairment the CCIR recommend minimum amplitude ratios of the wanted and unwanted signals for different offsets. These are known as 'protection ratios'. For the U.K. 625-line television standard (Standard I) with an offset of two-thirds (or five-thirds) line frequency, the recommended protection ratio is 30 dB, the assumption being made that this figure should be exceeded for all but a small percentage of the time (between 1% and 10%). The actual percentage time for which 30 dB or more is achieved is a measure of the degree of protection realized.

Using this protection ratio, the CCIR method for a single source is described by an example, which illustrates the basic requirements for an interference calculation. Such calculations aim at determining the 'protected field strength', that is, the minimum field strength that will provide the standard of service envisaged. For consistency, the CCIR propagation curves are used in the example.

3.1. Example

The e.o.p. of an interfering u.h.f. transmitter is 100 kW, the transmitting aerial height being 300 m. The distance from the transmitter to the
receiving site to be protected is 300 km over a land path. The offset of the interfering carrier relative to the wanted signal is two-thirds line frequency. The receiving aerial discrimination against the interference is 5 dB. What is the protected field strength for (a) 1% of the time; (b) 10% of the time?

From Figs. 7 and 8 of Recommendation 370 the field strength for an e.r.p. of 1 kW at a distance of 300 km is

\[ 5 \text{ dB}(\mu\text{V/m}) \quad -4 \text{ dB}(\mu\text{V/m}) \]

Add, for the e.r.p. of the interfering transmitter

<table>
<thead>
<tr>
<th>Time percentage</th>
<th>1%</th>
<th>10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Protected field strength</td>
<td>50 dB(\mu\text{V/m})</td>
<td>41 dB(\mu\text{V/m})</td>
</tr>
</tbody>
</table>

The field strengths obtained by these calculations are known as 'protected field strengths', and may be described as 99% (time) or 90% (time) protected field strengths for 50% of locations. They signify (within the limits of accuracy of the CCIR curves) that in an area where the median (location) field strength of the wanted signal is 50 dB(\mu\text{V/m}), the probable duration of the interference will be about 1% of the time; or where the median (location) field strength is 41 dB(\mu\text{V/m}), the probable duration of the interference will be about 10% of the time.

4. MULTIPLE CO-CHANNEL INTERFERENCE

4.1. Probability in Time-location

As may be noted from the above example, the degree of protection realized in an area is specified by time and location probabilities taken as a pair. This dual form of description reflects the situation that at any location in the area, signal amplitudes have a distribution in time, and that at any instant in time over the whole area, signal amplitudes have a distribution with location. However, before questions of protection involving the ratio of two signals are examined, the statistical behaviour of the field strength of one signal will be considered.

The signal variations in time and with location may reasonably be considered as uncorrelated, and two complementary statistical principles may therefore be applied to the situation. These are:

1. The total signal deviation due to both causes is the sum of the signal deviations due to each.

2. The standard deviation of the overall variations is the root-sum-square of the standard deviations of the variations due to the separate causes.

The total signal variations, when one has to consider any location at any instant, may be described as occurring in 'time-location'. If \( \sigma_T \) is the standard deviation of the time variations at a fixed location and \( \sigma_L \) the standard deviation of the location variations at a given moment, the standard deviation \( \sigma_{TL} \) of the variations in time-location is given by

\[
\sigma_{TL} = (\sigma_T^2 + \sigma_L^2)^{1/2}
\]

The datum to which the variations are referred is the median in time of the median location value, usually denoted \( E_{50,50} \). Fig. 1 has been drawn to illustrate the relationship between the variations in time, in location, and in time-location. In the figure \( M \text{ dB}(\mu\text{V/m}) \) is the common median, with

\[
\sigma_T = 6 \text{ dB} \quad \sigma_L = 8 \text{ dB}
\]

It follows then from Equation (1) that \( \sigma_{TL} = 10 \text{ dB} \).

The curves of Fig. 1 represent cumulative distributions. The upper scale gives the percentage probability (100 P) that the signal level indicated by the ordinate will not be exceeded, while the lower scale gives the complementary percentage probability 100(1 - P) that the level will be exceeded. These two aspects of the probability distributions will be referred to as protection probability and interference probability.

