RESEARCH DEPARTMENT

Optimum colour analysis characteristics for colour television picture sources with three receptors

RESEARCH REPORT No. T-175

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THE BRITISH BROADCASTING CORPORATION
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SUMMARY

The colour fidelity of a colour television signal source may be substantially improved if a linear matrix is included in the signal chain, moreover this improvement is greatest when the optical colour analysis characteristics of the source are chosen together with the matrix coefficients. This report describes computations made to determine the optimum practicable analyses and matrices for various signal sources that employ three receptors. The characteristics determined are shown to give an improvement not only in colour fidelity but also in noise performance, when compared with the characteristics giving best colour fidelity when a matrix is not permitted.

1. INTRODUCTION

For accurate reproduction of colour within the gamut of the primaries of a system of additive synthesis such as colour television, it is necessary that the signals controlling the reproducing primaries be derived using particular spectral sensitivities. The characteristics shown in Fig. 1, for instance, define the analysis necessary for accurate colour reproduction when the standard NTSC primaries are used.*

The ideal analysis characteristics are very difficult to realize in practice, however, because they have subsidiary positive and negative lobes, and because a high sensitivity is required for both the "red" and the "green" colour channels in the spectral region where the major lobes of the corresponding characteristics cross. Thus a completely accurate instrumentation of the ideal analysis characteristics would be difficult to achieve, and would involve a very inefficient photoelectric conversion.

The normal method of signal generation is by means of three receptors only, one for each colour channel. Such an arrangement is bound to involve errors of colour reproduction, because the analysis characteristics of the system can then have only positive lobes.

It was at one time thought that when analysis characteristics having only positive values were permissible they should coincide with the major positive lobes of the ideal analysis.[1] It was later realized that some advantage would result from using characteristics obtained by trimming the extremities of the major positive lobes.[2] An investigation to determine the effect of modifying the curve shapes led to the publication in BBC specifications TV/126 and TV/148 for television cameras of a set of optimum positive-only analysis characteristics for NTSC primaries. They are reproduced here as Fig. 2.

Although these characteristics give an improved colour fidelity as compared with the broader ones suggested by the major positive lobes of the ideal analysis, the improvement is gained at the expense of optical efficiency, and a degraded sensitivity is thereby incurred.

The accurate reproduction of colours portrayed by cine-film presents a somewhat different problem, since both the colour gamut and also the spectral transmission characteristics of cine-film are limited as compared with those encountered in natural scenes. Thus in the evaluation of a set of optimum positive-only analysis characteristics for the reproduction of film, attention can be restricted to a relatively limited range of colours. This subject has been investigated,[3] the additional restriction to analysis curves imposed by the flying-spot cathode-ray tube and photomultiplier characteristics

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* Note that the mode of operation of a colour television picture source effectively controls the way in which its analysis characteristics are normalized by adjustment of the channel gain controls. For instance a flying spot scanner whose colour channels are balanced to give equal outputs with no film in the gate has analysis characteristics that are in effect normalized so that the areas under the three curves are equal. A camera operating in a studio illuminated by a 3000°K source and adjusted to give equal outputs when pointed at a white card has analysis characteristics that are effectively normalized with respect to illuminant 3000°K (i.e. to give equal areas when multiplied by the spectrum of $E_{3000^\circ K}$). These differences in normalization were taken into account in the calculations to be described; however the resulting analysis curves have all been normalized with respect to illuminant C so that they may readily be compared with one another.

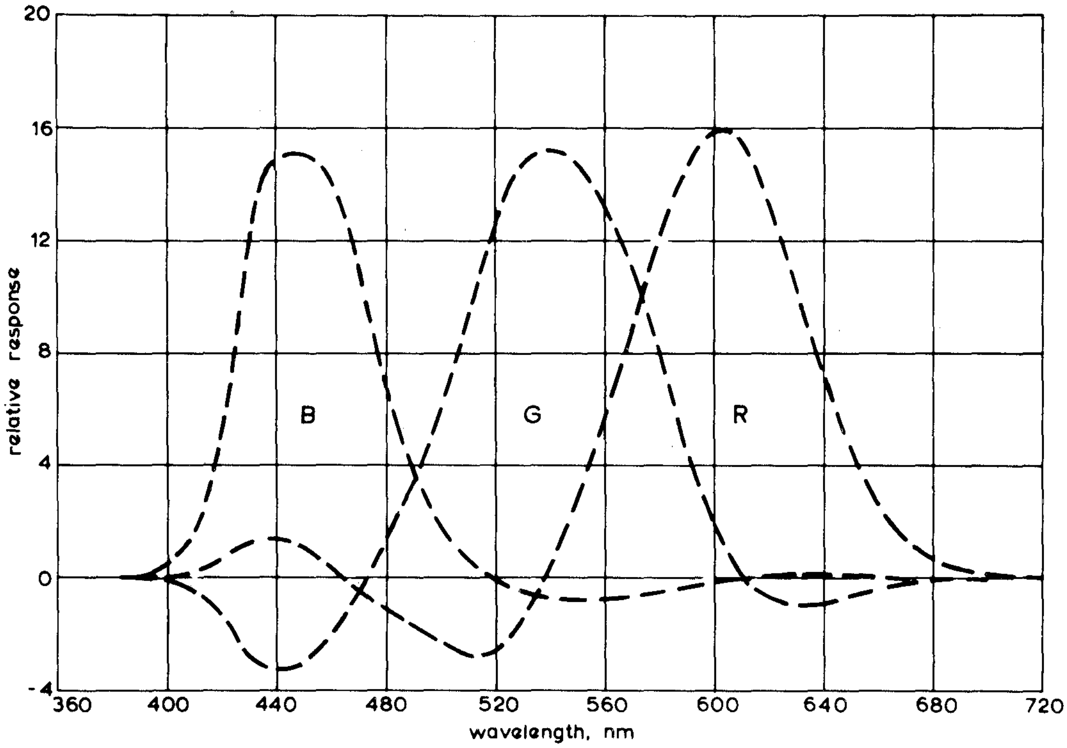

Fig. 1 - Ideal analysis characteristics for NTSC primaries

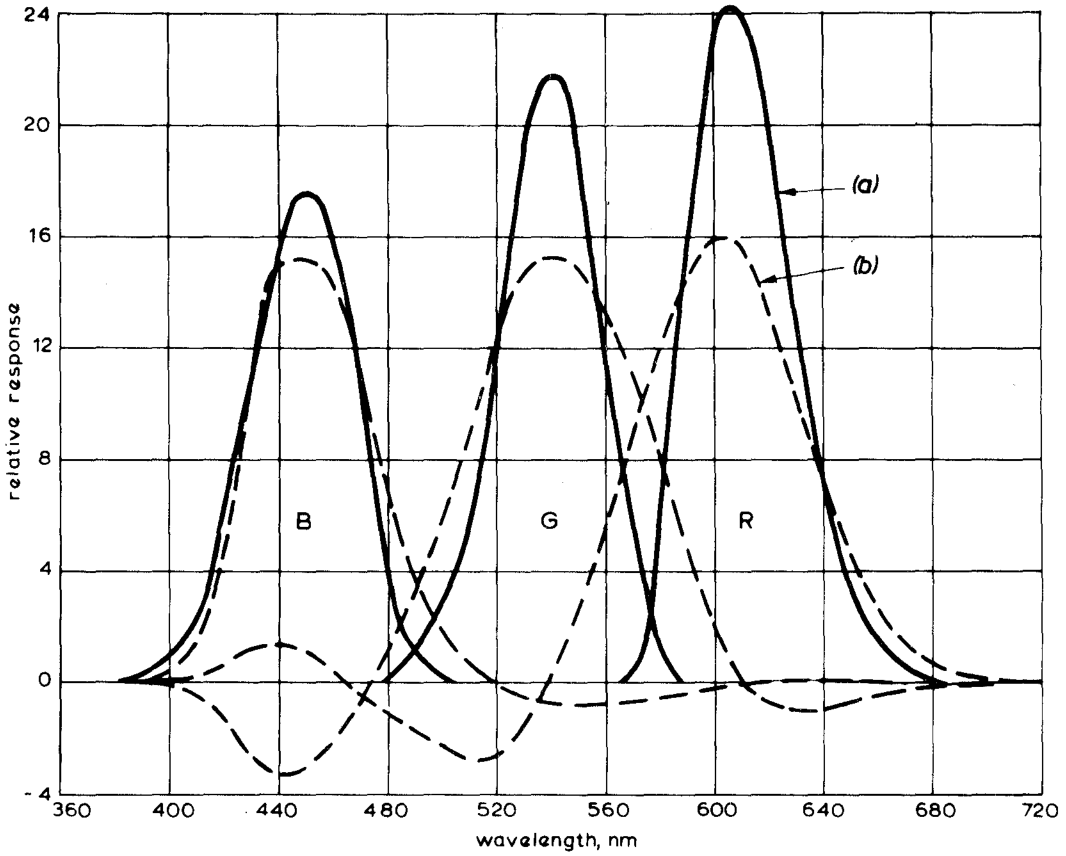

Fig. 2 - Comparison of analysis characteristics specified in TV 126, and TV 148 and ideal analysis characteristics for NTSC primaries

(*a*) TV Specification (*b*) Ideal analysis being included in the calculation, and the resulting optimum analysis characteristics are given as Fig. 3. (It appears possible that by processing the three colour signals in a manner analogous to photographic masking one may obtain from a film scanner a more accurate representation of the colours existing in the original scene than can be obtained by optical projection of the positive point. The development of the characteristics shown in Fig. 3 and of those described in this report did not, however, envisage the use of this technique.)

