RESEARCH DEPARTMENT

THE ESTIMATION OF IONOSPHERIC CROSS MODULATION BETWEEN BROADCASTING TRANSMITTERS

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Investigation by: E. Sofaer
Report written by: E. Sofaer

(W. Proctor Wilson)
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GENERAL

This note furnishes a set of curves and two nomograms to enable estimates to be made of the degree of ionospheric cross modulation to be expected under given conditions. It explains how they should be used, the significance of the factors involved, and the accuracy of the results obtained.

The nomograms are based on the theoretical analysis of ionospheric cross modulation by L.G.H. Huxley and J.A. Ratcliffe, and on an article by Dr. I.J. Shaw.

In this note, the mechanism by which ionospheric cross modulation occurs will be discussed qualitatively. The mathematical analysis will be found in the references quoted.

1. MECHANISM OF IONOSPHERIC CROSS MODULATION

The occurrence of cross modulation in the ionosphere implies that it behaves at such times as a non-linear medium to radio waves passing through it. The non-linearity is caused by another wave present in the same region of the ionosphere at the same time which, in its passage, modifies the degree of ionospheric absorption.

When a portion of the ionosphere is in thermal equilibrium, the energy lost by the electrons in some of their random collisions with molecules is equal to the energy gained by them in others. The mean energy in the electrons is then equal to the mean energy in the molecules.

A wave passing through the region transfers energy to the electrons. The mean level of energy in the electrons then rises above that in the molecules; in consequence energy flows to the molecules and is lost to the wave. The amount of energy lost will be greater, the greater the difference between the energy levels. It therefore follows that ionospheric absorption will increase as the power in the wave increases.
Second wave passing through the same region will encounter an ionosphere whose absorption is greater than it would have been if the first wave had not been present; it will, in consequence, suffer correspondingly greater attenuation.

If the first wave were suddenly switched off, ionospheric absorption would fall to the value appropriate to the presence of the second wave in an ionosphere in equilibrium, but the fall would occur in a finite time. If instead of being switched off, the first wave (called the disturbing wave) were modulated at a frequency, the period of which was greater than the time required by the ionosphere to recover equilibrium, the absorptive characteristic of the ionosphere would follow the modulation. The second wave (i.e. the wanted wave) would then be attenuated in step with the changes in absorption, and would therefore become amplitude-modulated by the modulation on the first wave.

The degree of attenuation to which the wanted wave will be subjected is naturally dependent on the constants of the ionosphere, and on its own frequency. It also depends on the angle between its direction and that of the earth's magnetic field. Shaw calls this attenuation $\alpha \left( r_w \right)$ and gives the expression for it at the end of Section 5 of his article.  

Summarising, it may be said that ionospheric cross-modulation is:

(a) dependent on the total power distribution of the disturbing wave at that part of the ionosphere traversed by the wanted wave. Modulation is accounted for in the above statement, since the total power at any instant is governed by the depth of modulation.

(b) dependent on the modulation frequency. Evidently the smaller the period of one cycle of modulation with respect to the recovery time of the ionosphere, the less effective will it be in causing increased absorption.

(c) dependent on the state of the ionosphere.

(d) dependent on the carrier frequency of the wanted wave and on the angle it makes with the earth's magnetic field.

2. COEFFICIENT OF TRANSFERRED MODULATION

The method adopted in Huxley and Ratcliffe's paper is to express ionospheric cross-modulation in terms of a coefficient of transferred modulation. Since the value of the coefficient has been shown in (b) above to be dependent on modulation frequency, its value $T_0$ at a frequency approaching zero, has been chosen as a reference. Furthermore,
the variables implicit in (a) above, namely, power modulation depth, and distance of the disturbing transmitter from the reflecting region are also referred to standard values of 100 kW, 100% and 100 km respectively.

Shaw has rewritten the Huxley and Ratcliffe expression for $T_o$ in a much simplified form. He gives:

$$T_o = \frac{\sigma(Pw)}{3Gk\vartheta} (P_1 - P_2)$$

Here, $P_1$ represents the power from the disturbing wave entering the ionosphere, and $P_2$ the power emerging from it. G and k are physical constants and $\vartheta$ is the absolute temperature.

For the present purpose, $P_2$ has been taken as negligibly small. This involves no great error, since, for a reflection coefficient of 0.25, i.e., $P_\alpha/P_1 = 0.25$, $P_\alpha/P_1 = 0.0625$ giving an error of a little over 6%. Since all the factors in the denominator are either constants or taken at a constant value, (they are the factors which determine the state of the ionosphere referred to in (c) above) $T_o$ is soon to be proportional to the power of the disturbing transmitter and to $\sigma(Pw)$.