To examine more closely how probabilities in time, location, and time-location are related, consider a signal deviation of any value, say +8 dB from the median. At a median location, i.e. a location at which the field strength has a value just sufficient to exceed that at 50% of locations, a value of \( (M + 8) \text{ dB} \) would be attributed entirely to time variations; such a value is exceeded for only 15.9% of time in Fig. 1. Taking this as the field strength due to an interfering signal, and assuming that \( (M + 8) \text{ dB} \) is the critical level for interference, the interference probability is 15.9%. It may therefore be said that for 50% of locations the time probability of protection is at least 84.1%. Equally, an instant of time may be chosen at which the time variations of the signal are at their median value.
In this case a field strength of \((M + 8)\) dB would arise entirely from location variation, and would not be exceeded at 90-7% of locations. In general, the -8 dB deviation may be attributed in part to time variations and in part to location variations. For example -6 dB for time and -2 dB for location variations gives protection probabilities, from Fig. 1, of 77.3% (time) and 63% (location). This means that 63% of locations have a field strength less than \((M + 8)\) dB for at least 77.3% of the time, and because the field strength considered is that of an interference we say that 63% of locations are protected for 77.3% of the time. A different pair of protection levels will be obtained by dividing the total deviation in a different way.

The overall deviation of \(+8\) dB considered in these examples represents a single level of protection, and the various pairs of time and location probabilities which may be derived from it also express this single level of protection. This level is also represented by a single probability in time-location, seen in Fig. 1 to be 78.8%. Thus any given probability in time-location represents an infinite number of pairs of time and location probabilities, all of which signify the same level of protection. This point is clarified in Table 1, which is based on Fig. 1 and records the probabilities for time and location deviations taken in steps of 2 dB. It is clear that the pairs of probabilities shown in the table are examples of an infinite number of possible pairs.

In the procedures used for summing interferences in the time-location method, the time and location variations are compounded into a single variation in time-location, and calculations are carried out in this form. The reverse sequence, from time-location to time and location may be carried out to express the final result. In this way the level of protection may be reduced to the separate probabilities in time and location to conform with established habits of thought.

The reverse sequence requires a known probability and a known standard deviation in time-location and additionally, the standard deviation of one of the separate distributions in time or location. The standard deviation of the other distribution can be found from Equation (1). In terms of Fig. 1 a knowledge of the time-location distribution and one of the other distributions enables the third distribution to be deduced; or what is more relevant to the interference problem, a knowledge of the time-location probability and standard deviation, together with either the time or location probability and standard deviation, enables the third probability to be calculated.

4.2. Correlation between Signals

As mentioned in Section 2.1 signal variations with changes of location are of the same order of magnitude, whether the transmitter is ten or a hundred kilometres from the receiving site; hence, the CCIR curves for location variations are independent of distance. An experiment\(^5\) has shown that when two signals are present in an area, the correlation coefficient for their location variations depends on the angular difference between the directions of the signals, the results obtained being a coefficient of 0.78 for 0\(^\circ\) and 0.36 for 180\(^\circ\) angular difference. For the purpose of calculation, 0.5 was considered a sufficiently good approximation to represent the location correlation coefficient in all cases. If at some later date it appears desirable to use instead a coefficient depending on angular separation of the signal directions, a linear interpolation between the extreme values would provide a useful first approximation.

The degree of correlation\(^5\) existing between the time variations of two signals depends on the period over which the signals are averaged. Short-term variations of two signals traversing different paths are not well correlated. On the other hand, correlation approaches unity for long-period mean values, such as, for instance, monthly means. For mean values taken over shorter periods, the correlation coefficient will have some value between zero and unity.

A coefficient of 0.5 is used also in the calculations for the time variations. This value resulted\(^5\) from an analysis of hourly medians of signal strength. A different coefficient would have been obtained had a different period been chosen, but the hour-to-hour variations are generally larger in extent than variations within the hour, so that the main effects of correlation should be covered by taking this value.

The protection achieved against any one interference of given offset is dependent on the difference (in decibels) between the median values of the wanted and unwanted signals and on their relative variation. The protection achieved has therefore a distribution equal to the difference between two Normally distributed signals, and is itself Normally distributed.\(^5\) When there is some correlation between the signals, the standard deviation of the difference distribution\(^6\) is given by

\[
\sigma = (\sigma_1^2 - 2\rho\sigma_1\sigma_2 + \sigma_2^2)^{1/2}
\]  

where \(\sigma\) is the standard deviation of the difference distribution,

\(\sigma_1, \sigma_2\) are the standard deviations of the two signals and \(\rho\) is the coefficient of correlation between the signals.