It has for a long time been realized[4] that the analysis obtained using three receptors only could be improved if a linear matrix were included in the circuitry at some point at which the three colour signals have magnitudes that are proportional to the incident light flux. For example, the ideal "green" analysis characteristic has a negative lobe peaking at about the same wavelength (448 nm) as does the major positive lobe of the ideal "blue" characteristic. Thus a receptor designed to produce the major portion of the "blue" signal could be arranged to feed a small amount of negative signal into the "green" channel; this would be equivalent to introducing a negative lobe into the characteristic of the "green" channel. Recent work[5] has shown that the colour fidelity of a currently available television camera may be appreciably improved by means of a linear matrix, and moreover the deterioration in signal-to-noise ratio incurred by insertion of the device is small.

The conclusion of the above work left unanswered the question whether a further significant improvement might be possible if one were free to adjust not only the constants of the linear matrix but also (within practicable limits) the transmission characteristics of the optical system by means of which the three input signals to the matrix are derived. This report outlines the method by which this question was investigated and presents sets of

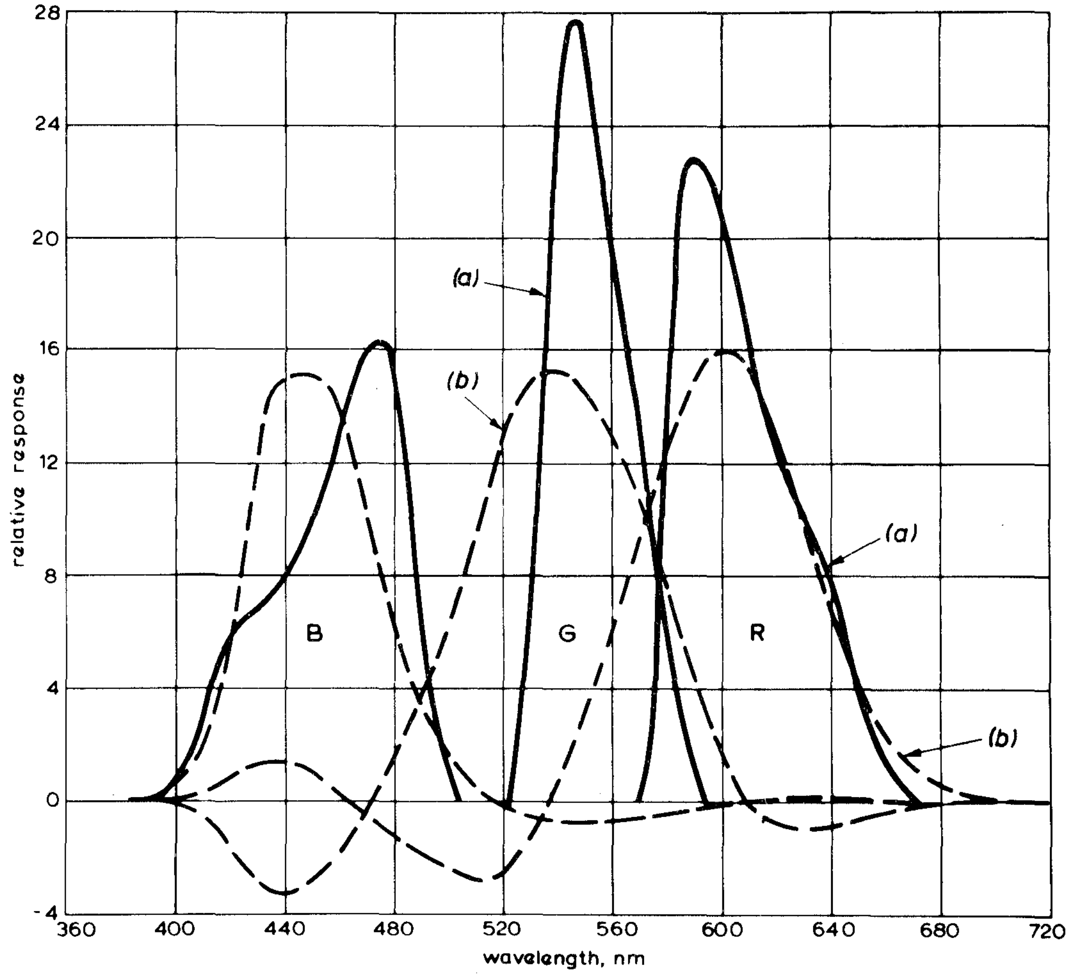

Fig. 3 - Comparison of optimum positive-only analysis characteristics for film scanners and ideal analysis characteristics for NTSC primaries

(*a*) Optimum positive-only analysis for scanners (*b*) Ideal analysis characteristics primary analysis characteristics* and matrix constants that enable a highly accurate reproduction of colour to be achieved.

2. METHOD

The method used to determine the optimum primary analysis characteristics and associated linear matrices was similar to that used to determine an optimum linear matrix when the primary characteristics had already been specified.[5] The calculations were made possible by the availability of a high speed computer.

A series of parameters was chosen to define a given set of characteristics and a matrix. The parameters were given initial values thought to be fairly close to the optimum ones, and they were then varied according to a series of successive approximations so as to minimize the errors in reproduction of a number of test colours. The test colours had been carefully chosen to constitute (it was thought) a fair representation of the very large range of colours encountered in practice. A description of the general technique, applicable to both camera and scanner analysis, will now be given.

The specification of a proposed analysis characteristic could involve the use of many independent parameters. For instance, if it is to be expressed by a series of quantities representing its magnitudes at particular wavelengths, these quantities may in theory be chosen independently of one another, and moreover the total number of possible values that could be ascribed to each one is inversely related to the accuracy with which the quantities need to be specified.

In practice, however, only certain analysis curve shapes can easily and efficiently be instrumented, and this effectively limits the number of parameters necessary to define a possible curve.

Fig. 4 shows four curve shapes that were used to assemble the transmission characteristics of the three optical paths forming part of the proposed colour analysis system. These particular shapes were chosen as being typical of what can be produced using available types of dichroic and shaping filters.

At the start of each optimization process, one of the four curves was selected. It was then stretched or compressed in the wavelength direction, and positioned against a wavelength scale in such a way as to build up independently the abutting "low-wavelength" and "high-wavelength" sides of each of the three peak-normalized optical transmission characteristics. Thus a total of six different scale factors, three positive and three negative, were applied to the one curve shape. The parameters used to control this process are defined in Fig. 5. They comprise the wavelengths λ_b, λ_g, λ_r, λ_{bg} and λ_{gr} at which the peaks and crossovers of the peak-normalized characteristics were to occur, the heights h_{bg} and h_{gr} of the crossovers, and the wavelengths λ_p and λ_q at which the low wavelength side of the "blue" curve and the high wavelength side of the "red" curve were to have 50% peak transmission. The choice of these particular nine parameters was made because previous calculations had indicated that they had a particularly sensitive influence on colour fidelity.

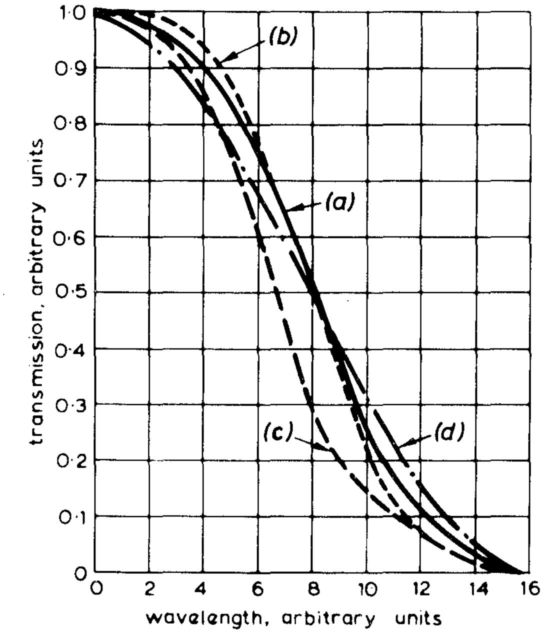

Fig. 4 - Curve shapes used to define transmission characteristics

(*a*) Shape No. 1 (*c*) Shape No. 3
(*b*) Shape No. 2 (*d*) Shape No. 4

Three separate sets of initial values were then proposed for the above parameters. It was felt that if the final values obtained by the optimization were independent of the initial values used, a genuine optimum reproduction of the test colours would have been reached. The initial values proposed were those best approximating to the ideal analysis (Fig. 1), to the optimum positive-only analysis (Fig. 2), and to the analysis of a typical colour television camera.[5]

Having been formed up in this manner, the initial transmission characteristics were first multi-

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* To prevent confusion, the expression "primary analysis characteristics" will be reserved for a description of the analysis made by the optical elements and receptors that generate the original colour signals, whilst the overall analysis obtained by the action of the matrix on these primary characteristics will be described in terms of "effective analysis characteristics".