The nomograms to which reference has been made have been worked out for a value of $T_o = 0.05$, which has been taken as a reasonable basis. This value was chosen as being near the average of the results of the experiments described in the Huxley and Ratcliffe paper. The effect of $\sigma(P_w)$ on the value of $T_o$ is derived, as explained later, as a correction in decibels to the solution obtained from the nomograms.

It may on occasion be required to find the degree of cross modulation at a single frequency other than that at which the nomograms have been calculated. A further correction to the basic solution will then be necessary. Fig.8 shows how a coefficient of transferred modulation $T_f$ varies with the modulation frequency $f$.

It will be noticed that in this figure the reference modulation frequency has been taken as 400 c/s and not the very low frequency associated with $T_o$. This frequency was chosen as one which would provide a better guide to a subjective assessment of the degree of cross modulation to be expected from a broadcast programme. Where broadcast interference is being considered, no correction need therefore be applied to the answer obtained from the nomograms. Where a specific frequency is concerned, however, the decibel correction obtained from Fig.8 must be made to the solution obtained from the nomogram.

3. **POWER FROM THE DISTURBING TRANSMITTER AT THE IONOSPHERE**

The region of the ionosphere where the wanted wave undergoes
reflection lies midway between the wanted transmitter and the receiver. The amount of power reaching this region from the disturbing transmitter depends, firstly, upon its distance from that transmitter and upon its height above ground, and, secondly, upon the effective power radiated by the aerial at the appropriate elevation angle.

The curves of Fig. 7 show the vertical radiation patterns of the four types of aerial, identified by their heights h, on which these calculations are based. They are:

\[
\begin{align*}
    h &= \ll \lambda \\
    h &= 0.50\lambda \\
    h &= 0.55\lambda \\
    h &= 0.60\lambda
\end{align*}
\]

The curves shown have been calculated for the direct and ground reflected waves only. The surface wave, which is not present at ionospheric heights, has been omitted.

The height of the ionosphere, H, measured vertically at the midpoint of the wanted path, has been given three values in the accompanying curves, namely, 80 km, 90 km and 100 km.

All the factors governing the amount of power reaching the reflecting region have been incorporated in the parameter X which, in the curves of Figs. 3, 4, 5 and 6, is shown as a function of \( d_s \), the Great Circle distance on the earth's surface between the disturbing transmitter and the midpoint of the path between the receiver and the wanted transmitter.

The equation for the coefficient of cross modulation in terms of power, modulation and the parameter X is:

\[
T_0 = XPM T_0
\]

where \( P \) is in kilowatts, \( M \) is percentage of modulation and \( T_0 \) is taken at its basic value of 0.05. In relating this value for \( T_0 \) to \( T_{400} \), we got a factor of 0.5, as may be seen from Fig. 8. Substituting, the equation becomes:

\[ T_{400} = 0.025 XPM \]

This is the equation of the nomogram of Fig. 1, from which \( T_{400} \) is obtained directly for the basic value of \( T_0 \), when X and PM are known.

The nomogram of Fig. 2 is the same as that of Fig. 1, except that the left-hand scale is for \( d_s \) instead of for X. In constructing this
nomogram, the height of the ionosphere was taken to be 90 km and the vertical radiation pattern to be that for a short aerial. These conditions would commonly apply when the disturbing transmitter is in the long waveband, and in such cases no reference need be made to the curves for \( X \). The coefficient of transferred modulation may be obtained in one operation by the use of this nomogram when \( d \), \( P \), and \( M \) are known, but here again it must be remembered that \( T \) has been taken as 0.05, and that a correction may be necessary for direction of the wanted wave relative to the earth's magnetic field.

The right-hand scale of the nomogram is marked both as a ratio and in decibels below 100% modulation at the wanted transmitter. The value of \( T \) may, therefore, be obtained in either of these terms. Note 2 gives the corrections necessary if the comparison is to be made with modulation depths other than 100%.

4. VALUE OF \( T_0 \)

The decision to use \( T_0 = 0.05 \) in the nomograms was made on the basis of information given by Huxley and Ratcliffe. This states that the experimental results show that the greatest transferred modulation is produced when the wanted wave is in the medium waveband (frequencies in the range 800 - 1000 kc/s were mostly used) and is radiated from a transmitter distant 300 or 400 km, and the disturbing wave is in the long waveband. An average value for \( T_0 \) reduced to the standard conditions is then about 0.05. For other combinations of frequencies, i.e. where both waves are in the long wave or in the medium wave band, or where the wanted wave is in the long wave band and the disturbing wave in the medium wave band, values of \( T_0 \) are found to be of the order of 0.02.