This equation gives the variation of one signal with respect to the other. It permits one signal to be treated as constant at the median value and the other as having the whole of the variation. In the calculating procedure, the wanted field strength is taken as the constant signal.
**Fig. 1 - Distribution of a signal in time, location and time-location**

**TABLE 1**

<table>
<thead>
<tr>
<th>Total deviation dB</th>
<th>Time-location protection probabilities, %</th>
<th>Separate time and location protection probabilities, %</th>
</tr>
</thead>
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<tr>
<td></td>
<td>T L T L T L T L T L T L T L</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>50.0 50.0 50.0</td>
<td>50.0 50.0 50.0 50.0 50.0 50.0 50.0 50.0 50.0</td>
</tr>
<tr>
<td>-2</td>
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</tr>
<tr>
<td>-4</td>
<td>65.5 74.7 63.0</td>
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</tr>
<tr>
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<td>72.6 84.1 74.7</td>
<td>69.1 74.7 69.1 69.1 74.7 69.1 69.1 69.1 69.1</td>
</tr>
<tr>
<td>-8</td>
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<td>90.9 90.9 90.9 90.9 90.9 90.9 90.9 90.9 90.9</td>
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<tr>
<td>+10</td>
<td>84.1 95.2 90.9</td>
<td>84.1 84.1 84.1 84.1 84.1 84.1 84.1 84.1 84.1</td>
</tr>
<tr>
<td></td>
<td>89.4 50.0</td>
<td>89.4 50.0 89.4 50.0 89.4 50.0 89.4 50.0 89.4 50.0</td>
</tr>
</tbody>
</table>
4.3. Protection in Time-Location against a Single Interfering Signal Partially Correlated with the Wanted Signal

In a Normal distribution the probability associated with a deviation \( x \) from the median is a function of \( x / \sigma \), where \( \sigma \) is the standard deviation of the distribution. Thus a value exceeded with a probability of 1% has a deviation of \( +2.33\sigma \), and that exceeded with a probability of 10%, a deviation of \( +1.28\sigma \). The interval between 1% and 10% is therefore 1.05\( \sigma \).

The 1% and 10% (time) interfering field strengths in the example in Section 3.1 are 5 and -4 dB(\( \mu \)V/m) respectively. These relate to 50% of locations. The standard deviation of the time variations of the unwanted signal (\( \sigma_{TU} \)) is therefore given by

\[
1.05\sigma_{TU} = 9 \text{ dB}
\]

i.e. \( \sigma_{TU} = 8.57 \text{ dB} \)

Since Normal distributions are assumed throughout these calculations, the deviation of the 1% (time) unwanted field strength from its median value is given by

\[
E_{1,50,U} - M_U = 2.33 \times 8.57 \text{ dB} \approx 20 \text{ dB}
\]

where \( E_{1,50,U} \) and \( M_U \) are respectively the 1% (time), 50% (location) and 50% (time), 50% (location) field strengths of the unwanted signal. Since \( E_{1,50,U} = 5 \text{ dB} \), it follows that

\[
M_U = -15 \text{ dB}(\mu \text{V/m})
\]

Consider next the effect of the (generally small) variations of the wanted signal. Let it be assumed that the receiving location for which the probability of protection is required is 50 km from the wanted station, whose transmitting aerial height is 300 m and whose e.r.p. is 1 kW. The 1% (time) and 10% (time) field strengths are given by the CCIR curves\(^4\) as 50 and 47.5 dB(\( \mu \)V/m). From these the median and standard deviation of the wanted signal are derived. Thus if \( \sigma_{TW} \) is the standard deviation of the time variations of the wanted signal

\[
1.05\sigma_{TW} = 2.5 \text{ dB}
\]

\( \sigma_{TW} = 2.38 \text{ dB} \)

The standard deviation of the time variations of the difference signal (\( \sigma_T' \)) is obtained by applying Equation (2).

\[
\sigma_T' = (\sigma_T^2 - 2\rho\sigma_T\sigma_{TU} + \sigma_U^2)^{1/2} \quad (3)
\]

Putting \( \rho = 0.5, \sigma_{TW} = 2.38 \text{ dB} \) and \( \sigma_{TU} = 8.57 \text{ dB} \)

\[
\sigma_T' = 7.65 \text{ dB}
\]

The standard deviation (\( \sigma'_L \)) of the location variations of the difference signal is calculated in the same way. Let the standard deviation of the location variations in this example be 5 dB, the value assumed in the BBC method of calculation. This figure is independent of distance, and is therefore applicable to both the wanted and unwanted signals. Writing

\[
\sigma'_L = (\sigma_L^2 - 2\rho\sigma_L\sigma_U + \sigma_U^2)^{1/2} \quad (4)
\]

it is easily verified that, for a correlation coefficient of 0.5

\[\sigma'_L = 5 \text{ dB}\]