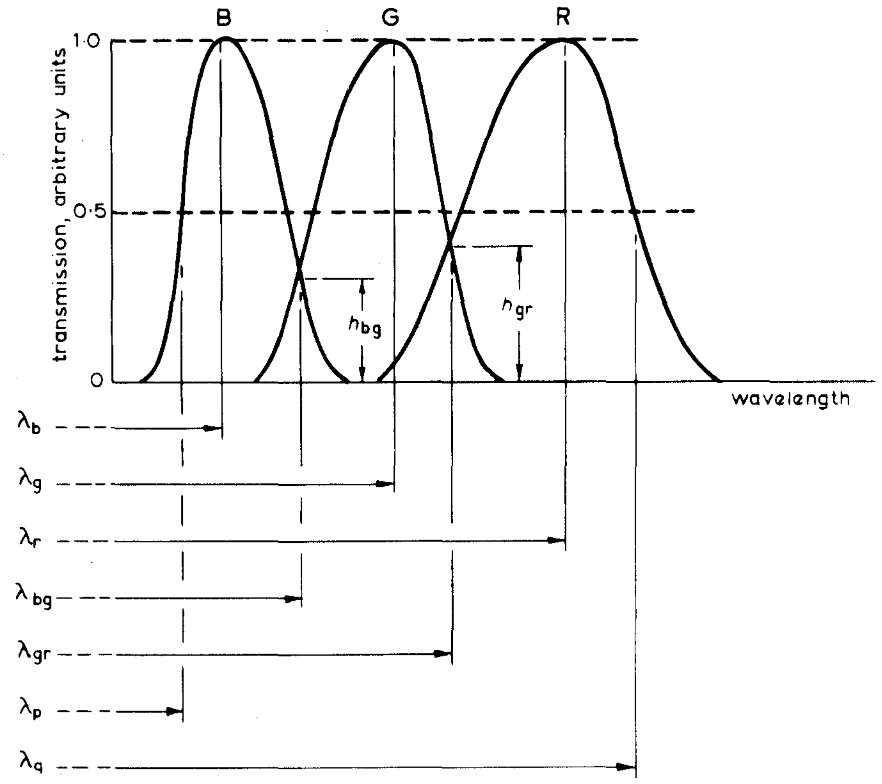

Fig. 5 - Diagram indicating parameters used to define transmission characteristics

plied by the response characteristics of the associated photoelectric devices (e.g. by the spectral sensitivity of a camera tube, or by the product of the spectral emission of a scanning tube and the spectral sensitivity of a photomultiplier tube), and the resulting primary analysis characteristics were then normalized so as to ensure that equal output signals would result from exposure to a white in the scene.

The next step was to obtain initial values for the matrix elements. The matrix transforms the signal voltages R_1, G_1 and B_1, produced by the primary analysis of a colour, into new values that we would like equal to R, G and B, the voltages necessary for an accurate reproduction of the colour using NTSC primaries.

Thus we seek to make:

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} a & b & (1 - a - b) \\ (1 - c - d) & c & d \\ e & (1 - e - f) & f \end{bmatrix} \begin{bmatrix} R_1 \\ G_1 \\ B_1 \end{bmatrix} \tag{1}$$

where a to f are constants. Initial values for a to f were found by first calculating values of R_1, G_1 and B_1 resulting from the analysis of each of the N test colours by the initial primary analysis, and values of R, G and B resulting from their analysis according to the ideal characteristics (Fig. 1). This enabled three sets each of N equations in two unknowns to be written down as indicated above in Equation (1). These sets of equations were then solved by a method of least squares,[6] and their solutions were used as starting values for the quantities a to f.

Thus the effective analysis was defined in terms of fifteen parameters, of which nine related to the optical transmission characteristics and six to the linear matrix. The initial values ascribed to these parameters were read into the computer together with a modified form* of the Elliott Autocode Library Programme for System Optimization. The object of optimization was to reduce as far as possible a quantity n that expressed in units relating to subjective impression the colour errors produced by analysis of the test colours according to the effective characteristics of the system. Thus n would be zero if the effective analysis coincided with the ideal analysis shown in Fig. 1.

n was defined as:

$$[\bar{n}_C{}^2 + \bar{n}_L{}^2]^{1/2} \text{ jnd**}$$

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* This work was carried out by R.W. Lee.

** just noticeable differences.

where $\bar{n}_c$ is the mean chromaticity error and $\bar{n}_L$ is the mean luminance error. For each colour:

$$n_c = [(u_0 - u_1)^2 + (v_0 - v_1)^2]^{½}/0{\cdot}00384 \text{ jnd},$$

(u_0, v_0) and (u_1, v_1) being the chromaticities of the original and reproduced colours respectively, expressed in terms of the 1960 CIE-UCS co-ordinates, and

$$n_L = |\log_e L_1 - \log_e L_0|/0{\cdot}0198 \text{ jnd},$$

L_0 and L_1 being the luminances of the original and reproduced colours respectively expressed relative to white.

The computer programme allowed small exploratory variations in the magnitudes of each of the given parameters taken separately, and calculated the alteration to n produced by these small changes. This information was then used to control an inverse interpolation or extrapolation procedure applied to all of the parameters together and enabling a new set of values closer (as judged by the value of n) to the optimum set to be determined. The process was repeated until no further reduction in n was obtained, when it was decided that the optimum set of values had been reached.

The minimum value of n was then printed out, together with the associated primary analysis and matrix coefficients, and the chromaticities and luminances of the test colours as reproduced using the optimum analysis.

In the choice of initial values, and in all but one of the optimizations a further practical limitation was imposed. An ideal optical system having no loss of light at wavelengths corresponding to the peaks of the transmission characteristics can only comply with the principle of the conservation of energy if the sum of the three characteristics is never greater than the height (100%) of their peaks. In a practical system, however, the dichroic filters used have regions of unwanted reflection and transmission, whilst the shaping filters have regions of unwanted absorption. Consequently the peaks of the transmission characteristics must always be less than 100%. Under certain circumstances, therefore, and in crossover regions, the sum of the three characteristics could exceed individual peaks. However an excess of this nature would be indicative of an inefficient analysis, whilst any attempt to promote or increase it could only be made at a substantial cost in terms of sensitivity or noise performance. It was therefore considered that, subject to colorimetric requirements, a high optical efficiency would be ensured by preventing the crossover heights h_{bg} and h_{gr} from rising above 50% of the peak height to which the transmission characteristics were normalized. (This did, in fact, permit the total transmission to rise somewhat above the normalized peaks in regions near to the crossover points, but the excess heights actually reached were no greater than might be considered permissible when unavoidable losses prevent the curves from peaking to 100% transmission.)

3. RESULTS

This Section will outline in general terms the main features of the results obtained and then discuss in detail those relating to particular television systems.

Note that the calculations have assumed that the display device would use NTSC primaries. When other primaries are envisaged, a close approximation to an optimum analysis may be obtained by using the optical transmission characteristics recommended in this section, and pre-multiplying the recommended linear matrices by the matrix necessary to convert signals suitable for NTSC primaries to signals suitable for the primaries to be used. If, for instance, sulphide phosphors are envisaged, the necessary conversion matrix is

$$\begin{bmatrix} R \\ G \\ B \end{bmatrix}_{\text{out}} = \begin{bmatrix} 1{\cdot}292 & -0{\cdot}207 & -0{\cdot}085 \\ -0{\cdot}052 & 0{\cdot}977 & 0{\cdot}075 \\ -0{\cdot}009 & -0{\cdot}097 & 1{\cdot}106 \end{bmatrix} \begin{bmatrix} R \\ G \\ B \end{bmatrix}_{\text{in}}$$

3.1. General Features

3.1.1. Dependence on Choice of Test Colours

The test colours used were largely those that had featured in earlier analyses[3,5] and which had consequently already been shown to be satisfactorily representative of the large range of colours encountered in practice. Nevertheless, and as a check of this, some of the calculations were made using a number of sets of test colours. Very similar results were found to occur, however, and this added weight to the belief that representative sets of colours were being used. Once this had been established, the calculations relating to camera analysis were done using a set of colours made by combining those which comprised sets 1 and 2 used in the determination of an optimum linear matrix to suit a given set of primary analysis characteristics.[5] Calculations relating to scanner analysis used a set of colours made by combining the thirteen "Eastman" and thirteen "Technicolor" colours used in the determination of optimum positive-only characteristics for scanners.[3]

3.1.2. Influence of Starting Values

It was found that the final values obtained for the fifteen parameters did depend to a small extent on the starting values used. Since n is a rather complicated function of the fifteen parameters, the search for a minimum value of n amounts to an exploration to find the lowest point of a fifteen dimensional hypersurface. The results obtained suggest that this surface in fact has a cluster of local minima situated within the region immediately surrounding its lowest point, so that the minimum actually reached depends on the direction from which the region is approached, and hence on the starting point. However the resulting differences in the final values of the parameters were well within instrumental tolerances, and were therefore negligible from a practical point of view.

3.1.3. Effect of Restriction on Crossover Heights

The final value reached by h_{bg} was found to be well below the specified upper limit of 50% of the peak to which the transmission curves were normalized. However, in all but one of the optimizations, h_{gr} reached this limit at an early point and subsequently remained there. Evidently, had the limit not been set, h_{gr} would have become greater than 50%. In order to ascertain how severe had been the restriction on h_{gr}, the 50% limit was removed during one of the calculations. h_{gr} then rose to 57·6% of the normalized peak transmission, the value of n being correspondingly reduced from 1·50 to 1·25.