5. ACCURACY OF RESULTS

The authors claim that the coefficient of cross modulation calculated in this way should be correct to a factor of 2 for a wanted wave transmitted at night over distances up to 500 km in the range of radio frequencies 150 kc/s to 1 Mc/s.

6. METHOD OF CALCULATING

The distances \( d \) and \( d_s \), and the magnetic bearing \( \beta \) of the wanted transmitter must first be measured on a map. The sites of the wanted transmitter and the receiver are joined by a straight line. The angle made by the intersection of this line with the magnetic meridian, measured at the midpoint of the line and in a clockwise direction, is angle \( \beta \). The length of half this line is \( d \) in km, and the distance of the disturbing transmitter from the midpoint, also in km, is \( d_s \).
For making the calculation, a value of $H$, the height of the ionosphere, must be decided upon. This value, together with the measured values of $d$ and $\beta$, and the frequency of the wanted wave will give the correction necessary in respect of direction. The procedure is as follows:

From $d$ and $H$ obtain, from Fig. 9, the angle $\eta$, an angle closely related to the angle of elevation of the wanted wave.

Now, the wanted wave in its upgoing path makes an angle with the earth's magnetic field which is different from that which it makes in its downcoming path. These two angles, which may be called $\theta_u$ and $\theta_d$, are derived from Fig. 10, $\theta_u$ being obtained from $\eta$ and $\beta$, and $\theta_d$ from $\eta$ and $(180^\circ - \beta)$. Where $\beta$ is greater than $180^\circ$, the result may be made positive by adding $360^\circ$, and $\theta_d$ is then obtained from $\eta$ and $(540^\circ - \beta)$.

The correction required is taken as the average of the corrections obtained for each of these values of $\varphi$. Thus, from Fig. 11, the decibel corrections are found for $\theta_u$ and $\theta_d$, and the average struck. This method of finding the mean correction is considered sufficiently accurate for the present purpose.

Fig. 10 has been calculated for a magnetic field with a dip angle of $70^\circ$ to the horizontal. This figure can therefore be used only for areas where the angle of dip is $70^\circ$. For other areas $\varphi$ must be calculated. It is given by the expression:

$$\cos \varphi = \sin \eta \sin \delta + \cos \eta \cos \delta \cos \beta$$

where $\delta$ is the angle of dip to the horizontal. This expression is derived in the appendix.

To obtain the basic answer from the nomogram of Fig. 1, the value of the parameter $X$ must be known. Using the same value of $H$, and choosing the aerial applicable to the disturbing transmitter, this parameter is found from one of the Figs. 3, 4, 5 and 6 for the measured distance $d$.

This value of $X$ applies to the left-hand scale of the nomogram of Fig. 1. For the centre scale of the nomogram it is necessary to use the product of the transmitter power in kilowatts and modulation depth as a percentage. A straight line joining the points on the left-hand and centre scales on the nomogram and extended to the right-hand scale gives the coefficient of transferred modulation for a modulation frequency of 400 c/s when $T = 0.05$, in terms both of equivalent depth of modulation and in decibels below 100% modulation from the wanted transmitter.
In cases where $h \ll \lambda$ and the ionosphere height may be taken as 90 km, the nomogram of Fig. 2 will give the coefficient for the same values of modulation frequency and $T_o$ directly from a knowledge of $d_s$, $P$, and $M$.

For other values of modulation frequency and $T_o$, the corrections are obtained from Figs. 8 and 11 respectively, as already described.

7. EXAMPLES

Shaw has given a number of measured and calculated results of his experiments. Two cases are worked out here, not only to illustrate the calculating procedure described above, but also to demonstrate the degree of accuracy that may be expected of this method. In both cases Lisnagarvey on 1050 kc/s is taken as the wanted transmitter, the disturbing transmitter in the first case being Ottringham on 167 kc/s, and in the second, Droitwich on 200 kc/s. The receiver was situated at Cambridge.

From a map we get, with Ottringham as the disturbing transmitter:

Distance between Lisnagarvey and Cambridge = 304 miles = 490 km

$\beta = 132^\circ$

Distance of Ottringham from the midpoint = 118 miles = 190 km = $d_s$

Power at Ottringham = 120 kW, modulated (during Shaw's measurements) at 80%. Therefore, $P_M = 9.6 \times 10^{-5}$.