The standard deviation of the variations of the difference signal in time-location is obtained by applying Equation (1)

\[
\sigma_{TL} = (\sigma_T^2 + \sigma_L^2)^{1/2} \quad (5)
\]

\[\sigma_{TL} = 9.1 \text{ dB}\]

With these data the protection probabilities in time-location, in time, and in location may now be calculated. Since only one interference is assumed, the median \( M_U \) is common to all three distributions. To provide a direct comparison with the example in Section 3.1, let the field strength to be protected be 50 dB(\( \mu \)V/m). Using the ratios of the previous example:

\[
M_U, \text{ the unwanted median for an e.r.p. of 1 kW} \quad -15 \text{ dB}(\mu \text{V/m})
\]

Add, for transmitter e.r.p. of 100 kW \[20 \text{ dB}\]

Add, for protection ratio (2/3-line offset) \[30 \text{ dB}\]

Deduct, for receiving aerial discrimination \[5 \text{ dB}\]

\[M', \text{ the 50\% protected field strength} \quad 30 \text{ dB}(\mu \text{V/m})\]

Excess protection for a protected field strength of 50 dB(\( \mu \)V/m) \[20 \text{ dB}\]

The protection probability in time-location related to this excess protection is a function of the ratio

\[
x = \frac{20}{9.1} \approx 2.2
\]

and is found from statistical tables to be 98.61%.
To resolve this protection into pairs of time and location probabilities, the standard deviation of one member of the pair must be known. It is usual, in evaluating the protection achieved at a site, to standardize the location probability at some figure like 50% or 70%, and to examine the variations in time probability with changes of protected field strength. The standard deviation of the location variations is therefore taken as the known member of the pair. Since \( \sigma_L^2 = 5 \) dB, it follows from what has gone before that \( \sigma_T^2 = 7.65 \) dB.*

For comparison with the example of Section 3.1, protection at 50% of locations is required, and therefore the whole of the excess protection is a deviation in time. The time probability is obtained from the ratio 20/7.65 and found from statistical tables to be 99.56%. The time-probability in the example of Section 3.1 is 99%, the increased protection in the present calculation being due to the inclusion of correlation.**

As another example, a calculation will be made for the BBC standard of protection, in which the location probability is 70%. The deviation for this probability is 0.0524 m which, in the present example, is 2.6 dB. The remaining part of the total deviation, namely 17.4 dB, is due to the time variations. The related protection probability in time is derived from the ratio 20/7.65 and is 98.86%. The two pairs of probabilities obtained from these calculations, namely 99.56% (time), 50% (location), and 98.86% (time), 70% (location) represent the same level of protection.

It should be noted that the protected field strength for which the calculation was made was arbitrarily chosen. If the protection probabilities resulting from a calculation at one value of protected field strength are considered too low (or too high) for the grade of service envisaged, the calculations are repeated for a higher (or lower) value until the required standard is met. The protected field strength is thus the minimum wanted field strength which satisfies the required standard of service.

5. MULTIPLE INTERFERENCE CALCULATIONS BY THE BBC METHOD

5.1. Probability Multiplication

When several interfering signals are present in the region of the receiving aerial, their 1%- and 10%-time field strengths are calculated as described in Part 1 of this report; the median value and standard deviation of each are then derived. The standard deviations of the differences (in decibels) between the wanted and each unwanted signal, in time and location, are next calculated, according to Equations (3) and (4) above. From these the standard deviation of each difference in time-location \( (\sigma_T L) \), is derived, using Equation (5).

The median value of each interference is adjusted by the appropriate protection ratio, depending on its offset frequency relative to the wanted signal, and on its direction of arrival relative to the orientation of the receiving aerial. The resulting median, given the symbol \( M' \), is the 50%-time, 50%-location protected field strength, applicable to each interference in the absence of the others.

The quantities \( M' \) and \( \sigma_T L \) thus represent each of the interfering signals in time-location. The method adopted for combining these distributions to obtain the field strength protected against multiple interference is illustrated in Fig. 2, where four such distributions are shown. A field strength is chosen for which the protection probability against multiple interfering sources is required; the protection probabilities relating to the individual interferences at this level are read from the figure* and multiplied together. The result is taken to be the probability that the chosen field strength will be protected against all the interfering sources. For instance, suppose 60 dB(\( \mu \)V/\( m \)) is chosen as the field strength for which the protection probability is required. In Fig. 2 the individual protection probabilities for this field strength are 99.995%, 99.91%, 99.88% and 99.50%. Their product is 99.29%. This gives the probability in time-location that 60 dB (\( \mu \)V/\( m \)) is protected against all the interferences. The upper curve in Fig. 2, which is the locus of protection probabilities calculated in this way for a range of field strengths, is called the probability product curve. Since it is not a straight line the distribution of the probability product is not Normal.