This represents a very small improvement in colour fidelity, and so the probable impairment of noise performance resulting from an insistance that h_{gr} should be 57·6% is unlikely to be justified.

It appears, therefore, that though the 50% limit does operate as a restriction, its small adverse effect on colour fidelity is outweighed by its benefit to noise performance.

3.1.4. Effect of Transmission Characteristic Curve Shape

The programme used was somewhat limited in that only one of the four curve shapes (Fig. 4) was considered in each calculation, whereas in practice a set of characteristics could make use of more than one shape. Nevertheless it seems reasonable to suppose that the shape of curves used is not critical, since the effect of changing throughout from one curve shape to another was quite small. It was concluded that a high standard of colour fidelity can be achieved using a fairly wide range of curve shapes, provided that the wavelengths λ_p, λ_b, λ_{bg}, etc. are set correctly. This tends to confirm the earlier belief that these particular parameters have a controlling influence on the colour fidelity of cameras. Further calculations described in Section 3.2.1, however, suggested that λ_r and λ_q are not subject to such a narrow tolerance as are the other wavelength parameters. Moreover, the magnitude of h_{bg} appears to be less critical, whilst the findings outlined in Section 3.1.3 suggest that a relatively wide tolerance may be ascribed to h_{gr} also.

The impression given by the results obtained was that a matrix can have a powerful corrective influence over a wide range of primary analyses. It is therefore recommended that whilst an attempt should be made to instrument the optimum transmission characteristics proposed in this report, for best results the method earlier described[5] should be used to compute the optimum matrix to work with the primary analysis characteristics actually achieved.

3.2. Results Applicable to Particular Television Systems

3.2.1. Optimum Characteristics For Use Where the Associated Photoelectric Devices do not Limit the Analysis

This result follows a calculation in which the photoelectric devices were ideally assumed to have a combined response characteristic that is rectangular for each channel, i.e. independent of wavelength within the channel. These results could also be used for a system in which the photoelectric devices has a non-uniform response that does not restrict the choice of low wavelength "blue" or high wavelength "red" characteristics. Thus the primary analysis characteristics themselves, as well as the transmission characteristics that helped to form them, were assumed to be subject to the curve shapes shown in Fig. 4.

It was assumed that the display device was adjusted in accordance with normal practice to give a white that matched illuminant C, and that this illuminant was also used in lighting the original scene.

The optimum transmission characteristics derived assuming these conditions are shown in Fig. 6. They are intended to be used in conjunction with the following linear matrix:

$$\text{Matrix No. 1} \quad \begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 1{\cdot}06 & -0{\cdot}09 & 0{\cdot}03 \\ -0{\cdot}03 & 1{\cdot}20 & -0{\cdot}17 \\ -0{\cdot}01 & -0{\cdot}04 & 1{\cdot}05 \end{bmatrix} \begin{bmatrix} R_1 \\ G_1 \\ B_1 \end{bmatrix}$$

This combination gave a minimum error figure n of 1·50 jnd. This error figure should be compared with the figure of 6·92 jnd that results from use of the optimum positive-only characteristics shown in Fig. 2.

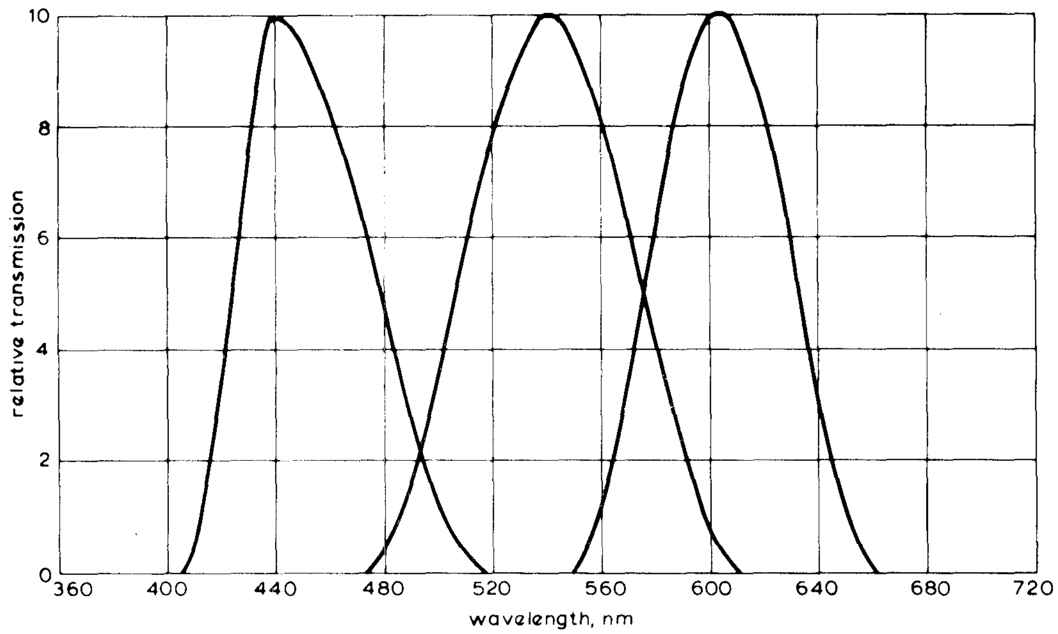

Fig. 6 - Optimum transmission characteristics calculated assuming photoelectric devices having uniform spectral responses

The resulting effective analysis characteristics are shown in Fig. 7, curves (*a*). It will be seen that a quite close approximation to the ideal curves (*b*) has been achieved. There is some discrepancy between the curves in the region of the negative lobe of the "red" characteristic, but this is to be expected since the primary analysis which, with the matrix is used to produce the negative "red" lobe, is also required in forming the major positive lobe of the "green" characteristic.

It is of some interest that the "red" analysis characteristic is not required to extend beyond about 660 nm, whereas the ideal "red" analysis extends to beyond 700 nm. A series of calculations was therefore made to ascertain the reason for this.

First the programme earlier developed[5] was used to calculate an optimum linear matrix for use with two sets of primary analysis characteristics based on those shown in Fig. 6 but with the high wavelength side of the "red" characteristic modified to extend first to 680 nm and then to 700 nm. It was found that these modifications resulted in increases in the error figure *n* finally reached of 0·66 jnd and 1·31 jnd respectively.

Secondly the analysis and matrix optimization was repeated using the 27 fictitious test colours described in Ref. 5 together with three other colours. These three were saturated reds of relatively high luminance: it is most unlikely that 10% of the colours encountered in any practical situation would be of this type, but it was thought interesting to see whether their presence in the set of test colours used would force an extension of the optimum "red" characteristic into the deep red region.

An extension of the "red" characteristic did in fact take place, its upper limit moving to about 675 nm. The error figure *n* was increased, but the increase was borne almost entirely by the three special colours, the other 27 being hardly affected. Moreover, the errors in reproduction of the three special colours were largely attributable to the fact that they lay outside the gamut of the display primaries; that is, errors of the order calculated would result from the inability of the display to reproduce such colours even if the ideal analysis characteristics were used.

It therefore appeared that there is a genuine preference for a red analysis characteristic that excludes the deep red spectral region, unless the scene contains an abnormal number of very deep red colours.

Two possible explanations were suggested at this point. The first was that the negative lobe in the optimum red characteristic could only be produced in the region of the main green lobe, and it was therefore at a higher wavelength than the ideal. The positive red lobe was therefore forced to commence at a higher wavelength than ideal, and to compensate for this the curve rose steeply to a high peak. This in turn could have necessitated a sacrifice of response in the deep red region.

The second explanation was that the C.I.E. distribution functions $\bar{x}$, $\bar{y}$ and $\bar{z}$ have reached quite low levels by 660 nm. Moreover the spectral reflectance of most objects does not change appreciably