$H$ will be taken as 90 km

From Fig. 9 $\eta = 21^\circ$

" " 10 $\vartheta_u = 56^\circ$

" " 10 $\vartheta_d = 83^\circ$

" " 11 correction for $\vartheta_u = -2.3$ db

" " 11 " " $\vartheta_d = +4.3$ db

average correction = 1.0 db

From Fig. 3 $X = 2.8 \times 10^{-5}$

" " $1T_{400} = -43.5$ db
Alternatively, $T_{400}$ may have been obtained direct from Fig. 2, since $H$ was taken as 90 km and $h<<\lambda$.

To convert $T_{400}$ to $T_0$ the correction is +6 dB (Fig. 8).

$$T_0 = -43.5 \text{ db} + 1.0 \text{ db} + 6 \text{ db} = -36.3 \text{ db} = 1.5\%$$

Shaw's calculated figure is 1.6% and his measured figure 1%.

For Droitwich as the interfering station, $d_s = 137$ km.

Power at Droitwich = 160 kW. For 80% modulation $PM = 1.44 \times 10^4$.

Taking $H$ again at 90 km.

From Fig. 3, $X = 4.45 \times 10^{-5}$

"""1 $T_{400} = -36.5 \text{ db}$

This result for $T_{400}$ could similarly have been obtained in one operation from Fig. 2.

The average correction for direction has already been found to be +1.2 db. Adding also the correction required for the modulation frequency:

$$T_0 = -36.5 \text{ db} + 1.2 \text{ db} + 6 \text{ db} = -29.3 \text{ db} = 3.5\%$$

Shaw's figures are 4.3% calculated and 4% measured.

It will be seen that the accuracy of the answers is well within the tolerance stated in paragraph 5.

If broadcast interference is being examined, the correction of +6 db should not be applied as we are not concerned with the value of $T_0$. Broadcast interference, the subjective effect of which may be assessed by the value of $T_{400}$, would have values one-half of those arrived at above, under the same conditions.

REFERENCES


(2) "Some Further Investigations of Ionospheric Cross Modulation" by Dr. I.J. Shaw, Proceedings of the Physical Society, Vol.64, Part I, Section B, January, 1951.
In Fig. 12:

ABCD is the horizontal plane.

T is the position of the transmitter.

R is the position of the receiver.

I is the point of reflection in the ionosphere at a height \( H \) above the plane.

IM is in the plane of the magnetic meridian containing I, and represents the direction of the magnetic field. Hence, \( \delta \) is the angle of dip to the horizontal.

\( \eta \) is the upgoing ray and IR the downcoming ray.

\( \eta \) is the angle made by the wave with the horizontal plane.

TOR is the ground projection of the wave, and the angles it makes with the magnetic meridian OM are \( \beta \) and \( (180^\circ - \beta) \).

\( \theta \) is the angle between the upgoing wave and the direction of the field.

\( \varphi \) is the angle between the downcoming wave and the direction of the field.

To find \( \theta \) in terms of \( \eta \), \( \delta \) and \( \beta \):

\[
TM^2 = TO^2 + OM^2 - 2TO\cdot OM \cos \beta \\
= H^2 \cot^2 \eta + H^2 \cot^2 \delta - 2H^2 \cot \eta \cot \delta \cos \beta \\
\]

(1)

Also, \( TM^2 = TI^2 + IM^2 - 2TI\cdot IM \cos \varphi \)

Now, \( TI^2 = TO^2 + I^2 = H^2 \cot^2 \eta + H^2 \)

and \( TI = H \csc \eta \)

Similarly, \( IM^2 = OM^2 + I^2 = H^2 \cot^2 \delta + H^2 \)

and \( IM = H \csc \delta \)
APPENDIX I (continued)

Substituting, we get:

\[ TM^2 = H^2 \cot^2 \eta + H^2 + H^2 \cot^2 \delta + H^2 - 2H^2 \cosec \eta \cosec \delta \cos \vartheta_u \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2) \]

Equating the right hand sides of equations (1) and (2):

\[-2H^2 \cot \eta \cot \delta \cos \beta = 2H^2 - 2H^2 \cosec \eta \cosec \delta \cos \vartheta_u \cosec \eta \cosec \delta \cos \vartheta_u = 1 + \cot \eta \cot \delta \cos \beta \]

Multiplying by \( \sin \eta \sin \delta \):

\[ \cos \vartheta_u = \sin \eta \sin \delta + \cos \eta \cos \delta \cos \beta \]

Similarly,

\[ \cos \vartheta_d = \sin \eta \sin \delta + \cos \eta \cos \delta \cos (180^\circ - \beta) \]
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NOTES:

(1) FOR AN INTERFERING MODULATION FREQUENCY OTHER THAN 400 c/s THE CORRECTION TO BE APPLIED CAN BE OBTAINED FROM FIG. 8.