The significance of this operation should be noted. The probability product gives only the probability that no individual signal exceeds in level the protected field strength being investigated. It neglects any possibility that the signals may add together to produce a greater degree of interference than each separately. When more than one interfering signal is present at a time, it may be expected that they do add in some fashion, and that if a number of them hover at a level just below the threshold of protection, their sum would occasionally cross this threshold. Probability multiplication disregards this joint contribution completely, and

* This value is of course known from Equation (3) but is derived merely by way of example, in preparation for the corresponding less straightforward calculations for multiple sources (see Section 3).

** Correlation is more fully discussed in the Appendix.
hence, it may be argued, underestimates the probability of interference. For signals subject to marked fading the error will be small; if the probability of interference by each signal in its own right is small, the probability of both being sufficiently high simultaneously will be very small.

5.2. Non-Normal Distributions

The operations described in Section 4.3 are rigorous when applied to Normal distributions, but non-Normal distributions also occur in problems of co-channel interference, as seen in Fig. 2. In order that such distributions may be amenable to the calculation procedures described, a form of approximation is used which is best explained by reference to Fig. 3. The curve of this figure, which is plotted on probability paper for a Normal distribution, has the general shape of the non-Normal distributions occurring in multiple interference problems. For the purpose of calculation such a curve is divided into segments of roughly equal interval along the ordinate and the chord of each segment taken as an approximation to the curve. Each chord is then regarded as part of a Normal distribution, having a hypothetical median value, and a standard deviation (in time-location) indicated by the slope of the chord. The segment between the signal levels* 

\((M + 5.5)\) and \((M + 8.2)\) \(\text{dB(μV/m)}\) in the figure is approximated by the chord extended to cut the 50% ordinate at \(M_a\); the segment between \((M + 3.0)\) and \((M + 5.5)\) \(\text{dB(μV/m)}\) is approximated by the chord extended through \(M_b\); the segment between \((M + 0.7)\) and \((M + 3.0)\) \(\text{dB(μV/m)}\) by the chord extended through \(M_c\); and the segment between \((M - 1.2)\) and \((M + 0.7)\) \(\text{dB(μV/m)}\) by the chord extended through \(M_d\). The signal levels \(M_a, M_b, M_c\) and \(M_d\) are used as the median levels for calculations relating to the respective segments. Suppose, for instance, that a calculation is being made with \(M_a\) as the median. This calculation is taken as valid only if the deviation and probabilities resulting from it lie within the limits of the first segment. Similarly, only those values of deviations and probabilities that lie within the limits of the second segment are taken as valid in calculations where \(M_b\) is used as the median; and so on. Thus the magnitude of the error

* The significance of these divisions becomes apparent in Section 5.4.
arising from this approximation depends on the extent to which the segments of the distribution curve depart from linearity on a Normal-distribution scale. This error can of course be reduced to any acceptably low figure by reducing sufficiently the interval over which the segment is taken.

The procedure for deriving the median value of a Normal distribution from the 1% and 10% probability field strengths, demonstrated in Section 4.3, is thus applicable to non-Normal distributions provided that the deviations and probabilities of interest in the problem lie in the interval between the given value.

5.3. Nearly Continuous Interference

An inevitable consequence of the large number of relay stations envisaged for the u.h.f. services is that many will be situated close to each other and close to areas covered by main transmitters. Fading over short distances is shallow, with only a few decibels between the field strengths exceeded for small and for large percentages of the time. Co-channel interference over such paths may be regarded as virtually continuous, and the problem of protecting wanted services against this kind of interference has become as important as protecting against the short-duration interference typical of long paths.

To take this into account the BBC method assumes that virtually continuous interference requires 10 dB* more protection at median level (i.e. for 50% (time)) than that normally given for 90% to 99% (time). If the fading range of an interfering signal is not large enough to provide this increase in protection, the protection ratio recommended by the CCIR for short-duration interference should be suitably increased. In each case of this sort, therefore, the protection ratio is increased by the number of decibels by which the fading range is deficient. If, for instance, the 50% to 95% fading range of an interference is 6 dB, the protection ratio required in calculating the 95% time protected field strength for that interference is increased by 4 dB.

* There is experimental evidence for this assumption.