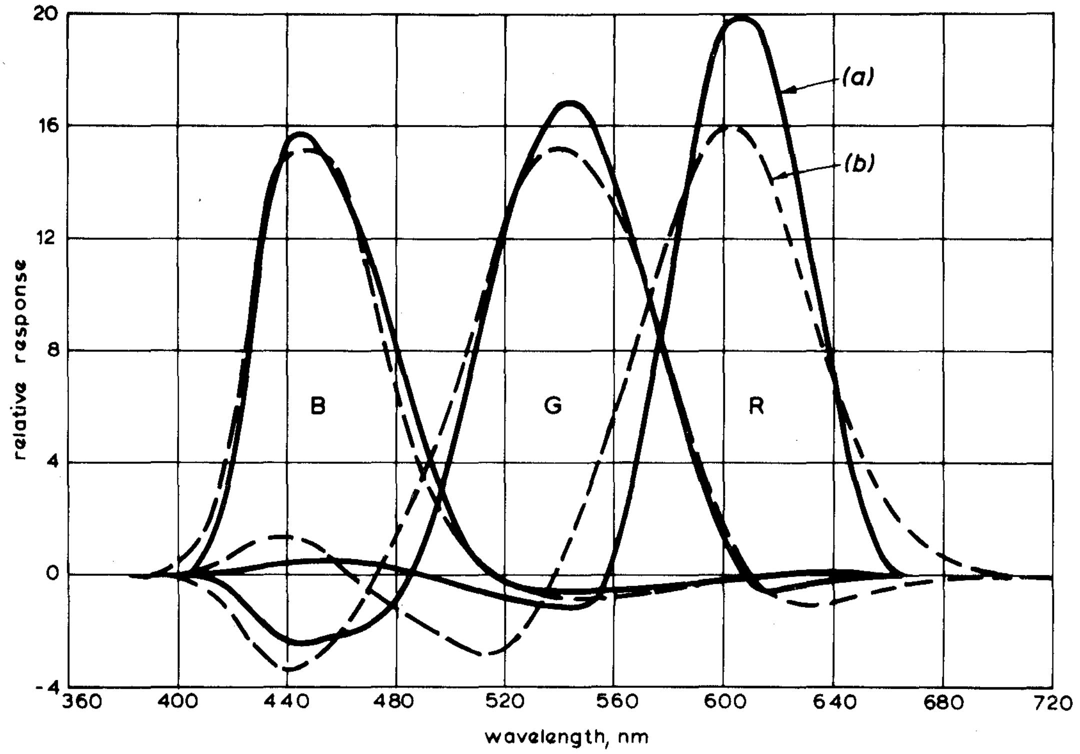

Fig. 7 - Comparison of effective analysis characteristics obtained using primary characteristics shown in Fig. 6 together with Matrix No. 1, and ideal analysis characteristics for NTSC primaries

(*a*) Effective analysis (*b*) Ideal analysis Error figure n = 1·50 jnd

within the range 660 to 700 nm. It therefore appears profitable to concentrate the red analysis around 600 nm (which is the position of the peak of $\bar{x}$) and to adjust the multiplying factor within the matrix to make some allowance for the deep red part of the spectrum that is not analysed. The error introduced by this procedure would in most cases be extremely small; colours whose spectrum was of zero magnitude until 660 nm would of course produce no response, but such colours are outside not only the colour gamut but also the contrast range of present day television systems.

Fig. 8 shows (curve (*a*)) the equivalent luminance characteristic obtained using the proposed analysis; it should be compared with curve (*b*) which is the ideal or photopic curve.

3.2.2. Optimum Characteristics For Use in Cameras Fitted With Plumbicon Tubes and Used With Interior Lighting

A typical plumbicon camera tube (Philips curve 55875 B) has a spectral sensitivity which when multiplied by the spectral emission of a tungsten source operated at 3000°K produces the characteristic shown in Fig. 9. When a camera employing such tubes is used together with a 3000°K illuminant, its primary analysis is given by the product of the above characteristic and the spectral transmission characteristics of its optical system. Thus these conditions of operation could be taken into account by replacing the illuminant C spectral energy characteristic that had been used to multiply the optical transmission characteristics as described in Section 3.3 by the characteristic shown in Fig. 9.

(Note that although a tungsten source at 3000°K was now supposed to illuminate the scene, the display was still assumed to be set up so that its white matched illuminant C, and the system was therefore required to reproduce the original scene as it would have appeared if illuminated by illuminant C. This cannot even in theory be done with complete accuracy, except with the aid of a colour temperature raising filter, but the unavoidable errors introduced are generally quite small.)

The optimum transmission characteristics derived assuming these conditions are shown in Fig. 10. They are intended to be used in conjunction with the following linear matrix:

$$\text{Matrix No. 2} \quad \begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 1{\cdot}14 & -0{\cdot}18 & 0{\cdot}04 \\ -0{\cdot}06 & 1{\cdot}23 & -0{\cdot}17 \\ -0{\cdot}03 & 0{\cdot}02 & 1{\cdot}01 \end{bmatrix} \begin{bmatrix} R_1 \\ G_1 \\ B_1 \end{bmatrix}$$

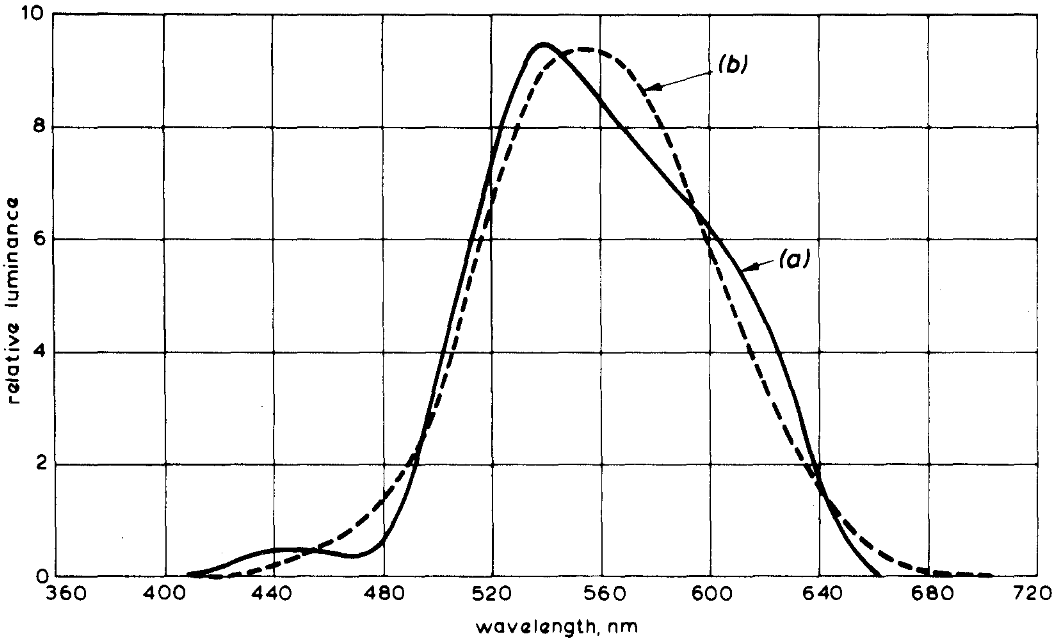

Fig. 8 - Comparison of (a) equivalent luminance characteristic obtained using effective analysis characteristics shown in Fig. 7, with (b) ideal (photopic) curve

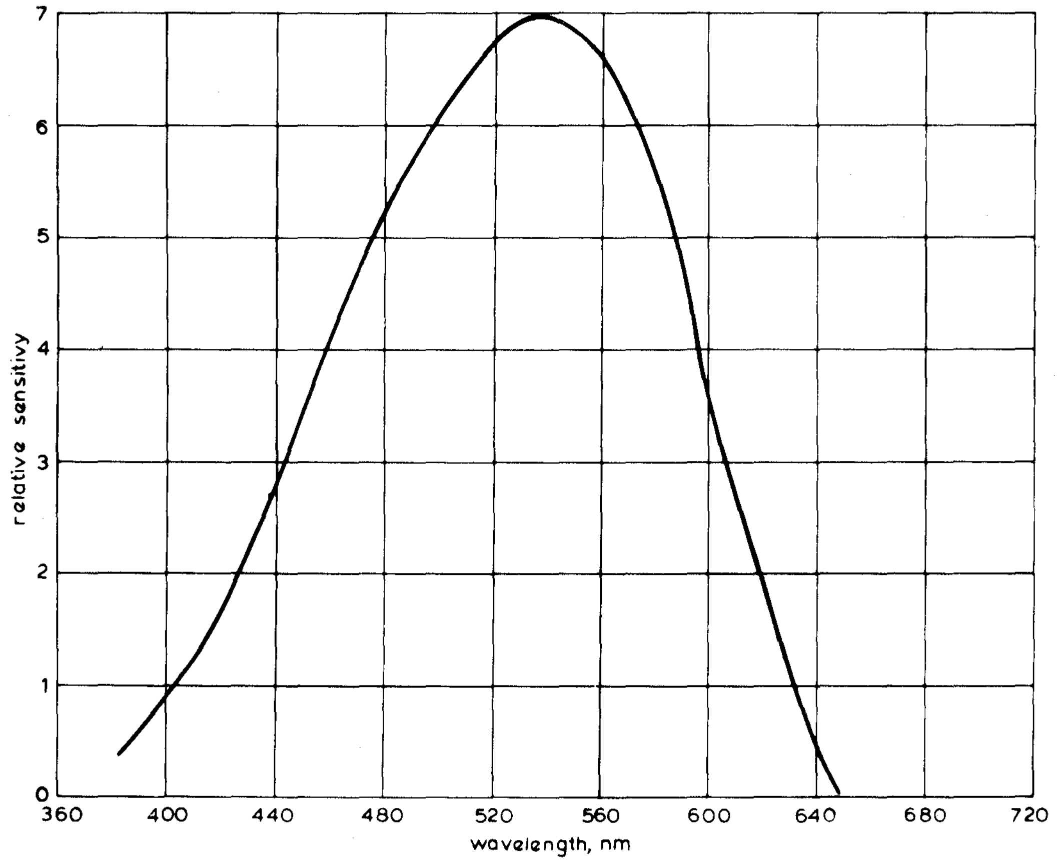

Fig. 9 - Product of response characteristic of typical plumbicon camera tube and spectral emission characteristic of 3000°K source