(2) FOR COMPARISON WITH OTHER MODULATION DEPTHS AT THE WANTED TRANSMITTER
   CORRECT BY +2 db FOR 80% MODULATION
   CORRECT BY +4.5 db FOR 60% MODULATION
   CORRECT BY +8 db FOR 40% MODULATION

FIG. 1
NOMOGRAM FOR CALCULATING PERCENTAGE OF CROSS MODULATION FROM VALUES OF X.
NOTES:

(1) FOR AN INTERFERING MODULATION FREQUENCY OTHER THAN 400c/s THE CORRECTION TO BE APPLIED CAN BE OBTAINED FROM FIG.B.

(2) FOR COMPARISON WITH OTHER MODULATION DEPTHS AT THE WANTED TRANSMITTER
CORRECT BY +2db FOR 80%/ MODULATION
CORRECT BY +4·5db FOR 60%/ MODULATION
CORRECT BY +8db FOR 40% MODULATION

FIG. 2
NOMOGRAM FOR CALCULATING PERCENTAGE OF CROSS MODULATION FROM VALUES OF $d_s$. 

$\gamma = \lambda$ & $H = 90\text{ km}$
FIG 3

$X$ AS A FUNCTION OF $d_s$ FOR AN AERIAL HEIGHT $\ll \lambda$

$H = 80\text{km}$

$H = 100\text{km}$

$H$ = HEIGHT OF IONOSPHERE

$10^3 X$ 7

$X$

$d_s$ (km)

0 50 100 150 200 250 300 350 400 450 500
FIG. 4

$X$ AS A FUNCTION OF $d_s$ FOR AN AERIAL HEIGHT OF $0.5 \lambda$. 
FIG. 5

\( x \) AS A FUNCTION OF \( d_s \) FOR AN AERIAL HEIGHT OF 0.55 \( \lambda \)
FIG. 6

$X$ AS A FUNCTION OF $d_s$ FOR AN AERIAL HEIGHT OF 0.6 $\lambda$.
FIG. 7

VERTICAL RADIATION CHARACTERISTICS FOR AERIALS OF DIFFERENT HEIGHTS h FOR THE SAME POWER TAKING THE RADIATION FROM A SHORT AERIAL OVER A PERFECT EARTH AT AN ELEVATION OF 0° AS UNITY.

(DIRECT & REFLECTED WAVE ONLY: SURFACE WAVE OMITTED.)

EARTH CONSTANTS
k = 5 x 10^8
\sigma = 10^8 c m
FREQUENCY 1 kc/s

RATIO OF INTENSITY

ANGLE OF ELEVATION
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$T_f$ in dB with reference to $T_{400}$

Variation of $T_f$ with frequency.

**FIG. 8**

- 400 800 1200 1600 2000 2400 2800 3200
- 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60
FIG. 9

ANGLE BETWEEN THE DIRECTION OF THE WAVE AT THE POINT OF REFLECTION & THE TANGENTIAL PLANE

\[ \theta = \begin{cases} \text{H=90 km} \\ \text{H=80 km} \\ \text{H=100 km} \end{cases} \]

\[ d(km) \]

\[ 10 \quad 15 \quad 20 \quad 30 \quad 40 \quad 50 \quad 60 \quad 70 \quad 80 \quad 90 \quad 100 \quad 150 \quad 200 \quad 300 \quad 400 \quad 500 \]
$\theta$ = ANGLE BETWEEN THE WAVE AND EARTH'S MAGNETIC FIELD FOR A DIP ANGLE OF 70°
$\beta$ = MAGNETIC BEARING OF DIRECTION OF WANTED WAVE.

FIG. 10
ANGLE BETWEEN THE DIRECTION OF THE WAVE & THE EARTH'S MAGNETIC FIELD.
CORRECTION FACTOR FOR $T_0$ (db)

FIG. II

CORRECTION FOR THE EFFECT OF THE EARTH'S MAGNETIC FIELD.
FIG. 12
SOLUTION FOR $\theta$