Fig. 3 - Division of a non-Normal distribution into segments
When the interfering signal has a small fading range, the distribution curve for the protected field strength related to it would, if plotted in the manner of Fig. 2, appear as a straight line of small gradient but relatively high median value. The presence of such a distribution in a group whose probability product is taken has the effect of raising the level of the probability product curve at the 50% and lower probability levels without affecting the curve at the levels where the protection probability is high.

5.4. Conversion of Protection in Time-Location to Time and Location Probabilities.

The probability product of the individual time-location distributions, plotted as in Fig. 2 is one form of expressing the protection available at different field strengths. The protection calculated in time-location may however be reduced to time and location probabilities, to conform with the practice of specifying protection in these terms.

As already stated the probability-product curve of Fig. 2 represents a non-Normal distribution which is typical of all probability products. To enable the use of methods of calculation applicable to Normal distributions, approximation by a series of chords, as described in Section 5.2, is adopted.

In the BBC method the probability product is divided into four parts comprising the probability intervals

a: 95% to 90%
b: 90% to 82%
c: 92% to 70%
d: 70% to 56%

The ordinate intervals resulting from this division of probabilities are approximately equal. When the probability-product curve of Fig. 2 is divided in this way, the four resulting Normal distributions are defined by the medians and standard deviations given in Table 2.

To reduce the protection probabilities in time-location to time and location probabilities, the standard deviation of one or other of the separate distributions must also be known. The location distribution is treated as the known distribution (Section 4.3) but the standard deviation requires derivation, since it will not, in general, have the value of 5 dB taken for a single interference.

For this purpose the interfering signals are assumed not to fade, that is to say, only location variations are assumed to exist. That is equivalent to putting $\sigma_T = 0$ in Equation (9). When this is done the four distributions depicted in Fig. 2 appear as in Fig. 4 with median values ($M_1$, $M_2$, etc.) unchanged but with standard deviations equal to 5 dB (see Section 2.1). The probability product of these distributions gives the probability of protection against the multiple effect of these four interferences in the absence of fading. The difference between this result and that for the time-location distributions can therefore only be due to the time variations of the signals.

### TABLE 2

<table>
<thead>
<tr>
<th>Median dB ($\mu$V/m)</th>
<th>Standard deviation, dB</th>
<th>Probability interval over which the values are valid %</th>
</tr>
</thead>
<tbody>
<tr>
<td>a 39.2</td>
<td>8.3</td>
<td>95 to 90</td>
</tr>
<tr>
<td>b 40.0</td>
<td>7.7</td>
<td>90 to 82</td>
</tr>
<tr>
<td>c 40.9</td>
<td>6.7</td>
<td>82 to 70</td>
</tr>
<tr>
<td>d 41.1</td>
<td>6.4</td>
<td>70 to 56</td>
</tr>
</tbody>
</table>

These distributions are shown as dashed lines in Fig. 2.

If this probability-product curve (Fig. 4) is divided into segments for the purpose of deriving medians and standard deviations in the way described for the time-location probability product, the medians obtained would not agree with the time-location medians.

The median obtained when location variations alone are considered will have a higher value than the median derived by taking account of the variations in time and location, since the combined variations have the greater standard deviation.

At the time when this difficulty arose it appeared that the location standard deviation of the multiple interference would best be deduced by the following procedure.

For each segment on the time-location curve of Fig. 2, the mid-value of field strength is taken as representative of the segment. The location probability for this field strength is read from the location probability-product curve of Fig. 4. Using the median associated with the segment of interest in Fig. 2 (listed in Table 2) the standard deviation giving this probability is calculated. This is the location standard deviation for all the protected
field strengths within the limits of the segment considered.

The values obtained for the example shown in Figs. 2 and 4 are tabulated below.

The standard deviations in this example are all less than 5 dB, and this is indeed so in practice in the vast majority of calculations made. Occasionally, however, a standard deviation exceeding 5 dB has been obtained. For this to occur, the time variations of at least one of the interferences must be very large. Its distribution, plotted as in Fig. 2 would be very steep, and the probability-product curve would be almost coincident with it at high dB levels.