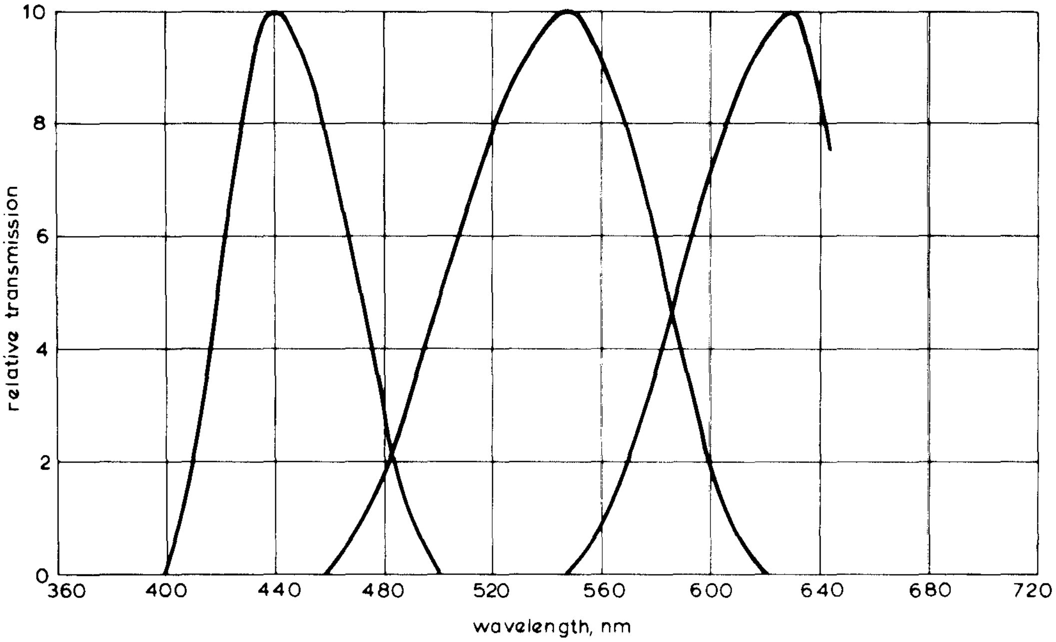

Fig. 10 - Optimum transmission characteristics calculated using characteristic shown in Fig. 9

This combination gave a minimum error figure *n* of 1·66 jnd. The resulting effective analysis characteristics are shown in Fig. 11 curves (*a*), these should be compared with the ideal characteristics reproduced as curve (*b*).

It is interesting to assess how the colour accuracy achieved using the above analysis and matrix compares with that which can be attained if a matrix is incorporated in a present day plumbicon camera. The optical splitting block was therefore removed from a production model camera for measurement. Its transmission characteristics were then multiplied by the characteristic shown in Fig. 9 to give a set of primary analysis characteristics applicable when interior lighting is used. An optimum matrix to match this analysis to the NTSC display primaries was then calculated using the method earlier described.[5] It was found that without the matrix the error figure *n* was 5·81 jnd, and that this figure was reduced to 2·07 jnd when the optimum matrix was inserted. The potency of the matrix is clearly demonstrated by this result, and it is clear also that the primary analysis carried out by this particular camera is close to the optimum for use with a matrix. An inspection of the optical transmission characteristics confirmed this point.

3.2.3. Optimum Characteristics For Use in Flying Spot Scanners

The zinc oxide scanning tube used in a flying spot scanner has a spectral emission characteristic which when multiplied by the spectral sensitivity characteristic of the associated photomultiplier tubes produces characteristics of the form shown in Fig. 12. Curve (*a*) of this figure relates to the blue and green channels of a typical scanner, whilst curve (*b*) relates to the red channel in which a different photomultiplier tube is used. The primary analysis characteristics of the scanner are given by the product of the characteristics shown in Fig. 12 and the spectral transmission characteristics of the optical system. Thus when the optimization procedure was applied to the analysis of scanners, the optical transmission characteristics described in Section 3.3 were multiplied by the characteristics shown in Fig. 12.

In carrying out this calculation it proved necessary to apply a further restriction to the parameters defined in Fig. 5. Because of the necessity for a high red-to-green crossover point, it is desirable to avoid the use of shaping filters to form the high wavelength side of the "green" transmission characteristic and the low wavelength side of the "red" transmission characteristic. Thus these two curves should correspond respectively to the transmission and reflexion characteristics of the dichroic filter that separates red from green. The curves should therefore exhibit symmetry about the wavelength λ_{gr}. An inspection of Figs. 6 and 10 will show that a fair degree of symmetry was previously produced in the optimization process, although for no obvious reason. It did not at first occur in the evaluation of characteristics for scanners, and therefore to obtain a practicable result it was necessary to ensure symmetry by specifying that λ_{gr} should occur mid-way between λ_g and λ_r. This additional restriction did not, however, seriously limit the colour fidelity achieved, and so it may be that a wider tolerance may be allowable for the

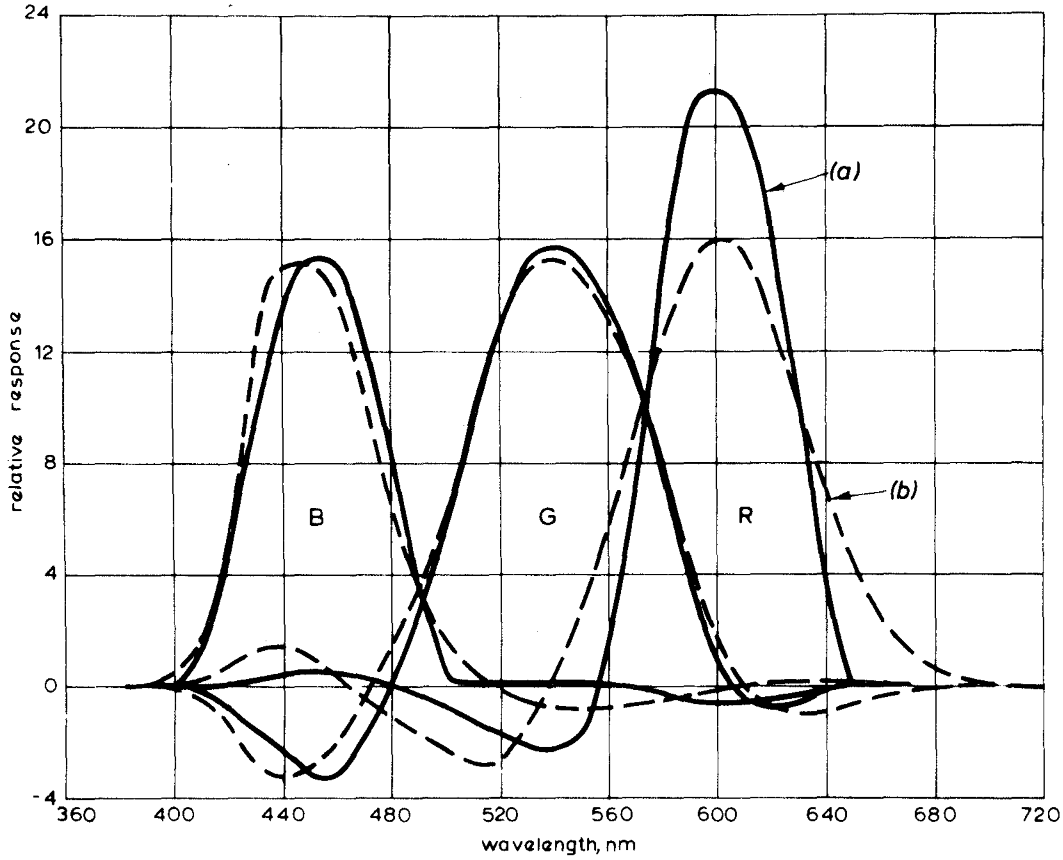

Fig. 11 - Comparison of effective analysis characteristics obtained using characteristics shown in Figs. 9 and 10 together with Matrix No. 2, and ideal analysis characteristics for NTSC primaries
(*a*) Effective analysis (*b*) Ideal analysis Error figure *n* = 1·66 jnd