TABLE 3
Calculation of Location Standard Deviation

<table>
<thead>
<tr>
<th>Mid-value of field strength in each segment of the time-location probability-product curve dB(\mu V/m)</th>
<th>Probability range covered by the segment, %</th>
<th>Time-location median dB(\mu V/m)</th>
<th>Probability on location probability-product curve %</th>
<th>Location standard deviation, dB</th>
</tr>
</thead>
<tbody>
<tr>
<td>51.3</td>
<td>95 to 90</td>
<td>39.2</td>
<td>99.75</td>
<td>4.3</td>
</tr>
<tr>
<td>48.4</td>
<td>90 to 82</td>
<td>40.0</td>
<td>98.40</td>
<td>3.9</td>
</tr>
<tr>
<td>45.75</td>
<td>82 to 70</td>
<td>40.9</td>
<td>95.50</td>
<td>2.8</td>
</tr>
<tr>
<td>43.25</td>
<td>70 to 56</td>
<td>41.1</td>
<td>88.00</td>
<td>1.8</td>
</tr>
</tbody>
</table>

The location distributions derived here are shown as dashed lines in Fig. 4.

Fig. 4 - Probability multiplication of location distributions
protection probabilities. The median associated with the segment in the probability-interval 95% to 90% would be low, and could cause the location standard deviation derived from it to exceed 5 dB. For these cases the computer programme has been adjusted to limit the location standard deviation to a maximum of 5 dB.

There is practical justification for this limit. When the group of time-location distributions whose product is taken includes one that is outstandingly steep when plotted as in Fig. 2, the other distributions at the high protection probabilities hardly affect the probability product, which is then virtually coincident with the steep distribution. The location variation content of this distribution has a standard deviation of 5 dB and it is therefore logical that the probability-product curve in this region should also have a standard deviation of 5 dB.

With the standard deviations of the location and time-location distributions available (Tables 2 and 3) the standard deviation of the time variations is calculated using Equation (5). From this the time-probability of protection at any field strength may be determined. The figure thus obtained relates to 50% of locations.

For a higher location probability, the increased protection required is calculated from the derived location standard deviation, using the appropriate factor (obtained from statistical tables) for the probability chosen. For protection at 70% of locations the factor is 0.524 and the number of decibels to be added to the 50% (location) field strength to obtain the increased protection is 0.524σL, where σL is the derived standard deviation (as, for instance, one of the four values in Table 3).

The approximation described for deriving the location standard deviation is not as satisfactory as one would wish. Although the error arising out of its use is not expected to be large, because the standard deviations of the time variations with which the location standard deviations are combined are usually very much greater, a simpler method is being considered as an alternative.

In the alternative method the probability product of the location distributions is derived as shown in Fig. 4. This is then divided into segments, and the chords of the segments taken. From the slopes of the chords are derived the location standard deviations for the probability intervals covered by the segments. Using these standard deviations with the respective standard deviations derived from the time-location probability-product curve, the standard deviations of the time variations — and hence the protection probability in time and location — may be derived.

6. DISCUSSION

The method developed by the BBC for estimating protection against multiple co-channel interference has been described, in the course of which various approximations have been introduced. The more important of these are now considered.

Two of the approximations made are such that one tends to overestimate and the other to underestimate the protection probability.

The overestimate results from the use of probability-multiplication, which ignores the effect of signals adding together when they occur simultaneously. If it is assumed that the effect of several signals occurring together is equivalent to that of a single signal of the same total power, signals may be combined by the method of convolution. This is a cumbersome method to apply in practice, but a few examples using signal distributions typical of long-range interference were calculated by this method and the method of probability-multiplication by way of comparison. The difference between the results was found to be negligible. It is believed that in practical cases the error will be less than 1 dB in the calculated protected field and, since probability-multiplication is much easier to carry out, is preferred.

The underestimate of the protection probability occurs because the variations of the individual interfering signals are assumed to be uncorrelated. This assumption is not entirely supported by fact, since experience indicates that there is partial correlation between the time variations of unwanted signals. Correlation between interfering signals increases the frequency with which they overlap in time, thus reducing the total duration of the interference. The probability of protection is thus greater in reality than in the estimate. The magnitude of this error in terms of the correlation coefficient is examined in the Appendix. In terms of signal strength it is expected to be of the order of 1 or 2 dB.

A third approximation to be noted is the method of obtaining the location standard deviation when reducing the probability product to separate time and location probabilities (Section 5.4). This is perhaps the least justifiable of the procedures used in the BBC method, and a less cumbersome alternative is being considered for adoption, but some estimate of the possible error it may introduce is however relevant.

The derived location standard deviation can have a value only in the range 0 to 5 dB (Section 5.4). In the presence of a standard deviation of the time variations equal to, say, 15 dB (a reasonable practical value) the time-location standard deviation
is 15 dB when the location standard deviation is 0, and 15.8 dB when it is 5 dB. As the error in the location standard deviation will never be the maximum, that is, the true value will never be 5 dB when the calculated value is 0, or vice versa, the error arising from this approximation is also seen to be small.

7. CONCLUSION

It is believed that the method described for interference calculations gives a reasonably reliable indication (within the accuracy of the propagation curves) of protected field strengths for the planning of television services. It is, however, proposed to review the method from time to time, and to adopt any simplifications or ways of improving its accuracy that experience might suggest.

8. REFERENCES


APPENDIX

Correlation between Interferences

The effect of taking correlation into the interference calculation, as described in Section 4.2, is to establish the distribution of the difference (in decibels) between the wanted signal and each interference. In the subsequent calculations, these separate distributions are treated as uncorrelated, any correlation between interferences being considered as taken into account through the correlation of each with the wanted signal. It is nevertheless of interest to examine the correlation between interferences implied by this procedure.

A simple approximate analysis of the problem is possible if only two interferences are assumed.

Let $\sigma_A$ be the standard deviation of the wanted signal, and $\sigma_B$ and $\sigma_C$ the standard deviations of the two interferences.

Let $\rho_{AB}$ and $\rho_{AC}$ be the coefficients of correlation between the wanted signal and each of the interferences, and let $\rho_{BC}$ be the coefficient of correlation between the interferences.

The relative variation of interference B with respect to the wanted signal, expressed by $\sigma_{AB}$, is

$$\sigma_{AB} = (\sigma_A^2 - 2\rho_{AB}\sigma_A\sigma_B + \sigma_B^2)^{1/2} \tag{1}$$

Similarly

$$\sigma_{AC} = (\sigma_A^2 - 2\rho_{AC}\sigma_A\sigma_C + \sigma_C^2)^{1/2} \tag{2}$$

The multiple interference calculations assume zero correlation between the distributions $AB$ and $AC$. The relative distribution of one of these distributions with respect to the other, expressed by $\sigma(AB)(AC)$, is

$$\sigma(AB)(AC) = \left[ (\sigma_A^2 - 2\rho_{AB}\sigma_A\sigma_B + \sigma_B^2) + (\sigma_A^2 - 2\rho_{AC}\sigma_A\sigma_C + \sigma_C^2) \right]^{1/2} \tag{3}$$

But $\sigma(AB)(AC)$ is also the standard deviation of the relative distribution of one interference with respect to the other i.e.

$$\sigma_{BC} = \sigma(AB)(AC) \tag{4}$$
Now
\[ \sigma_{BC} = (\sigma_B^2 - 2\rho_{BC}\sigma_B\sigma_C + \sigma_C^2)^{1/2} \]  \hspace{1cm} (5)

This leads to
\[ -2\rho_{BC}\sigma_B\sigma_C = 2\sigma_{BC} - 2\rho_{AB}\sigma_A\sigma_B - 2\rho_{AC}\sigma_A\sigma_C \]  \hspace{1cm} (6)

To reduce the number of independent variables in this expression,

\[ \text{let } \sigma_B = \sigma_C \text{ and } \sigma_B / \sigma_A = r > 1 \]

Then
\[ \rho_{BC} = \left[ \frac{1}{r} \left( \rho_{AB} + \rho_{AC} \right) - \frac{1}{r^2} \right] \]

Putting \( \rho_{AB} = \rho_{AC} = 0.5 \), the value used in the interference calculations

\[ \rho_{BC} = \left[ \frac{1}{r} - \frac{1}{r^2} \right] \]

Some values of \( \rho_{BC} \) for various values of \( r \) are tabulated below. This table is valid for two interferences having the same standard deviations, and for a correlation coefficient of 0.5 between the wanted signal and the interferences.

<table>
<thead>
<tr>
<th>( r )</th>
<th>( \rho_{BC} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0.250</td>
</tr>
<tr>
<td>4</td>
<td>0.167</td>
</tr>
<tr>
<td>6</td>
<td>0.139</td>
</tr>
<tr>
<td>8</td>
<td>0.109</td>
</tr>
<tr>
<td>10</td>
<td>0.099</td>
</tr>
</tbody>
</table>

Thus the correlation between interferences implied by the procedure adopted in the multiple-interference calculations has a small positive value.

An analysis of measurements made over two paths of about 130 km each in length resulted in a correlation coefficient of 0.85. The ratio of the standard deviations of the wanted signal to the interferences for this length of interference path (as implied in the c.c., propagation curves of Part 1 of this report) is of the order of 2.5, a value which suggests, on reference to the above table, that the 'reflected' correlation coefficient would be about 0.24 if the experimental paths were treated as interference paths in a calculation.