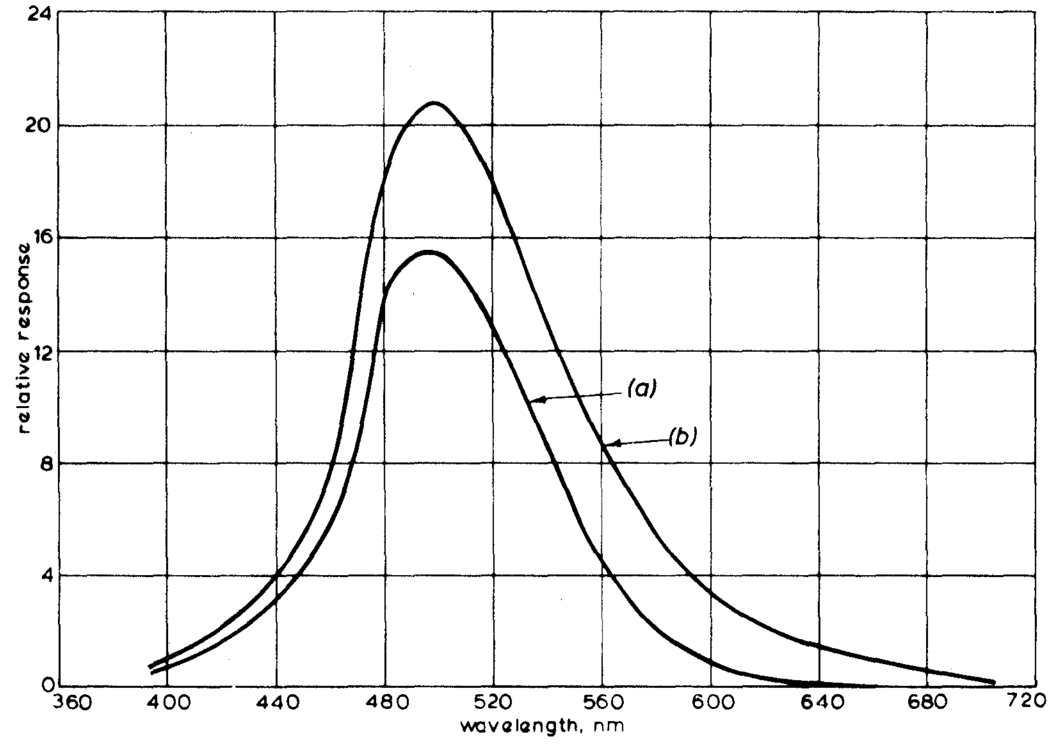

Fig. 12 - Typical combined responses of cathode-ray tubes and photomultipliers
(*a*) Blue and green channels (*b*) Red channel

parameters defined in Fig. 5 when only the restricted range of colours encountered in a film scanner is considered.

The optimum transmission characteristics obtained from this calculation are shown in Fig. 13. They are intended to be used in conjunction with the following linear matrix.

$$\text{Matrix No. 3} \quad \begin{bmatrix} R \\ G \\ B \end{bmatrix} = \begin{bmatrix} 1{\cdot}23 & -0{\cdot}31 & 0{\cdot}08 \\ 0{\cdot}06 & 1{\cdot}20 & -0{\cdot}26 \\ -0{\cdot}04 & -0{\cdot}12 & 1{\cdot}16 \end{bmatrix} \begin{bmatrix} R_1 \\ G_1 \\ B_1 \end{bmatrix}$$

This combination gave a minimum error figure n of 2·18 jnd. This error figure should be compared with the figure of 3·34 jnd that results from use of the optimum-positive only characteristics shown in Fig. 3.

Fig. 14 shows the effective analysis characteristics that result from the use of the transmission characteristics shown in Fig. 13 together with matrix No. 3. It will be seen that they approximate less closely to the ideal characteristics than do the curves shown in Fig. 7 and 11. This is not unexpected, as a rather restricted range of film-dye spectral characteristics is being considered; and in fact the reproduction of film colours is not seriously impaired as the final value of n indicates.

3.3. Noise Performance Attainable Using Recommended Analyses

It is of interest to determine what change in signal-to-noise ratio would be incurred by using the recommended transmission characteristics with their matrices instead of the optimum positive-only analyses shown in Figs. 2 and 3. In practice, the change in signal-to-noise ratio will depend not only on the shapes of the analysis characteristics used but also on the optical efficiencies achieved, and any differences in the sensitivities or noise performances of the three receptors. The comparisons made here are therefore based on certain assumptions that may not apply in a given practical situation, nevertheless the results obtained are believed to give a reasonable indication of the changes to be expected.

3.3.1. Calculation Relating to Characteristics Outlined in Section 3.2.1

It is here assumed that the photoelectric-devices have combined responses that are rectangular and equal for each colour channel and independent of wavelength within the channel, and that a given peak transmission is achieved in each of the optical paths irrespective of which characteristic is being implemented. It is further assumed that the three channels generate noise signals having magnitudes that are equal and independent of the light incident on the receptors.

When normalized with respect to illuminant C, the characteristics shown in Fig. 6 have peak heights of 1·87 units (red), 1·42 units (green), and 1·51 units (blue). If the same normalization procedure is applied to the optimum positive-only characteristics (Fig. 2), the resulting peak heights are 2·40 units (red), 2·15 units (green), and 1·74 units (blue).

Therefore, if equal peak optical efficiencies are maintained in each channel, the output signals R_1, G_1 and B_1 resulting from the analysis of a white in the scene according to the characteristics given in Fig. 6 are proportional respectively to:

$$\frac{1}{1{\cdot}87}\text{ units, }\frac{1}{1{\cdot}42}\text{ units, and }\frac{1}{1{\cdot}51}\text{ units}$$

The channel gain controls are now used to make $R_1 = G_1 = B_1 = 1$ for white. Therefore the noise signals observed at the outputs of the gain controls are proportional to:

1·87 units, 1·42 units, and 1·51 units

Similarly if the optimum positive-only characteristics (Fig. 2) are used, noise signals proportional to:

2·40 units, 2·15 units, and 1·74 units

respectively are produced.

These figures indicate that prior to matrixing, the broader analysis characteristics shown in Fig. 6 give signal-to-noise ratio improvements of 2·2 dB (red), 3·6 dB (green) and 1·2 dB (blue) as compared with the optimum positive-only analysis (Fig. 2), assuming the above conditions. The matrix, however, reduces these advantages.

The "red" signal R obtained from Matrix 1 is given by:

$$R = 1{\cdot}06R_1 - 0{\cdot}09G_1 + 0{\cdot}03B_1$$

The noise signal present in the "red" channel at the output of the matrix is therefore proportional to:

$$[(1{\cdot}06 \, . \, 1{\cdot}87)^2 + (-0{\cdot}09 \, . \, 1{\cdot}42)^2 + (0{\cdot}03 \, . \, 1{\cdot}51)^2]^{1/2}$$

i.e. to 1·99 units.

Similarly the noise signals present in the "green" and "blue" channels at the output of the matrix are proportional to:

1·72 units and 1·59 units

respectively.

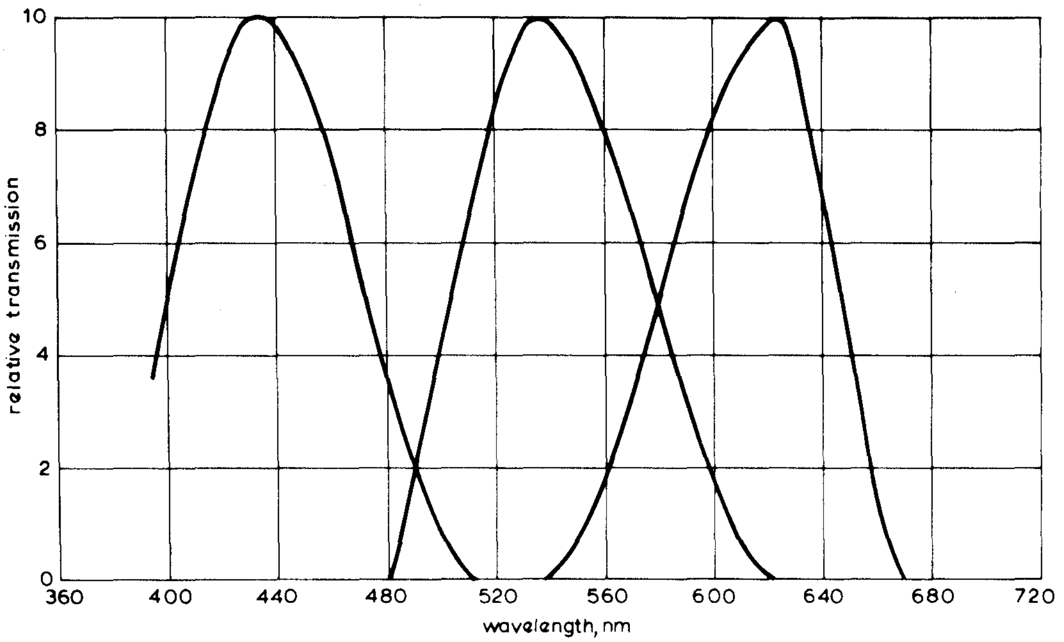

Fig. 13 - Optimum transmission characteristics calculated using characteristics shown in Fig. 12

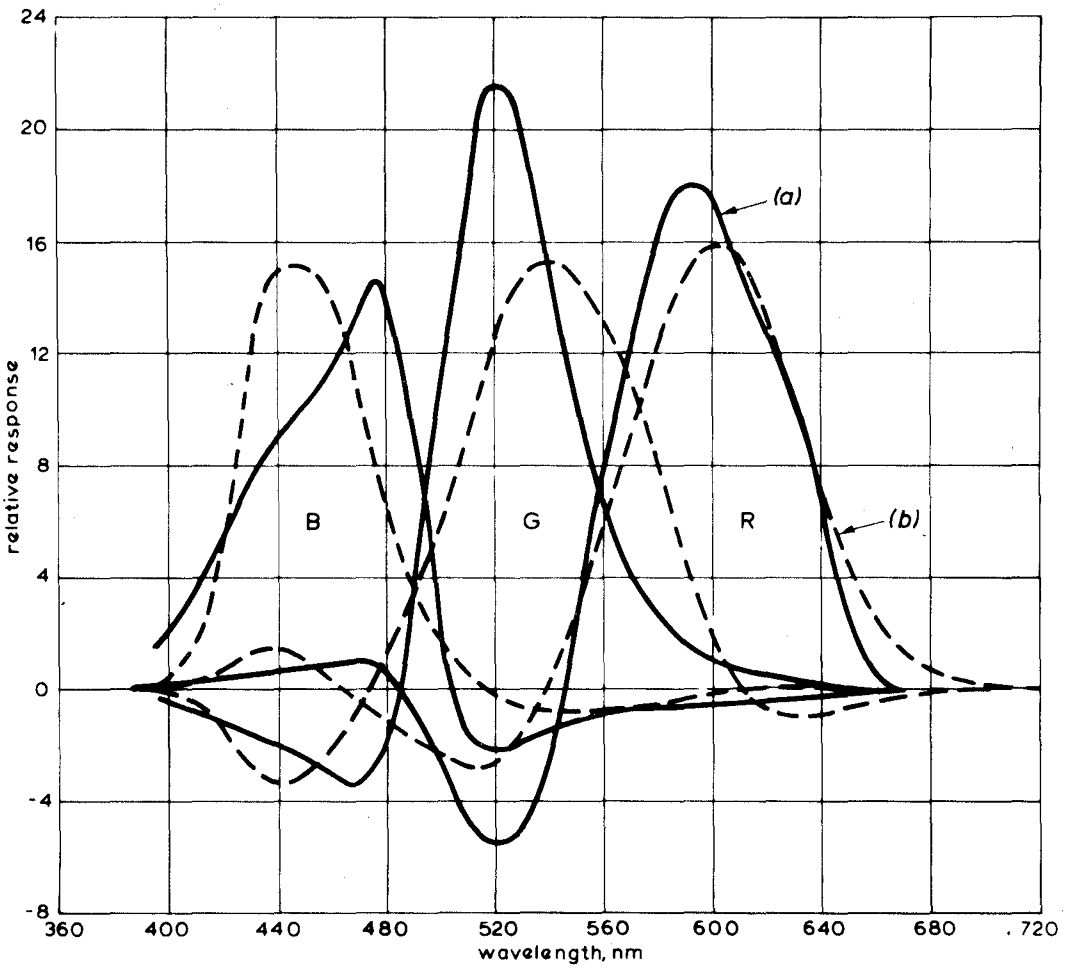

Fig. 14 - Comparison of effective analysis characteristics obtained using characteristics shown in Figs. 12 and 13 together with Matrix No. 3, and ideal analysis characteristics for NTSC primaries

(*a*) Effective analysis (*b*) Ideal analysis Error figure n = 2·18 jnd

Therefore when the matrix is connected, the analysis shown in Fig. 6 gives signal-to-noise ratio improvements of 1·7 dB (red), 2·0 dB (green), and 0·8 dB (blue) as compared with the optimum positive-only analysis (Fig. 2) assuming the above conditions.

These are the differences in signal-to-noise ratio that would be measured in the three colour channels at the inputs to the gamma correctors. If gamma correction were not necessary, the above figures would indicate also the improvements in luminance noise observed when saturated colours are displayed. In practice, however, the luminance noise at the output of a colour camera in regions of saturated colour is caused predominantly by noise originating in the channels carrying low level picture signals, because this noise is considerably amplified by the relatively little excited gamma correction circuits. The amount of noise actually observed depends on the characteristics of the circuits used. Calculations made assuming typical circuit parameters have indicated that a reduction in luminance noise of about 2 dB would be observed in high saturation regions if the recommended analysis characteristics were used in place of the optimum positive-only characteristics (Fig. 2).

We now calculate the luminance noise associated with desaturated regions. Assuming that no matrix is present, the luminance of the output display is given by:

$$0{\cdot}299R_1 + 0{\cdot}587G_1 + 0{\cdot}114B_1$$

Therefore if the optimum positive-only analysis is used the luminance noise present in a displayed white is proportional to:

$$[(0{\cdot}299\ .\ 2{\cdot}40)^2 + (0{\cdot}587\ .\ 2{\cdot}15)^2 + (0{\cdot}114\ .\ 1{\cdot}74)^2]^{1/2}$$

i.e. to 1·47 units.

The matrix modifies the proportions of R_1, G_1 and B_1 that make up the luminance of the displayed colour. Thus when matrix 1 is used, the luminance is given by:

$$0{\cdot}298R_1 + 0{\cdot}673G_1 + 0{\cdot}029B_1$$

Therefore if R_1, G_1 and B_1 are generated using the characteristics shown in Fig. 6, the luminance noise present in a displayed white is proportional to:

$$[(0{\cdot}298\ .\ 1{\cdot}87)^2 + (0{\cdot}673\ .\ 1{\cdot}42)^2 + (0{\cdot}029\ .\ 1{\cdot}51)^2]^{1/2}$$

i.e. to 1·11 units.

Comparing this level of noise with the level of 1·47 units earlier determined, we find that the proposed analysis gives an improvement of 2·5 dB in the signal-to-noise ratio in desaturated areas as compared with the optimum positive-only analysis.

3.3.2. Calculation Relating to Characteristics Outlined in Section 3.2.2

This calculation was based on the assumptions made in Section 3.3.1 with the exception that the three receptors were now assumed to have spectral responses subject to the characteristic shown in Fig. 9. Having been normalized so as to have equal areas when multiplied by this characteristic, the recommended transmission characteristics shown in Fig. 10 have peaks of 6·78 units (red), 1·90 units (green), and 6·00 units (blue). The transmission characteristics which when multiplied by the characteristic shown in Fig. 9 give the optimum positive-only analysis similarly normalized were found to have peaks of 8·68 units (red), 3·14 units (green), and 6·66 units (blue).

This data was used in a calculation similar to that described in Section 3.3.1, and it was deduced that the characteristics shown in Fig. 10 together with matrix No. 2 would give signal-to-noise ratio improvements as compared with the optimum positive-only characteristics (Fig. 2) of 1·0 dB (red channel), 1·7 dB (green channel), 0·8 dB (blue channel), and 2·6 dB (white).

3.3.3. Calculation Relating to Characteristics Outlined in Section 3.2.3

This calculation was based on the assumptions that a given peak transmission is achieved in each of the optical paths irrespective of which characteristic is being implemented, that the combined responses of the photoelectric devices are as shown in Fig. 12, and that each channel generates a noise signal whose magnitude is proportional to the square root of the light flux incident on its receptor, the constant of proportionality being the same for each channel. The transmission characteristics shown in Fig. 13 have peaks of 4·68 units (red), 1·60 units (green) and 3·31 units (blue) when normalized with respect to the characteristics shown in Fig. 12. Similarly normalized, the transmission characteristics necessary to give the optimum positive-only analysis (Fig. 3) have peaks of 5·00 units (red), 4·30 units (green) and 3·62 units (blue).

In this instance, the noise signal present in any of the three colour channels after the matrix is dependent on the picture signals in all three channels. Thus for a bright and saturated red, for which the blue and green signals are zero, the noise produced by matrix No. 3 in conjunction with the transmission characteristics shown in Fig. 12 is proportional to:

$$2{\cdot}16\ .\ 1{\cdot}23 = 2{\cdot}66 \text{ units}$$

in the "red" channel and zero in the "green" and "blue" channels. (In a well designed scanner the noise produced by dark current in the photomultiplier tubes and in the first stages of the head amplifiers should be of an insignificant magnitude.) Similarly the noise signals in the "green" and "blue" channels corresponding respectively to bright and saturated greens and blues are proportional to:

1·51 and 2·11 units

Thus the signal-to-noise ratio improvements in areas of saturated colour are:

-1·5 dB (red), 2·8 dB (green), -0·9 dB (blue)

assuming the above conditions.

Using a calculation similar to that described in Section 3.3.1, it was deduced that in white areas the characteristics shown in Fig. 12 together with matrix No. 3 would give a signal-to-noise ratio improvement of 1·8 dB as compared with the optimum positive-only characteristic (Fig. 3).

4. CONCLUSIONS

The calculations described have established that for accurate reproduction of colour it is preferable to use a relatively broad band analysis together with a linear matrix rather than the narrower band analysis required when a matrix is not permitted. Characteristics are recommended that give improvements in both colour fidelity and noise performance. The improvements are greatest in the case of colour cameras, which need to deal with a very wide range of colours. Film scanners, however, have also been shown to benefit from the technique proposed, where the objective is to produce colours identical to those obtained by optical projection.

The linear matrix has been shown to have a powerful influence on colour fidelity, and the following table indicates the degree of improvement that can be obtained:

TABLE 1

	Typical 3-tube plumbicon camera	Typical camera with optimized matrix	Optimized optical characteristics and matrix
n, jnd	5·81	2·07	1·66*

* This figure reduces to 1·50 jnd when the restriction of the plumbicon tube spectral sensitivity in the far red region (i.e. greater than 650 nm) is removed.

The camera quoted in this table has optical transmission characteristics very close to the optimum for use with a matrix. Nevertheless if a matrix is used, the shape of the optical transmission characteristics becomes much less critical, and differences in the colour performance of sources having differing transmission characteristics may be substantially reduced.

It is therefore proposed that after an attempt has been made to instrument the proposed transmission characteristics a matrix be computed to work with the characteristics actually achieved, using the method earlier described.[5]

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